4.1: Angles and Their Measures

1. Approximate each angle in terms of decimal degrees to the nearest ten-thousandth.
   a. $\theta = 56\degree 34\arcmin 53\arcsec$
   b. $\theta = -35\degree 48\arcmin$

2. Approximate each angle in terms of degrees, minutes, and seconds
   a. $\theta = 46.327\degree$
   b. $\theta = -1.6556\degree$

3. Convert the following angles to degree measure (decimal degrees)
   a. $\frac{17\pi}{5}$
   b. $-1.873$

4. Convert the following angles to radian measure
   a. $108\degree$
   b. $-244.48\degree$

5. Solve each of the following word problems. Show all work.
   a. A clock in Copenhagen, Denmark makes one complete revolution every 25,753 years. The clock is 20 ft in diameter. How far does a point on the outside of the clock move every decade?

   b. A building in Milwaukee has a clock with a minute hand 20 feet long. What is the linear speed of the tip of the minute hand in in./sec?

   c. The Tour de France is a bicycle race held in France. The average speed of the cyclists in one race was 15 mph. The diameter of a wheel is approximately 26 in. What is the average angular speed in radians/hr?

   d. A clock’s pendulum is 1 m long. From one side to the other it moves through an angle of 0.2 radians. Through what distance does the tip of the pendulum move?
1. Find the values of all six trigonometric functions of the angle theta.
   a. 
   b. 
   
2. Assume that theta is an acute angle in a right triangle. Find the five other trigonometric values of
   the angle theta.
   a. \[ \tan \theta = \sqrt{2} \]
   b. \[ \csc \theta = \frac{\sqrt{5}}{4} \]

7. Solve the following triangles. Standard position and lettering is used.
   a. \[ a = 11.7, b = 3.1 \]
   b. \[ A = 37^\circ 43', a = 1250 \]
8. Solve each of the following. Show all work.
   a. A surveyor wishes to measure the distance from a point on one side of a river to a point on
      the opposite side. She stakes out two points 100 ft apart on the same side of the river. She
      then uses a transit (a device for measuring angles) and determines that the angle between
      the line through the two stakes and a line through the second stake and the point across the
      river is 43°39'. What is the distance between the two opposite points across the river from
      each other?

   b. A housing developer wishes to create a triangular lot at the corner of a street. The sides
      bordering on the two sides of the corner are 135 ft and 210 ft, respectively. Determine the
      angles formed by the vertices of the lot.

   c. An airplane takes off from a runway at an angle of elevation of 4°20'. Two minutes later the
      plane is at an altitude of 2000 ft. How far is the airplane from the point of takeoff? How fast
      is the plane flying in miles per hour?

   d. Suppose a laser beam is directed toward the visible center of the moon and it misses its
      assigned target by 30 seconds. Approximate how far it is, in miles, from its assigned target.
      Assume that the distance from the earth to the moon is 234,000 miles and that, because of
      the extreme distance, the surface of the moon is flat.
1. Use transformations to describe how the graph of the function is related to a basic trigonometric graph.
   
   a. \( y = 2 + 2 \cos(x - \pi) \)  
   b. \( y = -\sin\left(\frac{x - \pi}{4}\right) - 4 \)

2. Graph \( y = -2\sin(2x) \) over the interval \( -\pi \leq x \leq 2\pi \).
   
   a. State the period ___________________  
   b. State the range _____________________

3. Graph \( y = \sin^2 x \) over the interval \( -2\pi \leq x \leq 2\pi \)

4. Determine the equation of the sinusoid shown below.

5. Graph the tangent, cotangent, secant, and cosecant functions over the interval \([-\pi, 2\pi]\)
6. Graph the functions \( f(x) = -2.5 \csc(x) \) and \( f(x) = -\tan(4x) \)

7. For each of the following equations, determine i-iii.

   A.) \( y = 2 + \sin \frac{x}{4} \)  
   B.) \( y = 2x + \cos \left( \frac{x}{4} \right) - 5 \)

   i.) Is it periodic?
   ii.) Is it a sinusoid?
   iii.) Find the equations of two linear functions \( y_1 \) and \( y_2 \) such that \( y_1 \leq y \leq y_2 \)

Solve for \( x \) in the given interval:

8. \( \sec(x) = 1; \quad -\pi \leq x \leq \frac{\pi}{2} \)

9. \( \csc(x) = 2; \quad -\frac{3\pi}{2} \leq x \leq -\pi \)

10. \( 4\sqrt{3} \csc(x) = -8; \quad \pi \leq x \leq 2\pi \)

11. \( 27 \cot(x) = 9\sqrt{3}; \quad -\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2} \)

12. Give the formula of a function that oscillates between the functions \( y = 3x - 7 \) and \( y = 3x - 2 \).

**CALCULATORS ALLOWED**

1. A person is seated on a Ferris wheel of radius 60 feet. The Ferris wheel makes one rotation every 30 seconds. The center of the wheel is 65 feet above the ground. Assume uniform speed from the beginning to the end of the ride. A passenger is half way to his maximum height when the ride begins.

   a. Express the person’s height above the ground as a function of time.

   b. Graph 2 minutes of the ride.
2. Without graphing the following functions, determine which will be a sinusoid. If one is, find values for \( a, b, \) and \( h \) (to the nearest hundredth) so that: \( g(x) = a \sin[b(x - h)] \)

A.) \( f(x) = 3 \sin(3x) - 2 \cos(3x) \) \hspace{1cm} B.) \( f(x) = -\sin\left(\frac{\pi}{3} x\right) + 1.5 \cos\left(\frac{\pi}{3} x\right) \)

3. Give the equation for the function with the following characteristics:
   
   a. \( \cot x \): period of \( 2\pi \), vertical shift up 3
   
   b. \( \sec x \): period of 6, phase shift 2 to the right, vertical stretch of 3

4. Determine the value(s) of \( k \) such that \( \sec(kx) \) has vertical asymptotes at every odd integer.

5. Solve for \( x \) in the given interval, support your answer (with a graph or algebraically)

   A.) \( \sec x = -2; \quad \frac{5\pi}{2} \leq x \leq \frac{7\pi}{2} \)

   B.) \( \tan x = -3; \quad 2\pi \leq x \leq 3\pi \)

   C.) \( \csc x = -1.8; \quad -\frac{5\pi}{2} \leq x \leq -2\pi \)
1. Sketch the graph of each function.
   a. \( f(x) = x + \cos x \)
   b. \( f(x) = \sqrt[3]{x} \sin x \)
   c. \( f(x) = 2^{-x} \cos x \)
   d. \( f(x) = |\csc 2x| \)

2. Verify that the function \( f(x) = \cos^2 x = (\cos x)^2 \) is periodic.

3. Decide if each of the following is a sinusoid. Then, find the period.
   a. \( f(x) = 3 \sin 5x - 4 \cos 3x \)
   b. \( f(x) = 7 \cos 4x - 8 \sin 4x \)

Calculator Part
4. Sketch a graph of each of the following functions. Decide if they are periodic, and if so, find the period.
   a. \( f(x) = \sin x \cos 2x \)
   b. \( f(x) = e^{\sin x} \)

5. Approximate the following sum as a single sinusoid (round to the nearest hundredth).
   \( f(x) = 2 \cos 3x - 3 \sin 3x \)
6. Solve the following problems.
   a. The profits of a company fluctuate during a 12-month period according to the formula
      \( P(t) = 50,000 (\sin t + \cos t) \) (\( t \) is measured in months).
      i. Sketch a graph of \( t \) over the interval [0,12].
      ii. Explain the meaning of the negative values of the function.

   b. The motion of a spring is given by \( D(t) = 7e^{-0.5t} \cos(2\pi t) \), where \( D(t) \) is measured in inches and time \( t \) is in seconds.
      a. Sketch a graph of \( D(t) \) on the interval [0,6].
      b. What is the maximum value of the function?
      c. What is the minimum value of the function?

   c. A spotlight 180 feet from a prison wall rotates in a circle once every 4 seconds. At \( t=0 \) seconds, the spotlight is pointing directly at point \( Q \). The spotlight is pointing at a distance \( d \) from \( Q \) after \( t \) seconds.
      a. Calculate the frequency of the rotation.
      b. Write a function for the \( d \) as a function of theta.
      c. Use the fact that \( \theta = 2\pi f t \) where \( f \) is the frequency (can you explain why?) in order to write a function for \( d \) as a function of \( t \).
      d. Sketch a graph of the function below
      e. Explain the meaning of the undefined values of \( d \).
      f. Explain the meaning of the negative values of \( d \).
A. Sketch a graph of each of the following functions, and write their domain and range:

1. \( f(x) = \tan^{-1}(x) \)
2. \( f(x) = \cos^{-1}(x) \)

3. \( f(x) = 4\sin^{-1}\left(\frac{x}{2}\right) + 2 \)
4. \( f(x) = \frac{1}{2}\cos 3x \)

B. Calculate the exact value for each of the following:

1. \( \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \)
2. \( \cos^{-1}\left(-\frac{1}{2}\right) \)
3. \( \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) \)

4. \( \tan^{-1}\left[\tan\left(\frac{2\pi}{3}\right)\right] \)
5. \( \tan^{-1}\left[\cos(\pi)\right] \)
6. \( \cos\left[\tan^{-1}(\sqrt{3})\right] \)

C. Find an expression for each of the following:

1. \( \sin\left(\cos^{-1} 2x\right) \)
2. \( \csc\left(\tan^{-1} 3x\right) \)
D. Solve each of the following problems.

1. An airplane takes off from a runway at an angle of elevation of $42^\circ$. Two minutes later the plane is at an altitude of 2000 ft. How far is the airplane from the point of takeoff?

2. A weight is attached to a spring and set in motion by compressing the spring from its rest position and releasing it. The maximum displacement of the spring is 8 cm, and it takes 4 seconds to complete one cycle.
   a. Write an equation for the height of the weight as a function of time.
   b. What is the distance below the starting point of the spring after 5 seconds?
   c. How far has the weight traveled during the first 5 seconds?
   d. After how many seconds will the spring be 9 inches below the starting point?

3. A person’s blood pressure changes in a rhythmic fashion, with a periodicity set by the besting of the heart. Suppose a person’s blood pressure at time $t$ minutes is given by the formula $P(t) = 101 + 24 \cos(160\pi t)$
   a. Sketch the graph of $P(t)$ below (show at least two full cycles of the periodic motion, as well as an accurate scale)
   b. Determine the rate at which the person’s heart is beating (beats per minute)
   c. What are the maximum and minimum blood pressure readings?
   d. After how many minutes will the blood pressure be about 110?