Name

LESSON MASTER 1-1

Questions on SPUR Objectives
See pages 73–79 for objectives.

Uses Objective D

In 1 and 2, consider the following scenario: A soft-drink company tests its new strawberry lemonade by releasing it to a mid-size city. After a 2-month trial period, the acceptance of the lemonade is evaluated.

1. a. Identify the population. National or world population

   b. Identify the sample. Population of mid-size city

   c. Identify the variable. Strawberry lemonade

2. Give one reason why the company might survey a sample rather than the entire population.
Sample: It would be risky to distribute a new product to such a large group.

Uses Objective E

In 3–5, use this table of percents.

Improper Driving as Factor in Accidents, 1993

Kind of improper driving	Fatal accidents			Injury accidents			All accidents		
	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural
Improper driving	**57.7**	**54.7**	**59.4**	**72.7**	**74.3**	**69.6**	**68.6**	**69.8**	**66.1**
Speed too fast or unsafe	16.5	14.4	17.7	13.5	11.8	17.6	12.2	11.1	15.4
Right of way	12.7	17.0	10.1	25.0	28.8	15.5	20.6	23.2	13.7
Failed to yield	7.8	9.4	6.8	17.3	19.3	12.3	15.1	16.6	11.3
Passed stop sign	2.7	2.7	2.7	2.7	3.0	1.9	2.0	2.1	1.4
Disregarded signal	2.2	4.9	0.6	5.0	6.5	1.3	3.5	4.5	1.0
Drove left of center	7.6	3.2	10.1	2.1	1.3	4.0	1.8	1.1	3.4
Improper overtaking	1.2	0.6	1.5	1.0	0.8	1.4	1.3	1.1	1.7
Made improper turn	2.9	2.7	3.0	3.4	3.3	3.7	4.5	4.6	4.2
Followed too closely	0.5	0.4	0.6	6.2	7.2	3.7	5.5	6.2	3.6
Other improper driving	16.3	16.4	16.4	21.5	21.1	23.7	22.7	22.5	24.1
No improper driving stated	**42.3**	**45.3**	**40.6**	**27.3**	**25.7**	**30.4**	**31.4**	**30.2**	**33.9**
Total	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

Source: *1996 Information Please Almanac*

3. Which numbers in the column for rural accidents resulting in injury total 15.5? 12.3, 1.9, 1.3

4. What percent of all accidents involved improper turns? 4.5%

Name

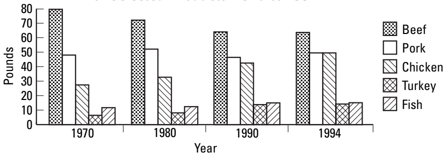
▶ **LESSON MASTER 1-1** *page 2*

5. In 1993, there were approximately 42,000 deaths due to motor-vehicle accidents. Estimate the total number of deaths caused by unsafe speeds or right-of-way accidents. ≈ 12,000 deaths

Representations Objective G

In 6 and 7, use the graph below.

Per-Capita Food Consumption of Selected Products: 1970 to 1994



Source: *Statistical Abstract of the United States*, 1996

6. Which food types have shown a consistent increase in consumption? Why do you think their consumption has increased?
Chicken and turkey; people have become more health-conscious and prefer leaner meats.

7. In 1994, what was the approximate total per-capita food consumption in pounds for these selected products? ≈ 190 pounds

Representations Objective J

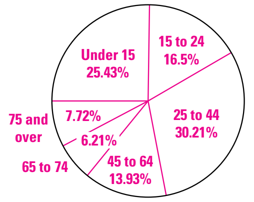
8. Use the table below. Draw a circle graph showing the distribution of age groups visiting emergency rooms in 1994.

Hospital Emergency-Room Visits by Age Group in 1994 (in thousands)

Under 15 years old	23,751
15 to 24 years old	15,411
25 to 44 years old	28,219
45 to 64 years old	13,011
65 to 74 years old	5,797
75 years old and over	7,214

Source: *Statistical Abstract of the United States*, 1996

Emergency-Room Visits by Age Groups, 1994



Name

LESSON MASTER 1-2

Questions on SPUR Objectives
See pages 73–79 for objectives.

In 1–6, use the stemplot below, which gives the results of Ginnie Davis's survey of a group of college students majoring in music. Ginnie asked the number of music CDs each person owned.

Underclassmen		Upperclassmen
4	0	0
7 4 2	1	0 5
3 0	2	6
8 6 5 4 1 1	3	2 2 4 8
9 7 7 2 1 0 0	4	3 3 5 7 8
7 5 0	5	0 2 4 4 5 5 7
	6	3
	7	2 8
2	8	5

Skills Objective A

1. For each data set, identify each statistic.

   a. the minimum Undercl. – 4 Uppercl. – 0

   b. the maximum Undercl. – 82 Uppercl. – 85

   c. the range Undercl. – 78 Uppercl. – 85

Uses Objective F

2. How many more underclassmen should Ginnie survey to have equal numbers of participants in each group? 1 more

3. What does the first row 4|0|0, represent?
4: units; 1st zero: tens; 2nd zero: units

4. How many upperclassmen in the survey have fewer than 30 CDs? 4 uppercl.

5. Describe any similarities and differences between the two groups. Sample: Most people in both groups have 30–60 CDs. In general, upperclassmen have more CDs.

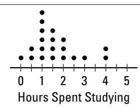
6. Which values, if any, appear to be outliers in each population? Undercl. – 82

Name

▶ **LESSON MASTER 1-2** *page 2*

Representations Objective I

In 7 and 8, use the dotplot at the right, which shows the distribution of total time spent studying one weekend by students in Mr. Bell's morning class.



7. What is the frequency of students who study for 2 hours? 3 students

8. Which time has the greatest frequency? 1 hour

Representations Objective J

9. The following sets of data show the average number of points scored by players on the boys' and girls' basketball teams.

Boys: 4.7 0.3 11.6 0.3 3.6 6.2 1.3 1.1 3.1 7.6 4.0 20.5 0.8 2.5 3.6

Girls: 7.0 2.6 9.8 6.3 5.7 0.8 6.5 8.5 12.4 7.2 5.3 7.9 9.1 7.6 6.9

a. At the right, make a back-to-back stemplot of these data.

Boys		Girls
8 3 3	0	8
3 1	1	
5	2	6
6 6 1	3	
7 0	4	
	5	3 7
2	6	3 5 9
6	7	0 2 6 9
	8	5
	9	1 8
	10	
6	11	
	12	4
	13	
	14	
	15	
	16	
	17	
	18	
	19	
5	20	

b. Which scores, if any, appear to be outliers in each data set?
Boys: 20.5; girls: none

c. Identify the range for both sets of data.
Boys: 20.2; girls: 11.6

10. Write several sentences comparing and contrasting the scores of the two teams. Include how the characteristics found in Exercises 9b and 9c describe each basketball team.
Sample: Scoring on the boys' team is done primarily by one or two players. Scoring on the girls' team is more evenly distributed, with most players scoring 5 to 10 points.

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Name

LESSON MASTER 1-3

Questions on SPUR Objectives
See pages 73–79 for objectives.

Skills Objective A

1. Find two different measures of center for the data given in the stemplot at the right.

4	3 7 9
5	0 1 4 4 7
6	1 5
7	4 6 8 9
8	2

Mean: 61.3; median: 57;
mode: 54

2. Stuart Dent has scored 75, 85, 76, 92, and 87 on his first five tests.

a. What score does Stu need on the next test in order to raise his mean score to 85? **95**

b. What score does Stu need on the next test in order to have a median score of 85? **85**

3. Practicing for an upcoming bowling tournament, the Turkeys kept track of their individual averages. For 9 games, John had a 168 average; for 12 games, Dennis had a 175 average; for 8 games, Chris had a 153 average; and for 10 games, Mark had a 161 average. What is the average score of all the Turkeys' games? **≈ 165.3**

Properties Objective B

In 4–6, x_i equals the normal precipitation in inches in the ith month of the calendar year in Memphis, TN.

$x_1 = 3.7, x_2 = 4.4, x_3 = 5.4, x_4 = 5.5, x_5 = 5.0, x_6 = 3.6,$
$x_7 = 3.8, x_8 = 3.4, x_9 = 3.5, x_{10} = 3.0, x_{11} = 5.1, x_{12} = 5.7$

Source: *The World Almanac and Book of Facts*, 1996

4. a. Write an expression using Σ-notation to represent the yearly precipitation in Memphis, TN. $\sum_{i=1}^{12} x_i$

b. Evaluate your expression in part **a**. **52.1 inches**

5. Consider the expression $\frac{1}{3}\sum_{i=7}^{9} x_i$.

a. What does this expression represent?
Average precipitation during July, Aug., Sept.

b. Evaluate this expression. **$3.5\overline{6}$ inches**

Name

6. a. Find $\bar{x}$. **≈ 4.3**

b. Find $\sum_{i=1}^{4}(x_i - \bar{x})^2$. **≈ 2.9**

Properties Objective C

7. Find a counterexample to the following statement: *For any set of three numbers, the mean is equal to the median.*
Sample: 1, 10, 11

8. *True or false.* For any set of consecutive integers, the mean is equal to the median. Give examples to illustrate your answer.
True; sample examples: {5, 6, 7} – mean, 6; median, 6; {5, 6, 7, 8, 9, 10} – mean, 7.5; median, 7.5

Uses Objective F

In 9 and 10, use the data below, which give the weights in pounds of the crews participating in a rowing race between Oxford and Cambridge.

Cambridge: 188.5, 183, 194.5, 185, 214, 203.5, 186, 178.5, 109
Oxford: 186, 184.5, 204, 184.5, 195.5, 202.5, 174, 183, 109.5

Source: *The Independent*, March 31, 1992

9. On the average, which team has the lighter crew members? Use measures of center to justify your answer.
The Oxford team is lighter with a mean of 180.4 and a median of 184.5, while the Cambridge mean is 182.4 and median is 186.

10. Each crew has an outlier when it comes to weight. What is the effect of this outlier on the measures of center of the data sets? Tell the purpose of this person on the crew team, if you know.
The outlier affects the mean more than the median. This person, the "coxswain," does not row, but keeps the rowers' rhythm steady.

Name

LESSON MASTER 1-4

Questions on SPUR Objectives
See pages 73–79 for objectives.

Skills Objective A

1. The stemplot at the right displays the team batting averages of all Major League Baseball teams for the 1995 season. The stem represents the first two decimal places of the averages. Identify each of the following.

Source: *1996 Information Please Almanac*

24	7 7 9
25	2 8 9 9
26	0 1 1 1 3 3 4 5 5 6
27	0 1 5 5 6 6 9 9
28	0 1
29	9

a. the first quartile **259.5**

b. the third quartile **275.5**

c. the median **264.5**

d. the interquartile range **16**

e. the number closest to the 60th percentile **266**

Uses Objective F

2. The stemplot at the right gives the prices, rounded to the nearest dollar, of the 30 stocks in the Dow Jones Industrials on January 2, 1996, and December 31, 1996.

January 2		December 31
	0	9
7 4 3	1	
	2	0 2
9 8 8	3	
9 7 6 5 2 0	4	1 1 4 5 6
4 3 2 1	5	1 3 6 7
9 8 4 1 0	6	4 5 6 7
9 4 2	7	0 5
3 1 1 0	8	0 0 3
2 1	9	4 8 8 8 9
	10	6 8
	11	3
	12	
	13	
	14	
	15	

a. Find the five-number summaries for each date.
Jan. 2: min., 13;
Q_1, 42; med., 53.5;
Q_3, 74; max., 92;
Dec. 31: min., 9;
Q_1, 46; med., 66.5;
Q_3, 98; max.,151

b. Find any outliers using the 1.5 × IQR criterion.
None

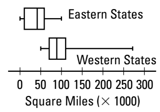
Name

c. The only two stocks which posted a decrease in price for 1996 were McDonald's, which started at 46 and dropped to 45, and Bethlehem Steel. If Bethlehem Steel stock opened the year at 14, what was its change for the year? **-35.7%**

d. The Dow Jones Industrials is one of many indices used to gauge the entire stock market. Based on the above data, do you think the stock market increased or decreased for the 1996 year? Justify your answer.
The mean and the median both increased; in general, the shape has shifted upward. So, it seems the market increased during 1996.

Representations Objective H

3. Refer to the box plots at the right, which represent the areas, in thousands of square miles, of the 48 contiguous states east and west of the Mississippi River.



Source: *1996 Information Please Almanac*

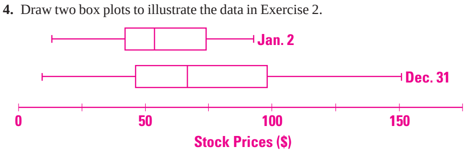
a. Which is greater, the maximum eastern-states area or the upper quartile of the western states? **Upper Q, west.**

b. There are 26 states east of the Mississippi River. How many states have areas which are at or below the lower quartile? **7**

c. Use your knowledge of geography to answer this question: If Alaska and Hawaii were included with the western-states data, which values of the five-number summary would change in the western-states box plot? **Min., 1st Q, 3rd Q, max.**

Representations Objective J

4. Draw two box plots to illustrate the data in Exercise 2.



Home

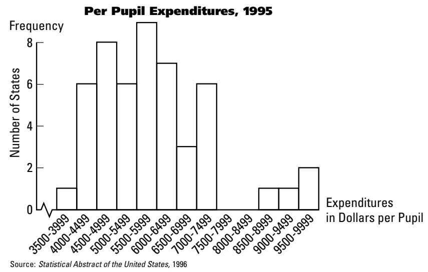
Name

LESSON MASTER 1-5

Questions on SPUR Objectives
See pages 73–79 for objectives.

Representations Objective I

1. Below is a frequency histogram displaying average expenditures per pupil for the 50 states in 1995.



Source: *Statistical Abstract of the United States*, 1996

a. How many states spend between \$6000 and \$6999 per pupil? **10 states**

b. In what interval is the median? Justify your answer.
\$5500–\$5999; the median is the mean of the 25th and 26th states, which is in the interval containing the 22nd–30th states.

c. How could the frequency histogram be changed to become a relative frequency histogram?
Each frequency could be divided by 50 to find a percent.

d. What percent of states spend more than \$8000 per pupil? **8%**

Name

2. The table at the right gives the relative frequency of drivers by age group in 1995.

a. What percent of the driving population are 45 years of age or older?
42%

b. If there were about 16,900,000 drivers between the ages of 45 and 49, how many drivers are 24 years old or younger?
≈ 24,600,000 drivers

Age	Relative Frequency
15–19	5.2
20–24	8.8
25–29	10.2
30–34	11.5
35–39	11.7
40–44	10.7
45–49	9.6
50–54	7.3
55–59	5.8
60–64	5.1
65–69	4.8
70–74	4.1
75–80	2.9
80–84	1.6
85 and over	0.8

Source: U.S. Dept. of Transportation

Representations Objective J

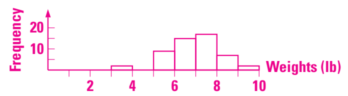
3. Below are the birth weights in pounds of a group of babies.

7.10	7.14	8.00	6.10	7.00	6.82	7.12	8.10	8.23
6.20	5.66	6.66	5.90	7.50	6.42	5.81	5.43	6.26
7.40	7.00	5.20	6.80	7.33	5.91	6.05	6.22	8.80
3.25	6.20	3.66	7.20	7.91	6.37	8.72	9.15	7.33
6.98	7.25	8.20	7.10	8.02	7.25	7.75	5.67	
9.22	7.78	5.36	6.50	5.55	6.88	7.55	6.70	

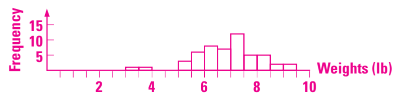
a. Determine each statistic from this data set.

i. minimum **3.25 lb** ii. maximum **9.22 lb** iii. range **5.97 lb**

b. Use intervals of size 1 to draw a histogram representing the data.



c. Use intervals of size 0.5 to draw a histogram representing the data.



d. Babies born weighing less than 5.5 pounds are at a higher risk of having developmental problems. What percent of the babies in the data set are at risk? **≈ 9.6%**

Name

LESSON MASTER 1-6

Questions on SPUR Objectives
See pages 73–79 for objectives.

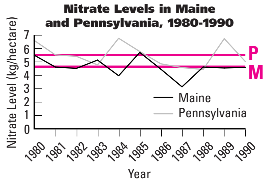
Representations Objective G

1. One measurement of acid rain is the level of nitrate deposits. The following data for Maine and Pennsylvania from 1980 to 1990 are displayed in the line graph below.

Nitrate Levels

Year	Maine	Pennsylvania
1980	5.51	6.61
1981	4.62	5.53
1982	4.52	5.42
1983	5.14	4.78
1984	3.97	6.77
1985	5.71	5.78
1986	4.46	4.86
1987	3.14	4.58
1988	4.62	4.44
1989	4.55	6.74
1990	4.60	5.03

Source: U.S. Department of Agriculture



a. Calculate the average rate of change of nitrate levels between 1984 and 1988 in each state.

i. Pennsylvania **-0.5825 kg/h** ii. Maine **0.1625 kg/h**

b. Which answer to part a better represents what happened between 1984 and 1988? Explain why.
Pennsylvania; between 1984 and 1988, Pennsylvania showed consistent decrease, but Maine nitrate levels were erratic.

c. Give two reasons why line graphs are good displays for this set of data.
Sample: It is easy to see the changes from year to year; it is easy to compare the two states' levels.

d. Calculate the mean nitrate levels for Maine and Pennsylvania from 1980 to 1990. **M: 4.62; P: 5.50**

e. Draw horizontal lines on the graph to show the mean for each state. Then explain why this is helpful in reading the graph.
It illustrates how each year compares to the mean.

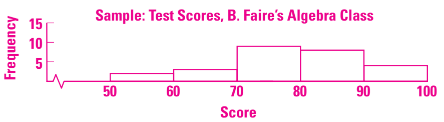
Name

Representations Objective J

2. The following list displays the scores of the latest test in Ben Faire's algebra class.

83	76	82	62	57	82	83	72	76	74	90	84	76
88	91	54	79	75	72	67	93	87	80	68	95	72

Draw a graph that you feel best displays the range of scores in B. Faire's algebra class.

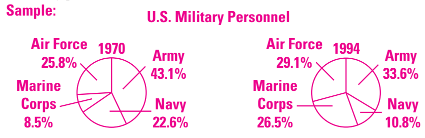


3. Refer to the table below which shows the number of active military personnel from 1960 to 1994 in each branch of the United States armed forces.

Year	Army	Navy	Marine Corps	Air Force
1960	873,078	616,987	170,621	814,752
1965	969,066	669,985	190,213	824,662
1970	1,322,548	691,126	259,737	791,349
1975	784,333	535,085	195,951	612,751
1980	777,036	527,153	188,469	557,969
1985	780,787	570,705	198,025	601,515
1990	732,403	579,417	196,652	535,233
1994	541,343	468,662	174,158	426,327

Source: *1996 Information Please Almanac*

a. Draw a graph that you feel best compares the distribution of military personnel over the branches in 1970 and 1994.



b. Explain why you chose the type of graph you used.
Sample: Circle graphs show the relationships among the categories; you can compare the two years by comparing the sizes of the sectors.

Name ______________________

LESSON MASTER 1-7

Questions on SPUR Objectives
See pages 73–79 for objectives.

Skills Objective A

1. Find the variance and standard deviation of each data set.
   a. 5, 9, 10, 3, 2, 4, 5, 7, 2, 5 — Var.: ≈7.51; *s*: ≈2.74
   b. -6, 1, -2, 0, -1, 8, 3, 1 — Var.: ≈16.29; *s*: ≈ 4.04

2. Consider the following two data sets.
   {1, 2, 3, 4, 5, 6, 7, 8} {1, 1, 1, 1, 8, 8, 8, 8}
   a. Without using a calculator, tell how the means and the standard deviations of the two data sets compare.
   Means are equal; standard deviation of second set is greater.
   b. Use a calculator to find the mean and the standard deviation of each set to check your answer to part a.
   $\bar{x}$: 4.5; *s*: ≈ 2.45 $\bar{x}$: 4.5; *s*: ≈ 3.74

Properties Objective B

In 3–7, match each expression with a descriptor of the data set $\{x_1, x_2, \ldots, x_n\}$.

I. mean
II. sum of the deviations
III. sum of the deviations squared
IV. variance
V. standard deviation

3. $\sum_{i=1}^{n}(x_i - \bar{x})^2$ — III
4. $\sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$ — V
5. $\dfrac{\sum_{i=1}^{n} x_i}{n}$ — I
6. $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ — IV
7. $\sum_{i=1}^{n}(x_i - \bar{x})$ — II

Properties Objective C

8. Tell whether the statistic may be negative. Write *yes* or *no*.
   a. the mean — yes
   b. a deviation — yes
   c. the variance — no
   d. the standard deviation — no

Name ______________________

▶ **LESSON MASTER 1-7** *page 2*

Uses Objective F

9. The following data give the total amount of snowfall, in inches, recorded at New York's JFK Airport in the month of January for the years 1965 to 1996.

1965	17.4	1971	11.6	1977	13.4	1983	1.0	1989	4.7	1995	0.1
1966	10.1	1972	1.7	1978	20.1	1984	10.1	1990	1.4	1996	23.0
1967	2.8	1973	0	1979	7.4	1985	12.4	1991	5.7		
1968	4.5	1974	6.7	1980	3.0	1986	3.0	1992	1.9		
1969	0.6	1975	0.6	1981	7.7	1987	11.8	1993	0.8		
1970	5.5	1976	6.9	1982	12.5	1988	15.7	1994	7.1		

Source: National Climate Data Center

   a. Find the mean and the standard deviation of the snowfall data using a statistics utility. — $\bar{x}$: 7.225; *s*: ≈ 6.164
   b. What percent of these data are within 1 standard deviation of the mean? — ≈ 66%
   c. The blizzard of January, 1996, which hit the East Coast was one of the worst in history. How many standard deviations above the mean was the snowfall for January of 1996? — ≈ 2.6

In 10 and 11, use the following table, which lists the percents of on-time flight arrivals and departures at major U.S. airports in 1994.

Airport City	1st qtr.	3rd qtr.	Airport City	1st qtr.	3rd qtr.
Atlanta	75.2	78.2	Newark	53.5	74.3
Boston	59.0	75.1	NY (Kennedy)	67.0	70.2
Charlotte	78.7	82.2	NY (LaGuardia)	70.3	77.9
Chicago (O'Hare)	73.8	85.9	Orlando	72.8	80.2
Cincinnati	77.7	83.7	Philadelphia	70.0	77.3
Dallas/Ft. Worth	77.5	84.9	Phoenix	80.7	87.4
Denver	71.9	86.8	Pittsburgh	69.9	82.0
Detroit	80.3	86.9	Raleigh/Durham	82.0	87.2
Houston	77.1	85.9	St. Louis	79.0	89.9
Las Vegas	79.5	84.1	Salt Lake City	82.3	86.0
Los Angeles	75.0	83.7	San Diego	78.5	87.5
Miami	73.3	78.7	San Francisco	71.4	84.3
Minneapolis/St. Paul	81.4	87.0	Seattle-Tacoma	72.9	84.4
Nashville	84.2	88.8	Tampa	72.5	78.6
			Washington, D.C.	72.4	81.6

Source: *Statistical Abstract of the United States*, 1995

10. Find the mean and the standard deviation
   a. of the first-quarter percents. — $\bar{x}$: ≈ 74.48; *s*: ≈ 6.72
   b. of the third-quarter percents. — $\bar{x}$: ≈ 82.78; *s*: ≈ 4.82
   c. Which set of percents is more variable? Explain why this seems reasonable.
   1st-quarter; weather is more severe in winter.

Name ______________________

LESSON MASTER 2-1

Questions on SPUR Objectives
See pages 152–157 for objectives.

Vocabulary

1. *True or false.* Justify your answer. The following set of ordered pairs is a function: {(1, 1), (1, 2), 1, 3), 1, 4)}.
   False; the *x*-value is paired with more than one *y*-value.

Skills Objective A

In 2 and 3, let $h(x) = \frac{1}{2}x^2 + 1$.

2. Evaluate.
   a. $h(4)$ — 9
   b. $h(-4)$ — 9
   c. $h\left(\frac{1}{4}\right)$ — $\frac{33}{32}$

3. *True or false.* Justify your answer.
   a. $0 \cdot h(4) = h(0 \cdot 4)$ — False; $0 \cdot h(4) = 0$, $h(0 \cdot 4) = 1$
   b. For all a, $h(-a) = h(a)$. — False; $h(-4) = 9$, $-h(4) = -9$
   c. If $a > b$, then $h(a) > h(b)$. — False; $4 > -4$, $h(4) \ngtr h(-4)$

4. Let $g(x) = \frac{12}{x^2}$. Evaluate.
   a. $g(2) + g(1)$ — 15
   b. $g(2 + 1)$ — $\frac{12}{9}$, or $\frac{4}{3}$
   c. $g(2) \cdot g(3)$ — 4
   d. $g(2 \cdot 3)$ — $\frac{12}{36}$, or $\frac{1}{3}$

Properties Objective B

In 5–7, an equation for a function is given.
a. State the function's domain. b. State the function's range.

5. $y = 7x - 1$
   a. All real numbers b. All real numbers

6. $y = |7x - 1|$
   a. All real numbers b. $\{y: y \geq 0\}$

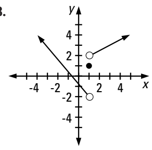
7. $f(x) = \frac{1}{x^2 - 1}$
   a. All real numbers except 1 and -1 b. $\{y: y > 0$ or $y \leq -1\}$

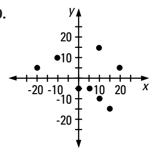
Name ______________________

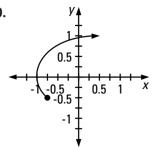
▶ **LESSON MASTER 2-1** *page 2*

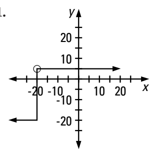
Representations Objective J

In 8–11, a relation is graphed.
a. State the relation's domain. b. State the relation's range.
c. Is the relation a function?

8. 
   a. All real numbers
   b. $\{y: y > -2\}$
   c. Yes

9. 
   a. {-15, -10, 0, 5, 10, 15, 20}
   b. {-15, -10, -5, 5, 10, 15}
   c. No

10. 
   a. $\{x: x \geq -1\}$
   b. $\{y: y \geq -0.5\}$
   c. No

11. 
   a. All real numbers
   b. $\{y: -20 \leq y \leq 5\}$
   c. No

Name

LESSON MASTER 2-2 **Questions on SPUR Objectives** See pages 152–157 for objectives.

Properties Objective C

1. Suppose r is a correlation coefficient for a line of best fit. Give each value.
   a. the greatest possible value for r **1**
   b. the least possible value for r **-1**
   c. the greatest possible value for r^2 **1**
   d. the least possible value for r^2 **0**

2. For a set of data, the line of best fit is $y = -3x - 4$ and $r^2 \approx 0.22$. What is the correlation coefficient? **≈ 0.5**

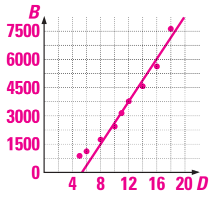
3. Dinah conducted an experiment in which she measured the time it took to boil a certain quantity of water. She noticed that the greater the volume of water, the longer it took to boil. Using her statistics utility, she calculated a regression line for her data and found that $r^2 \approx 0.87$. However, she was not sure whether $r \approx \sqrt{0.87}$ or $r \approx -\sqrt{0.87}$. How could Dinah determine which value of r is correct?
   Since boiling time and volume increase together, the slope of the line is positive and $r \approx \sqrt{0.87}$.

Uses Objective E

4. The table below contains breaking strength data for new 3-strand polypropylene fiber rope. **Sample answers are given.**

Diameter (mm)	5	6	8	10	11	12	14	16	18
Breaking strength (lb)	780	1,125	1,710	2,430	3,150	3,780	4,590	5,580	7,650

   a. Make a scatterplot of the data with the diameters on the horizontal axis.
   b. Draw a line that fits the data reasonably well.
   c. Use two points on the line in part b to write an equation for the line in the form $y = mx + b$. **$y = 562.5x - 3000$**
   d. Use the equation found in part c to estimate the breaking strength of a 25-mm-diameter rope. **11,062.5 lb**



Name

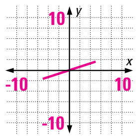
▶ **LESSON MASTER 2-2** *page 2*

   e. Give a reason why your estimate in part d might not be accurate.
   Sample: The model is not linear for ropes with large diameters.

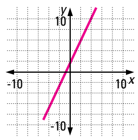
Representations Objective I

In 5–7, graph each function over the domain {x: -5 ≤ x ≤ 5}.

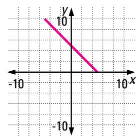
5. $y = \frac{1}{3}x$



6. $y = 2x + 1$

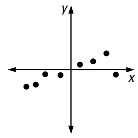


7. $y = -x + 5$

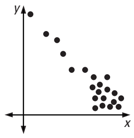


Representations Objective K

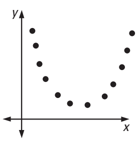
In 8–11, suppose a linear relation is used to model the data in the given scatterplot. State whether the correlation coefficient is likely to be *negative, positive,* or *approximately zero.*

8. 

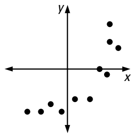
Positive

9. 

Negative

10. 

Approximately zero

11. 

Positive

Name

LESSON MASTER 2-3 **Questions on SPUR Objectives** See pages 152–157 for objectives.

Vocabulary

1. Why is the process of finding the line of best fit sometimes called the "method of least squares"?
   Sample: Because the line of best fit will give the least value for the sum of the squares of the errors.

2. Use $\sum$-notation to write an expression for the center of gravity of the data set $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$. **$\left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n}\right)$**

Properties Objective C

In 3 and 4, suppose a data set is modeled by two lines m_1 and m_2. Line m_1 represents the linear regression model and line m_2 represents some other linear model. Complete the sentence.

3. The sum of the squares of the deviations for m_1 is __?__ (greater than, less than, equal to) the sum of the squares of the deviations for m_2. **less than**

4. If all the points in the data set lie on m_1, then the sum of the squares of the deviations for this model is __?__ (greater than 0, less than 0, equal to 0). **equal to 0**

Uses Objective E

5. The table below contains breaking-strength data for polypropylene rope.

Diameter (mm)	5	6	8	10	11	12	14	16	18
Breaking strength (lb)	780	1,125	1,710	2,430	3,150	3,780	4,590	5,580	7,650

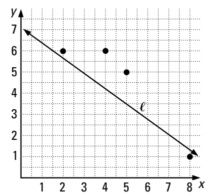
   a. Use a statistics utility to find an equation for the line of best fit to model this data. **$y \approx 501.7x - 2153.2$**
   b. Find the error in the values predicted by the linear regression model for the breaking strength of ropes with diameters of 12 mm and 18 mm.
   12 mm **-87.2 lb** 18 mm **772.6 lb**
   c. Use the equation in part a to estimate the breaking strength of ropes with diameters of 13mm and 25 mm.
   13 mm **4370 lb** 25 mm **10,389 lb**
   d. Which estimate in part c do you think is more accurate? Why?
   Sample: The 13-mm estimate, because it is an interpolation rather than an extrapolation

Name

▶ **LESSON MASTER 2-3** *page 2*

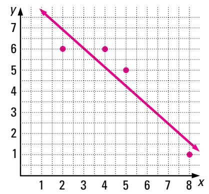
Representations Objective K

In 6 and 7, a student fit a line ℓ to the data points (2, 6), (4, 6), (5, 5), and (8, 1), as shown below.



6. a. What is the observed value of y at $x = 4$? **6**
   b. What is the predicted value of y at $x = 3$? **5**
   c. Estimate the error of each of the four points from line ℓ.
   (2, 6): 0.3; (4, 6): (1, 7); (5, 5): 1.5; (8,1): -0.4
   d. Find the sum of the squares of the deviations of the four points from line ℓ.
   $0.3^2 + 1.7^2 + 1.5^2 + (-0.4)^2 = 5.39$

7. Using a statistics utility, the student found that an equation of the line of best fit is $y = -0.88x + 8.68$.
   a. Graph the four data points and the line of best fit.



   b. Find the sum of the squares of the deviations. **2.48**
   c. Verify that the center of gravity is on this line.
   C. of g. is $\left(\frac{19}{4}, \frac{18}{4}\right)$
   $\frac{18}{4} = -0.88\left(\frac{19}{4}\right) + 8.68$
   d. How do you know that this line is a better fit line than line ℓ?
   The sum of the squares of the deviations is less for this line than for line ℓ.

Home

Name

LESSON MASTER 2-4 **Questions on SPUR Objectives** See pages 152–157 for objectives.

Properties Objective B

In 1 and 2, an equation for an exponential function is given.
a. State the function's domain. b. State the function's range.

1. $f(x) = 0.32(12.6^x)$
   a. All real numbers
   b. $\{y: y > 0\}$

2. $g(x) = 4(0.15^x)$
   a. All real numbers
   b. $\{y: y > 0\}$

Properties Objective D

In 3–6, consider an exponential function given by the equation $f(x) = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. *True or false.*

3. If $b = \frac{1}{2}$, then the graph of the function never crosses the x-axis. True
4. If $b = \frac{1}{2}$, then the graph of the function never crosses the y-axis. False
5. If $a = 0.6$ and $b = 3$, then f is strictly decreasing. False
6. If $a = 1.37$ and $b = 0.85$, the function can model exponential decay. True

Uses Objective F

7. In 1994, the population of the Las Vegas metropolitan area was about 1,076,000, with an average annual growth rate of 6.5%. Assume this growth rate continues into the future.
   a. Estimate the population of Las Vegas in each year.

1995	1996	1997
1,146,000	1,220,000	1,300,000

   b. Express the population P as a function of n, the number of years after 1994. $P = 1,076,000(1.065^n)$
   c. Estimate the population of Las Vegas in the year 2020. 5,532,000

8. A paticular prescription drug has an initial concentration in the blood of 50 mg/ml and is absorbed by the body so that each day its concentration drops by 68%. What is the drug's concentration in the blood after the given amount of time?

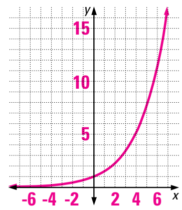
1 day	2 days	d days
16 mg/ml	5.12 mg/ml	$50(0.32^d)$ mg/ml

Name

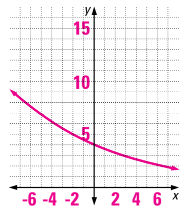
Representations Objective I

In 9 and 10, graph the exponential function over the domain $\{x: -7 \leq x \leq 7\}$.

9. $y = 1.5^x$

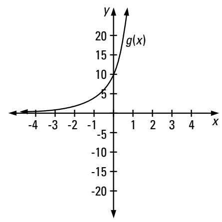


10. $y = 4(0.9^x)$



Representations Objective J

11. The equation graphed at the right is of the form $g(x) = ab^x$.



   a. *True or false.* The function is strictly decreasing. False
   b. Give a range of possible values for b. $b > 1$
   c. Find a. $a = 10$
   d. Does g represent exponential growth or exponential decay? Exponential growth
   e. Give an equation for an asymptote of the graph of g. $y = 0$

Name

LESSON MASTER 2-5 **Questions on SPUR Objectives** See pages 152–157 for objectives.

Uses Objective F

1. The table below contains breaking strength data for new 3-strand polypropylene fiber rope.

Diameter (mm)	5	8	12	14	16	22	30	36	40	48
Breaking strength (lb)	780	1,710	3,780	4,590	5,580	10,350	19,350	27,350	31,950	46,800

   a. Use the data points (12, 3780) and (14, 4590) and a system of equations to determine an exponential model for the data. $y \approx 1179(1.1019^x)$
   b. Using the entire data set and a statistics utility, determine an exponential model for the data. $y \approx 1042(1.0922^x)$
   c. Which of these models better represents the data? Defend your answer.
   Sample: The equation in part b, because it is the exponential regression model for the entire data set
   d. Use the model you chose for part c to estimate the breaking strength of 44-mm-diameter 3-strand polypropylene fiber rope. Is your estimate consistent with the data in the table? Explain your answer.
   $\approx$ 50,500 lb; Sample: Yes, since the breaking strength of 44-mm rope is not between those for 40-mm and 48-mm ropes

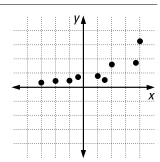
2. In 1995, Edith purchased a \$50 U.S. Savings Bond for \$25. Assume the bond has a constant annual yield of 4.75%. (Note: The annual yield on bonds is not always constant. \$50 is the amount the bond is worth when it reaches maturity.)
   a. Express the value of the bond A as a function of n, the number of years after 1995. $A = 25(1.0475^n)$
   b. Use a calculator and the equation found in part a to estimate the doubling time for the value of the bond. $\approx$ 15 years

Name

3. The half-life of one isotope of the element lithium (^{8}Li) is 0.855 second.
   a. How many seconds are in three half-life periods? 2.565 sec
   b. How much of an 8-gram sample of ^{8}Li will be left after three half-life periods? 1 gram
   c. Use a statistics utility to find the regression equation which models the decay of an 8-gram sample of ^{8}Li. $y \approx 8(0.445^x)$
   d. Use the equation found in part c to determine how much of an 8-gram sample will be left after 15 seconds. 4.25×10^{-5} grams

Representations Objective K

4. Is a linear or exponential model more suitable for the data graphed at the right? Justify your answer.



Sample: An exponential model, because the data points seem to approach the x-axis asymptotically

Home

Name ________________

LESSON MASTER 2-6

Questions on SPUR Objectives
See pages 152–157 for objectives.

Properties Objective B

In 1 and 2, a quadratic function is described. Identify each.
a. the independent variable b. the dependent variable
c. its domain d. its range

1. $z = 4x^2 - 8$
a. x
b. z
c. All real numbers
d. $\{z: z \geq -8\}$

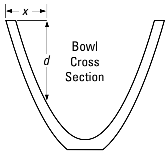
2. $T = 6.4 - v^2$
a. v
b. T
c. All real numbers
d. $\{T: T \leq 6.4\}$

Properties Objective D

3. Consider the function f with equation $f(x) = 2x^2 + x - 15$.
a. Find its y-intercept. -15
b. Find its x-intercepts. $\frac{10}{4}$, or $\frac{5}{2}$
c. Tell whether the graph has a maximum or minimum point and find its coordinates. Min., $\left(-\frac{1}{4}, -\frac{121}{8}\right)$

Uses Objective G

4. The inner surface of a round wooden bowl is carved so that the depth measured from the top of the bowl is given by $d = 0.5x^2 - 4x + 2$, where x (in inches) is the horizontal distance from the outer edge of the bowl.



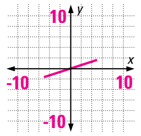
a. Graph the equation for the inner surface of the bowl on an automatic grapher. What is an appropriate domain for this function? Sample: $\{x: 0 \leq x \leq 8\}$
b. How deep is the bowl at $x = 2$? 4 inches
c. How deep is the bowl at its deepest point? 6 inches
d. How wide (thick) is the wood at the top of the bowl? $\approx \frac{1}{2}$ inch
e. What is the interior diameter at the top of the bowl? $\approx$ 7 inches

Name ________________

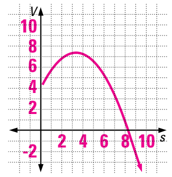
Representations Objective I

In 5 and 6, graph the function over the given domain.

5. $g(x) = 0.2x^2 + x - 3$, $\{x: -5 \leq x \leq 5\}$



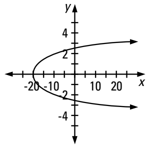
6. $V = -0.3s^2 + 2s + 4$, $\{s: 0 \leq s \leq 10\}$



Representations Objective J

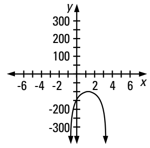
In 7 and 8, a quadratic relation is graphed. a. State its domain.
b. State its range. c. Tell whether the relation is a function.

7.



a. $\{x: x \geq -20\}$
b. All real numbers
c. No

8.

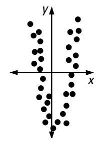


a. All real numbers
b. $\{y: y \leq -100\}$
c. Yes

Representations Objective K

9. *Multiple choice.* Which equation best models the data in the scatterplot at the right? b

(a) $y = -x^2 - 5x - 2$
(b) $y = 3x^2 - 2x - 4$
(c) $y = 6x^2 + 7$
(d) $y = x^2 + 5x + 6$



Name ________________

LESSON MASTER 2-7

Questions on SPUR Objectives
See pages 152–157 for objectives.

Skills Objective A

1. Let $f(x) = \lfloor x \rfloor + \lfloor x - 0.5 \rfloor$. Evaluate.
a. $f(1)$ 1
b. $f(3)$ 5
c. $f(2.3)$ 3
d. $f(-0.1)$ -2

2. Let $c(x) = \lceil x \rceil + \lfloor x - 1 \rfloor$. Evaluate.
a. $c(75)$ 149
b. $c(75.3)$ 150
c. $c\left(\frac{13}{4}\right)$ 6
d. $c\left(-\frac{3}{5}\right)$ -2

Properties Objective B

In 3 and 4, an equation for a step function is given.
Identify each.
a. its domain b. its range c. any points of discontinuity

3. $m(x) = \lceil x^2 \rceil$
a. All real numbers
b. Nonnegative integers
c. Integral values of x

4. $y = 3\lfloor 2x + 1 \rfloor$
a. All real numbers
b. Integral multiples of 3
c. Integral multiples of $\frac{1}{2}$

Uses Objective H

In 5 and 6, *multiple choice.*

5. Which of the following gives the number B of 40-seat buses that a field trip for s students will require? b

(a) $B = \lfloor 40s \rfloor$
(b) $B = \left\lceil \frac{s}{40} \right\rceil$
(c) $B = \left\lfloor \frac{s}{40} \right\rfloor$
(d) $B = 40\lceil s \rceil$

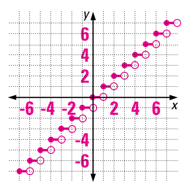
6. A phone company charges 49 cents per minute for calls made from the U.S. to Manchester, England, and rounds all calls up to the nearest 6 seconds. Which formula gives the cost $c(t)$ of a phone call to Manchester lasting t seconds? c

(a) $(t) = 0.49\left\lceil \frac{t}{6} \right\rceil$
(b) $c(t) = 0.49\left\lceil \frac{t}{60} \right\rceil$
(c) $c(t) = 0.49\frac{\left\lceil \frac{t}{6} \right\rceil}{10}$
(d) $c(t) = 0.49\frac{\left\lceil \frac{t}{10} \right\rceil}{6}$

Name ________________

Representations Objective I

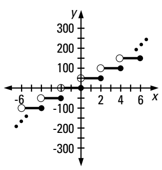
7. Sketch a graph of the function over the given domain.
$y = \lfloor x + 1 \rfloor - 1$, $\{x: -6 \leq x \leq 6\}$



Representations Objective J

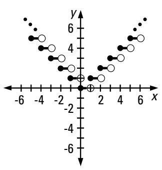
In 8 and 9, a graph of a step function is given.
a. State the domain of the function. b. State the range of the function. c. Identify any points of discontinuity.

8.



a. All real numbers
b. Integral multiples of 50
c. Integral multiples of 2

9.



a. All real numbers
b. Non negative integers
c. Integral values of x

Home

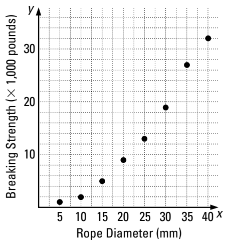
Name

LESSON MASTER 2-8

Questions on SPUR Objectives
See pages 152–157 for objectives.

Representations Objective K

1. Use the scatterplot at the right showing the relation between diameter and breaking strength of 3-strand polypropylene rope.



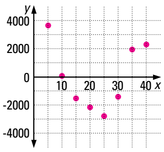
a. Use a statistics utility to determine the regression equation for each model of the data.

linear model: $y \approx 924x - 7285$

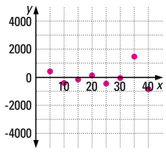
quadratic model: $y \approx 18.57x^2 + 88.1x - 321.4$

b. Plot the residuals for each model in part a.

linear model:



quadratic model:



c. From the residual plots in part b, which do you believe is a more appropriate model for this data? Justify your answer.

Sample: The quadratic model, since the residuals are much closer to zero and since there appears to be a pattern in the residuals for the linear model

Name

LESSON MASTER 3-1

Questions on SPUR Objectives
See pages 225–229 for objectives.

Vocabulary

1. Explain what is meant by a *parent function.*

Sample: a function from which other related functions can be derived.

2. Describe the asymptotes and point of discontinuity of the graph of the function $f(x) = \frac{1}{x^2 - 3x - 4}$. Use an automatic grapher if needed. vertical asymptotes: $x = 4$ and $x = -1$; horizontal asymptote: $y = 0$; points of discontinuity: $x = 4$ and $x = -1$

Representations Objective J

In 3–5, give an equation of a parent function whose graph has the given features. Samples are given.

3. an asymptote but no points of discontinuity $y = b^x$

4. points of discontinuity but no asymptotes $y = \lfloor x \rfloor$

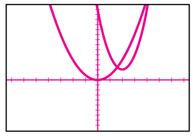
5. two asymptotes $y = \frac{1}{x}$ or $y = \frac{1}{x^2}$

6. a. Give an equation for the parent function of a parabola with equation $y = 3(x - 2)^2 + 2$. $y = x^2$

b. Graph $y = 3(x - 2)^2 + 2$ and its parent function on an appropriate viewing window of an automatic grapher. Give the intervals of x- and y-values for your window. Samples are given.

$-5 \le x \le 5$ $-2 \le y \le 15$

c. In the screen at the right, sketch what you see on your window.



$-5 \le x \le 5$, x-scale = 1
$-2 \le y \le 15$, y-scale = 1

d. Describe the relationship between the two graphs.

The graph of $y = 3(x - 2)^2 + 2$ is shifted 2 units right and 2 units up from the graph of its parent function, $y = x^2$.

Name

LESSON MASTER 3-2

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective C

1. Let T be the transformation $T: (x, y) \rightarrow (x + 5, y - 6)$. Find an equation for the image of $y = x^3 + \frac{3}{2}$ under T. $y = (x - 5)^3 - \frac{9}{2}$

2. Give an equation for the transformation T which moves each point 9 units down and 3 units to the right. $T: (x, y) \rightarrow (x + 3, y - 9)$

3. What transformation maps the graph of $y = |x|$ onto the graph of $y = |x - 8| + 15$? $T: (x, y) \rightarrow (x + 8, y + 15)$

Properties Objective D

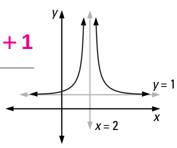
4. What are the zeros and the asymptotes of the graph of $y = \frac{1}{x}$ under the translation $T(x, y) = (x + 3, y - 1)$?

zero at $x = 4$; asymptotes $x = 3$ and $y = -1$

5. *True or false.* A translation does not change the number of asymptotes of a graph. True

Representations Objective K

6. a. Use the Graph-Translation Theorem to write an equation for the graph at the right. An equation for the parent function is $y = \frac{1}{x^2}$. $y = \frac{1}{(x - 2)^2} + 1$



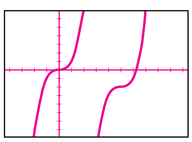
b. Use your equation in part a to find the value of the graphed function at $x = 11$. $\frac{82}{81}$

7. Consider the function t given by $t(x) = (x - 5)^3 - 2$.

a. Give an equation for the parent function of t. $y = x^3$

b. What transformation maps the parent function onto t? $T: (x, y) \rightarrow (x + 5, y + 2)$

c. Use an automatic grapher to graph t and its parent function on the same window. Choose an appropriate window for viewing key features of both graphs. In the screen at the right, sketch what you see on your window.



$-2 \le x \le 8$, x-scale = 1
$-5 \le y \le 5$, y-scale = 1

d. Identify the x- and y-intercepts of t and its parent function.

t: $x \approx 6.26$; parent: $x = 0$, $y = 0$

Name

LESSON MASTER 3-3

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective E

1. A data set has a mean of 5 and a standard deviation of 2. Suppose 1,000 is added to each observation. What are the new mean and standard deviation? mean: 1005; st. dev.: 2

2. A data set has a median of 35 and a mode of 30. Suppose 15 is added to each observation. What are the new mode and median? mean: 45; st. dev.: 50

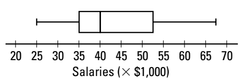
Uses Objective I

3. A meteorologist takes a number of air-temperature readings and finds that the mean temperature is -24.66°C with a standard deviation of 2.27°C. He then decides to convert all of his measurements from degrees Celsius to degrees Kelvin. To do this, he uses the formula K = C − 273.15, where C is the temperature in degrees Celsius and K is the temperature in degrees Kelvin.

a. What is the mean air-temperature reading in degrees Kelvin? -297.81°K

b. What is the standard deviation of air-temperature readings in degrees Kelvin? 2.27°K

4. The box plot below displays the annual salaries of employees at Transformation Technologies, Inc., a small biotech company involved in cloning research.



Suppose, due to profit sharing, each employee receives an end-of-year bonus of $5,000. Which, if any, of the following descriptive statistics will change due to this bonus? If they change, give their new values.

a. median annual income $45,000

b. interquartile range no change

c. range no change

d. outliers $30,000, $72,000

Home

Name

LESSON MASTER 3-4

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective F

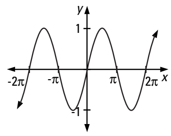
In 1–4, decide whether the function with the given equation is *even*, *odd*, or *neither*. Justify your answer algebraically.

1. $s(t) = 8t^7$ Odd; $8(-t)^7 = -8t^7 = -(8t^7)$
2. $f(x) = 7x^5 - 5x^2$ Neither; $7(-x)^5 - 5(-x)^2 = -7x^2 + 5x^2$
3. $g(h) = -9h^2 + 5$ Even; $-9(-h)^2 + 5 = -9h^2 + 5$
4. $v(m) = |7m + 2| - 5$ Neither; $|7(-m) + 2| - 5 = |-7m + 2| - 5$

Representations Objective L

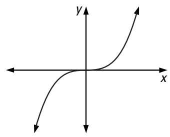
In 5 and 6, decide whether the function whose graph is given is *even*, *odd*, or *neither*.

5.



odd

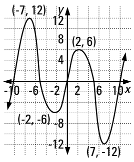
6.



odd

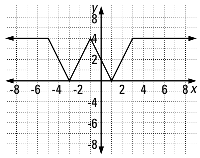
In 7 and 8, describe the symmetries of the graphed function.

7.



180° rotation symmetry about the origin

8.



reflection symmetry about $x = 1$

Name

LESSON MASTER 3-5

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective C

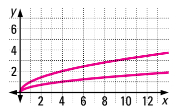
1. Find the scale change S which shrinks a graph horizontally with a factor of $\frac{1}{6}$ and stretches it vertically with a factor of 8. $S: (x, y) \rightarrow \left(\frac{x}{6}, 8y\right)$
2. Find an equation for the image of $y = \sqrt{x^2 + 1}$ under the scale change $S: (x, y) \rightarrow \left(\frac{x}{3}, 3y\right)$. $y = 3\sqrt{9x^2 + 1}$
3. Describe two different transformations S_1 and S_2 which map the graph of $y = x^2$ onto the graph of $y = \frac{9}{4}x^2$. Samples are given.
$S_1: (x, y) \rightarrow \left(\frac{2}{3}x, y\right)$ $S_2: (x, y) \rightarrow \left(x, \frac{9}{4}y\right)$
4. *Multiple choice.* Which scale change will map $y = \frac{\pi}{2}x^2$ so that the transformed graph includes the point (1, 1)? c
(a) $S(x, y) = \left(\sqrt{\frac{2}{\pi}}x, \frac{2}{\pi}\right)$ (b) $S(x, y) = \left(\sqrt{\frac{2}{\pi}}x, y\right)$
(c) $S(x, y) = \left(x, \frac{2}{\pi}y\right)$ (d) $S(x, y) = \left(x, \frac{\pi}{2}y\right)$

Properties Objective D

5. The graph of an equation has x-intercepts -1.5, 1, and 2, and y-intercept -3. Give the x- and y-intercepts for the image of the graph under the transformation $S: (x, y) \rightarrow (2x, 3y)$. x-intercepts: -3, 2, 4; y-intercept: -9
6. Describe the points of discontinuity on the image of the graph of $y = [x]$ under the scale change $S: (x, y) \rightarrow \left(2x, \frac{1}{3}y\right)$. integral multiples of 8
7. Suppose the scale change $S: (x, y) \rightarrow (4x, 3y)$ is applied to the graph of $y = \frac{x}{x^2 - 9}$. What effect does this transformation have on the graph's asymptotes? Horizontal asymptote $y = 0$ is unchanged; vertical asymptotes $x = 3$ and $x = -3$ move to $x = \frac{3}{4}$ and $x = -\frac{3}{4}$

Representations Objective K

8. Sketch graphs of $y = \sqrt{x}$ and its image under the transformation $S: (x, y) \rightarrow \left(\frac{1}{4}x, y\right)$.



Name

LESSON MASTER 3-6

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective E

In 1–4, suppose each element in a data set is multiplied by -7. Describe the effect of this transformation on each measure.

1. mean multiplied by -7
2. mode multiplied by -7
3. median multiplied by -7
4. range multiplied by 7
5. A data set is rescaled so that its variance is multiplied by 4. What are two possible values for the scale factor? $a = 2$ and $a = -2$

Uses Objective I

4. Neil Vestor is trying to decide whether he should purchase stock in an American or a Japanese manufacturing company. He recorded the price of each stock over a 3-week period and computed the mean and standard deviation for each.

	American Company	Japanese Company
Mean stock value	$39.60	¥6734
Standard deviation	$ 2.50	¥ 187

To compare the two stocks, Neil rescales his raw data by converting the stock prices in yen to dollars, using the exchange rate $1 = ¥127. If Neil is trying to minimize his risk by choosing the stock with the least variability, which stock should he buy? Justify your answer.
Sample: the Japanese company, as its standard deviation is $\frac{187}{127} = \$1.47$ so it is less variable than that of the American company.

Name

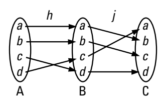
LESSON MASTER 3-7

Questions on SPUR Objectives
See pages 225–229 for objectives.

Skills Objective A

In 1 and 2, let $f(x) = x^2 + 2x + 7$ and $g(x) = 5x - 3$.

1. Evaluate each composite.
a. $f(g(1))$ 15 **b.** $g(f(1))$ 47
2. Find a formula for each composite.
a. $f(g(x))$ $y = 25x^2 - 20x + 10$ **b.** $g(g(x))$ $y = 25x - 18$
3. Let $F = \{(1, 7), (2, 4), (3, 2), (4, 1)\}$ and $G = \{(7, 6), (1, 3), (2, 2), (4, 1)\}$. Find each composite.
a. $F \circ G$ {(1, 2), (2, 4), (4, 7)} **b.** $G \circ F$ {(1, 6), (2, 1), (3, 2), (4, 3)}
4. Consider the functions h mapping A to B and j mapping B to C.



Evaluate each composition.
a. $h(j(a))$ b **b.** $j(h(b))$ c **c.** $(h \circ j)(d)$ c

Properties Objective G

5. Let $s(x) = \sqrt{x - 1}$ and $n(x) = x^2 - 2$. Give the domain of each composite.
a. $n \circ s$ set of all reals > 1 **b.** $s \circ x$ set of all reals > $\sqrt{3}$
6. Let $p(t) = \frac{1}{t} - 1$. *True or false.* The domain of p is the same as the domain of $p \circ p$. Justify your answer.
False; The domain of p is the set of all real numbers except 0; the domain of $p \circ p$ is the set of all real numbers except 0 and 1.

Name

LESSON MASTER 3-8

Questions on SPUR Objectives
See pages 225–229 for objectives.

Skills Objective B

In 1–3, a function is described. a. Give an equation for the inverse of the function. b. State whether the inverse is a function.

1. $y = 3 - 2x$ a. $y = -\frac{1}{2}x + \frac{3}{2}$ b. a function

2. $g(x) = \frac{1}{x^2}$ a. $y = \pm\frac{\sqrt{x}}{x}$ b. not a function

3. $f = \{(-2.5, 0), (0, -2.5), (1, 3), (3, 1)\}$
a. $f^{-1} = \{(0, -2.5), (-2.5, 0), (3, 1), (1, 3)\}$ b. a function

Properties Objective G

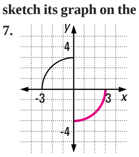
4. Let $f(x) = \lfloor x \rfloor$ and $g(x) = \lceil x \rceil$. Are functions f and g inverses? Justify your answer.
$f(g(x)) = \lfloor \lceil x \rceil \rfloor$; if $x = 2.5$, $f(g(2.5)) = 3$; so $f(g(x)) \neq x$ and f and g are not inverses.

5. *True or false.* If a function is an even function, then its inverse is not a function. True

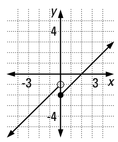
6. Suppose f is a function such that for all x, $f(x) = f(x + 2)$. Is the inverse of f a function? Justify your answer.
No; if $x = 0$, then $f(0) = f(2)$ and the line $y = f(0)$ intersects two points on the graph: $(0, f(0))$ and $(2, f(2))$.

Representations Objectives L and M

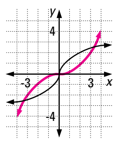
In 7–9, determine whether the inverse of the graphed function is a function. If the inverse is a function, sketch its graph on the same set of axes.

7. 

a function

8. 

not a function

9. 

a function

Name

LESSON MASTER 3-9

Questions on SPUR Objectives
See pages 225–229 for objectives.

Properties Objective H

1. Explain how a *z*-score is calculated.
The mean is subtracted from the raw score; the difference is divided by the standard deviation.

2. A data set has a mean of 25.6 and a standard deviation of 2.3. Find each for the data set's *z*-scores.
a. the mean 0
b. the standard deviation 1

In 3 and 4, a *z*-score is given. Explain what it means in terms of the mean and standard deviation of the original data set.

3. $z = 0.75$ $\frac{3}{4}$ of a standard deviation above the mean

4. $z = -1.25$ $\frac{5}{4}$ of a standard deviation below the mean

Uses Objective I

5. The following sets of data show the average number of points scored per game by players on the boys' and girls' basketball teams.

Boys	4.7	0.3	11.6	0.3	3.6	6.2	1.3	1.1
	3.1	7.6	4.0	20.5	0.8	2.5	3.6	
Girls	7.0	2.6	9.8	6.3	5.7	0.8	6.5	8.5
	12.4	7.2	5.3	7.9	9.1	7.6	6.9	

a. Convert the above data for the 15 boys and 15 girls to *z*-scores. (When calculating *z*-scores, use the population standard deviation, not the sample standard deviation.)

Boys	-0.1	-0.86	1.33	-0.86	-0.22	0.28	-0.67	-0.71
	-0.32	0.55	-0.14	3.06	-0.77	-0.44	-0.22	
Girls	0.03	-1.61	1.08	-0.23	-0.45	-2.28	-0.15	0.59
	2.05	0.11	-0.60	0.37	0.82	0.26	0	

b. Who did better relative to the rest of the team, the boy who averaged 6.2 points per game or the girl who averaged 7.9 points per game? Justify your answer in terms of *z*-scores.
the girl; her *z*-score was 0.37 and boy's was 0.28

6. A student took two tests. On the first, she scored 87 and on the second she scored 80. If the class mean was 80 and the standard deviation was 10 on the first test and the class mean was 72 with a standard deviation of 5 on the second, on which test did she do better compared to the other students?
second test (1st *z*-score: 0.7; 2nd *z*-score: 1.6)

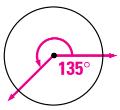
Name

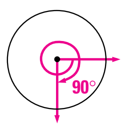
LESSON MASTER 4-1

Questions on SPUR Objectives
See pages 303–307 for objectives.

Skills Objective A

In 1 and 2, the measure of a rotation is given. a. Convert the measure to revolutions. b. On the circle draw a central angle showing the given rotation.

1. 225°
a. $\frac{5}{8}$, counterclockwise
b. 

2. $-\frac{5\pi}{2}$ radians
a. $1\frac{1}{4}$, clockwise
b. 

3. Give two other degree measures, one positive and one negative, for a rotation of 138°.
498°, -222°

4. Give two other radian measures, one positive and one negative, for a rotation of $\frac{4\pi}{3}$.
$\frac{10\pi}{3}$, $-\frac{2\pi}{3}$

In 5–7, convert to a radian measure without using a calculator.

5. 60° $\frac{\pi}{3}$ 6. 135° $\frac{3\pi}{4}$ 7. -210° $-\frac{7\pi}{6}$

In 8–10, convert to a degree measure without using a calculator.

8. $\frac{11\pi}{6}$ 330° 9. $-\frac{\pi}{10}$ -18° 10. 3.14159 ≈ 180°

In 11–14, use a calculator to convert the given angle measure to the indicated units. Give your answer correct to the nearest thousandth.

11. -42° a. to revolutions ≈ 0.12, clockwise b. to radians ≈ -0.733

12. 19π a. to revolutions 95, counterclockwise b. to degrees 3420°

13. 19 a. to revolutions ≈ 3.02, counterclockwise b. to degrees 1088.620°

14. 0.33 revolution clockwise
a. to radians ≈ 2.073 radians b. to degrees -118.8°

Name

LESSON MASTER 4-2

Questions on SPUR Objectives
See pages 303–307 for objectives.

Skills Objective B

1. Find the length of an arc of a circle of radius 8 m if the central angle of the arc is $\frac{5\pi}{6}$.
$\frac{20\pi}{3}$, or ≈ 20.94, m

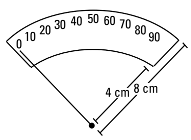
2. Find the area of a sector of a circle of diameter 22 in. if the central angle of the sector is 315°.
$\frac{847\pi}{8}$, or ≈ 332.62 in^2

3. The arc of a circle of radius 4 cm has a length of $\frac{2\pi}{3}$ cm. Find the measure of the central angle in radians and degrees.
$\frac{\pi}{6}$, 30°

4. A sector in a circle with central angle $\frac{7\pi}{12}$ has an area of 14π m^2. Find the exact length of the radius of the circle.
$4\sqrt{3}$ m

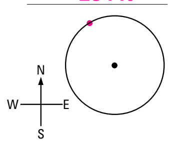
Uses Objective G

5. James needs to replace the glass of the speedometer on his old car. If the needle can maximally rotate $\frac{5\pi}{12}$, find the area of the glass that James needs.
10π, or ≈ 31.4, cm^2



6. Austin, TX, and Oklahoma City, OK, have approximately the same longitude, 97°30′W. Austin has latitude 30°16′N. Oklahoma City has latitude 35°28′N. Use 3,960 miles for the radius of the earth to estimate the air distance from Austin to Oklahoma city.
≈ 359 mi

7. Kaitlin watched her son Dizzy ride a horse 22 ft from the center of a merry-go-round. Dizzy completed one revolution in 45 seconds.
a. How far did Dizzy travel in one revolution? ≈ 138 ft
b. How far did Dizzy travel in one minute? ≈ 184 ft
c. Kaitlin noted that her son started at the easternmost position. If the merry-go-round rotates counterclockwise and the ride lasts 4 minutes, sketch the position of her son when the ride ended.



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LESSON MASTER 4-3

Questions on SPUR Objectives
See pages 303–307 for objectives.

Skills Objective C

In 1–6, give the exact value of each without using a calculator.

1. cos (-90°) **0**
2. sin(-90°) **-1**
3. cos 6π **1**
4. tan $\frac{\pi}{2}$ **undefined**
5. sin π **0**
6. tan 1260° **0**

In 7–9, use a calculator to evaluate to the nearest thousandth.

7. sin 112° **0.927**
8. tan $\frac{\pi}{4}$ **1**
9. cos (-16°) **0.961**
10. The point (1, 0) is rotated $\frac{5\pi}{4}$ about the origin. Find the coordinates of its image correct to three decimal places. **(-0.707, -0.707)**

Properties Objective D

11. *True or false.* For some integer values of k, $\tan\left(k \cdot \frac{\pi}{2}\right) = 1$. Justify your answer. **False; for integral values of k, $\sin\left(k \cdot \frac{\pi}{2}\right)$ and $\cos\left(k \cdot \frac{\pi}{2}\right) = 0$ or 1; when $\sin\left(k \cdot \frac{\pi}{2}\right) = 0$, $\cos\left(k \cdot \frac{\pi}{2}\right) = 1$ and when $\sin\left(k \cdot \frac{\pi}{2}\right) = 1$, $\cos\left(k \cdot \frac{\pi}{2}\right) = 0$; so $\tan\left(k \cdot \frac{\pi}{2}\right) = 0$ or undefined.**

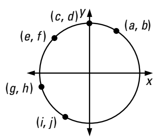
In 12–15, describe an interval between 0 and 2π in which θ satisfies the given requirements. **Sample answers are given.**

12. $\cos\theta > 0$ and $\sin\theta < 0$ **$\frac{3\pi}{2} < \theta < 2\pi$**
13. $\sin\theta > 0$ and $\tan\theta < 0$ **$\frac{\pi}{2} < \theta < \pi$**
14. $\cos\theta = 0$ and $\sin\theta > 0$ **$\theta = \frac{\pi}{2}$**
13. $\tan\theta > 0$ and $\cos\theta < 0$ **$\pi < \theta < \frac{3\pi}{2}$**

Representations Objective J

In 16–20, refer to the unit circle shown. Which letter best represents the value given?

16. cos -270° ***c***
17. sin $\frac{13\pi}{12}$ ***h***
18. sin 135° ***f***
19. cos $\frac{\pi}{3}$ ***a***
20. sin $-\frac{2\pi}{3}$ ***j***



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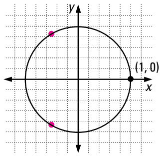
LESSON MASTER 4-4

Questions on SPUR Objectives
See pages 303–307 for objectives.

Properties Objective D

1. The point (1, 0) is rotated about the origin such that $\cos\theta = -\frac{8}{17}$.
   a. In what quadrant(s) could $R_\theta(1, 0)$ lie? **II and III**
   b. Justify your answer to part a by graphing $R_\theta(1, 0)$ on the unit circle at the right
   c. Find all possible values of $\sin\theta$. **$\frac{15}{17}$, $-\frac{15}{17}$**



Properties Objective E

2. If $\sin\theta = \frac{\sqrt{17}}{7}$, find all possible values for the following.
   a. $\cos\theta$ **$\pm\frac{4\sqrt{2}}{7}$**
   b. $\tan\theta$ **$\pm\frac{\sqrt{34}}{8}$**
3. If $\cos\theta = 0.68$, evaluate the following.
   a. $\cos(-\theta)$ **0.68**
   b. $\cos(\pi - \theta)$ **-0.68**
4. If $\sin\theta = -0.368$, and $\pi < \theta < \frac{3\pi}{2}$, evaluate the following.
   a. $\sin(\pi + \theta)$ **0.368**
   b. $\sin\left(\frac{\pi}{2} - \theta\right)$ **≈ -0.930**
   c. $\cos(-\theta)$ **≈ -0.938**
   d. $\tan(\pi - \theta)$ **≈ -0.396**
5. *True or false.* $\tan(k \cdot \pi + \theta) = \tan\theta$ for all integers k. Justify your answer. **True; by the Half-Turn Theorem, $\tan(\pi + \theta) = \tan\theta$; another half-turn gives $\tan(\pi + (\pi + \theta)) = \tan(\pi + \theta) = \tan\theta$. So, $\tan(2\pi + \theta) = \tan\theta$. Repetition shows that $\tan(k \cdot \pi + \theta) = \tan\theta$ for all integers k.**

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LESSON MASTER 4-5

Questions on SPUR Objectives
See pages 303–307 for objectives.

Skills Objective C

In 1–8, give the exact value.

1. $\cos\left(-\frac{\pi}{3}\right)$ **$\frac{1}{2}$**
2. $\sin\left(\frac{3\pi}{4}\right)$ **$\frac{\sqrt{2}}{2}$**
3. sin 30° **$\frac{1}{2}$**
4. $\tan\left(\frac{2\pi}{3}\right)$ **$-\sqrt{3}$**
5. cos 240° **$-\frac{1}{2}$**
6. $\tan\left(-\frac{\pi}{4}\right)$ **-1**
7. $\cos\left(\frac{17\pi}{6}\right)$ **$-\frac{\sqrt{3}}{2}$**
8. sin (-2115°) **$\frac{\sqrt{2}}{2}$**

Properties Objective E

9. Find all values of θ between 0 and 2π such that $\cos\theta = \frac{\sqrt{3}}{2}$. **$\frac{\pi}{6}$, or 30°; $\frac{11\pi}{6}$, or 330°**
10. Find four values of θ between -2π and 2π such that $\cos\theta = -\frac{1}{2}$. **$\pm\frac{4\pi}{3}$, or 240°; $\pm\frac{2\pi}{3}$, or 120°**
11. Give all possible values for $\cos\left(k \cdot \frac{\pi}{6}\right)$ or $\sin\left(k \cdot \frac{\pi}{6}\right)$ for all integers k. **0, ± 1, $\pm\frac{1}{2}$, $\pm\frac{\sqrt{3}}{2}$**
12. Give all possible values for $\cos\left(k \cdot \frac{\pi}{4}\right)$ or $\sin\left(k \cdot \frac{\pi}{4}\right)$ for all integers k. **0, ± 1, $\pm\frac{\sqrt{2}}{2}$**

Name

LESSON MASTER 4-6

Questions on SPUR Objectives
See pages 303–307 for objectives.

Properties Objective D

1. Complete the following table.

	$f(\theta) = \sin\theta$	$g(\theta) = \cos\theta$	$h(\theta) = \tan\theta$
Domain	all reals	all reals	all reals but odd multiples of $\frac{\pi}{2}$
Range	$-1 \le y \le 1$	$-1 \le y < 1$	all reals
Zeros	$k\pi$, for all integers k	$k\pi$, for all integers k	$k\pi$, for all integers k
Period	2π	2π	π
Even, odd, or neither	odd	even	odd

2. For what values of x between 0 and 2π are both cos x and tan x negative? **$\frac{\pi}{2} < x < \pi$**
3. One solution to the equation $\sin\theta = 0.732$ is $\theta \approx 0.821$. Find the three other solutions closest to this value. **≈ 2.32, ≈ -3.96, ≈ 7.10**

Representations Objective K

In 4–11, identify which, if any, of the parent circular functions have graphs with the given characteristic.

4. symmetry with respect to the origin **sine, tangent**
5. symmetry with respect to the x-axis **none**
6. symmetry with respect to the y-axis **cosine**
7. vertical asymptotes **tangent**
8. horizontal asymptotes **none**
9. points of discontinuity **tangent**
10. x-intercepts at integral multiples of π **sine, tangent**
11. y-intercept -1 **none**

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LESSON MASTER 4-7

Questions on SPUR Objectives
See pages 303–307 for objectives.

Properties Objective F

In 1 and 2, a circular function is given. a. State the period of the function. b. State the amplitude of the function.

1. $4y = \sin\left(\frac{x}{2}\right)$ **a.** 4π **b.** $\frac{1}{4}$

2. $\frac{y}{6} = \frac{\sin x}{4}$ **a.** 2π **b.** $\frac{3}{2}$

3. Consider the image of the graph of $y = \cos x$ under the transformation $S(x, y) = \left(\frac{x}{4}, 5y\right)$.
   a. Find the amplitude of the image. 5
   b. Find the period of the image. $\frac{\pi}{2}$
   c. Find an equation for the image under this transformation. $y = 5 \cos 4x$

4. How many cycles does the graph of $y = \sin 3x$ make for each cycle of the graph of $y = \sin x$? 3 cycles

5. How many cycles does the graph of $y = 3 \sin x$ make for each cycle of the graph of $y = \sin 3x$? $\frac{1}{3}$ cycle

Uses Objective H

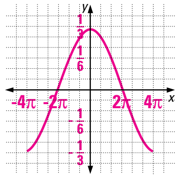
6. Suppose a tuning fork vibrates with a frequency of approximately 370 cycles per second. If the vibration displaces air molecules by a maximum of 0.22 mm, give a possible equation for the sound wave that is produced. $y = 0.22 \sin (740\pi t)$

7. A certain sound wave has equation $y = 15 \cos (110\pi t)$. Give an equation of a sound wave with pitch one octave lower and three times as loud as this one. $y = 45 \cos (55\pi t)$

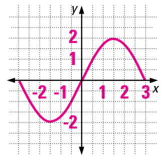
Representations Objective L

In 8 and 9, sketch one cycle of the graph without an automatic grapher.

8. $y = \frac{\cos\left(\frac{x}{4}\right)}{3}$



9. $y = 2 \sin\left(\frac{\pi}{3}x\right)$



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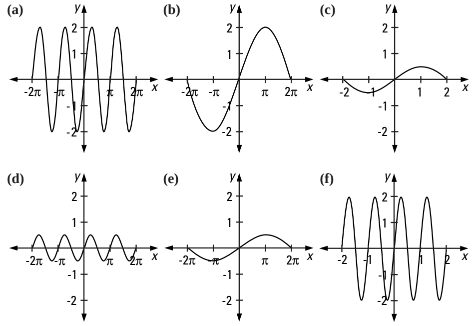
▶ **LESSON MASTER 4-7** *page 2*

Representations Objective M

In 10–15, match each equation with its graph below.

10. $\frac{y}{2} = \sin\left(\frac{x}{2}\right)$ b
11. $2y = \sin\left(\frac{x}{2}\right)$ e
12. $2y = \sin 2x$ d
13. $\frac{y}{2} = \sin 2x$ a
14. $2y = \sin\left(\frac{\pi x}{2}\right)$ c
15. $\frac{y}{2} = \sin 2\pi x$ f



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LESSON MASTER 4-8

Questions on SPUR Objectives
See pages 303–307 for objectives.

Properties Objective F

1. Consider the graph of the function $f(x) = \cos\left(x - \frac{9\pi}{4}\right) - 4$. Find each of the following for f.
   a. the phase shift $\frac{\pi}{4}$
   b. the period 2π
   c. the amplitude 1
   d. the maximum and minimum values max.: -3, min.: -5

2. *True or false.* The graphs of the functions $g(x) = \sin (x + \pi)$ and $h(x) = \sin (x - \pi)$ are identical. Justify your answer.
   True; $\sin (x - \pi) = \sin (x - \pi + 2\pi) = \sin (x + \pi)$

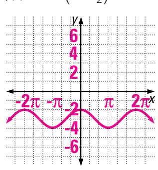
Uses Objective H

3. For an electrical-power supply, the output potential (in volts) and current (in amps) as functions of time (in seconds) are given by $V = 25 \cos t + 25$ and $I = 0.3 \cos\left(t - \frac{5\pi}{4}\right)$, respectively.
   a. What are the maximum and minimum output voltages? 50 volts, 0 volts
   b. What are the maximum and minimum output currents? 0.3 amp, -0.3 amp
   c. What is the phase shift between output current and output voltage? $\frac{5\pi}{4}$ or $\frac{-3\pi}{4}$
   d. By about how many seconds does the maximum current lag behind the maximum voltage? ≈ 2.36

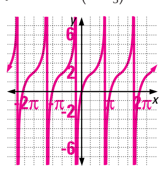
Representations Objective L

In 4 and 5, sketch a graph of the function.

4. $f(x) = \sin\left(x + \frac{\pi}{2}\right) - 3$



5. $y - 2 = \tan\left(x - \frac{\pi}{3}\right)$



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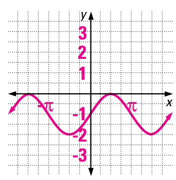
▶ **LESSON MASTER 4-8** *page 2*

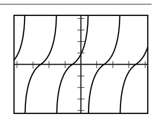
6. Consider the translation $T: (x, y) \rightarrow \left(x - \frac{3\pi}{2}, y - 1\right)$.
   a. Graph the image of the cosine function under T.



   b. Write an equation for the image of $y = \cos x$ under T. $y = \cos\left(x + \frac{3\pi}{2}\right) - 1$

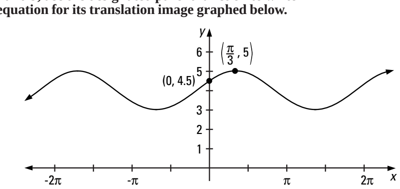
Representations Objective M

7. Write an equation for the translation image of $y = \tan x$ shown at the right. $y = \tan\left(x + \frac{3\pi}{2}\right)$



$-2\pi \le x \le 2\pi$ x-scale $= \frac{\pi}{2}$
$-4 \le y \le 4$ y-scale $= 1$

In 8 and 9, use the designated parent function to write an equation for its translation image graphed below.



8. parent function: $y = \cos x$ $y = \cos\left(x - \frac{\pi}{3}\right) + 4$

9. parent function: $y = \cos x$ $y = \sin\left(x + \frac{\pi}{6}\right) + 4$

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LESSON MASTER 4-9 **Questions on SPUR Objectives** See pages 303–307 for objectives.

Properties Objective F

In 1 and 2, a circular function is described.
a. State the amplitude. b. State the period.
c. State the phase shift.

1. $y = 2\sin\left(\frac{x-4}{4}\right)$
a. 2
b. 8π
c. π

2. $y = \cos\left(\frac{2x+\pi}{3}\right) - 4$
a. 1
b. 3π
c. $-\frac{\pi}{2}$

3. Describe a scale change S and translation T whose composite maps the graph of $y = \sin x$ onto the graph of $y = 3\sin\left(2x - \frac{\pi}{2}\right) + 1$.

$S(x, y) \to \left(\frac{1}{2}x, 3y\right);\ T(x, y) \to \left(x + \frac{\pi}{4}, y + 1\right)$

4. Suppose the rubber band transformation $B: (x, y) \to \left(\frac{x-a}{h}, \frac{y}{k}\right)$ is applied to the graph of $y = \cos x$.

a. State an equation for the image. $y = \frac{1}{k}\cos(xh + a)$

b. Find the amplitude, period, phase shift, and vertical shift of the image.

$\frac{1}{k}, \frac{2\pi}{h}, \frac{-a}{h}$, no vertical shift

In 5 and 6, write a function whose graph will have the given characteristics.

5. parent $y = \sin x$, phase shift $\frac{\pi}{6}$, period π, amplitude $\frac{1}{2}$

$y = \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)$

6. parent $y = \cos x$ phase shift π, period $\frac{\pi}{3}$, amplitude 4

$y = 4\cos(6x - 6\pi)$

Name

Uses Objective H

7. Dizzy, who loves the rides, is positioned on a Ferris wheel such that once the ride begins, his height d (in feet) above the ground after t seconds is given by $d = 65 + 60\cos\left(\frac{\pi}{90}(t - 30)\right)$.

a. At the start of the ride, how high is Dizzy off the ground? 95 ft

b. How long does it take Dizzy to make one complete revolution? 180 sec = 3 min

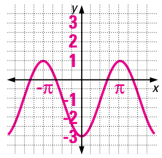
c. What is Dizzy's maximum height above the ground? 125 ft

d. If the ride lasts for 9 minutes, find all times at which Dizzy is at his maximum height. 30 sec, 210 sec, 390 sec

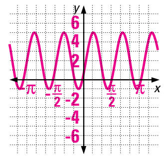
Representations Objective L

In 8 and 9, sketch a graph of the function described.

8. $y = 2\cos(x + \pi) - 1$



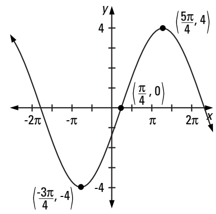
9. $\frac{y-2}{3} = \sin\left(4x - \frac{\pi}{6}\right)$



Representations Objective M

10. Give an equation for the sine wave at the right.

$y = 4\sin\left(\frac{x}{2} - \frac{\pi}{8}\right)$



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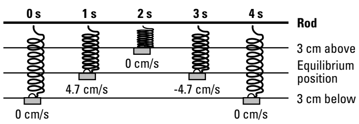
LESSON MASTER 4-10 **Questions on SPUR Objectives** See pages 303–307 for objectives.

Uses Objective I

1. When a weight suspended from a spring is caused to oscillate, not only is its distance from equilibrium a sinusoidal function of time, but so is its velocity. The diagram below shows the velocity of the weight in cm/s at 1-second intervals for the same oscillating spring as pictured in Lesson 4-10. (Note: A positive velocity means that the weight is moving up, and a negative velocity means that the weight is moving down.)



Suppose the velocity v of the weight as a function of time t is to be modeled by an equation of the form $\frac{v-k}{b} = \cos\left(\frac{t-h}{a}\right)$.

a. Find the amplitude of the velocity. 4.7

b. Find the period of the velocity. 4s

c. Write an equation for the velocity function. $v = 4.7\cos\left(\frac{\pi t}{2} - \frac{\pi}{2}\right)$

d. What is the weight's velocity at $t = 3.5$s? $-\frac{4.7\sqrt{2}}{2} \approx -3.32$ cm/s

2. The following table gives the average normal temperatures in °F for six months of a year for Dodge City, KS.

Month	Jan	Feb	May	July	Aug	Nov
Average temperature (°F)	30	35	64	80	78	43

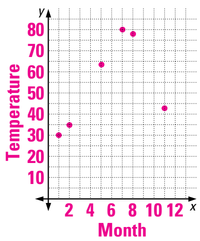
a. Plot these data using the month (Jan = 1, Feb = 2, and so on) as the independent variable.

b. Determine an equation for a periodic function to model these data.

$y = 24.4\sin(0.534x - 2.244) + 55.321$

c. What does your model predict as the average normal temperature for September in Dodge City?

≈ 56°F



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LESSON MASTER 5-1 **Questions on SPUR Objectives** See pages 364–367 for objectives.

Skills Objective A

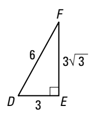
In 1–4, refer to △DEF at the right. Find each.

1. cos D $\frac{1}{2}$

2. sin F $\frac{1}{2}$

3. cos F $\frac{\sqrt{3}}{2}$

4. tan D $\sqrt{3}$



In 5–8, approximate to the nearest hundredth.

5. tan 11.1° 0.20

6. cos 165°12′ -0.97

7. $\sin\frac{8\pi}{7}$ -0.43

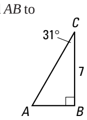
8. $\cos\frac{3\pi}{8}$ 0.38

Skills Objective C

9. Refer to △ABC. Find AC and AB to the nearest hundredth.

AC = 8.17

AB = 4.21

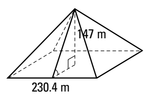


10. Find the measure of ∠D in △DEF above. 60°

Uses Objective G

11. The largest of the ancient Egyptian pyramids, built for the king Khufu, is a regular square pyramid with base edges of length 230.4 m and a height of 147 m. What angle do the faces of the pyramid make with the ground?

≈ 51.9°



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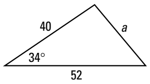
LESSON MASTER 5-2

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective C

1. Refer to $\triangle ABC$ at the right. Find a to the nearest tenth.
29.2

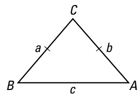


In 2 and 3, consider $\triangle DEF$ where $DE = 38$, $EF = 48$, and $DE = 70$. Find the measure of the given angle to the nearest tenth of a degree.

2. $\angle D$ 40.6°
3. $\angle F$ 108.4°

Properties Objective E

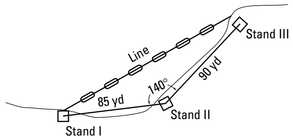
4. Use the Law of Cosines to show that for isosceles $\triangle ABC$ pictured at the right $c = a\sqrt{2 - 2\cos C}$.



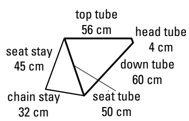
$a = b$, so $c^2 = a^2 + a^2 -$
$2a^2 \cos C = 2a^2 - 2a^2 \cos C =$
$a^2(2 - 2\cos C)$, so $c = a\sqrt{2 - 2\cos C}$.

Uses Objective H

5. Three lifeguard stands are positioned as shown in the diagram below. The lifeguards would like to have a buoyant line that would run from stand I to stand III for nonswimmers. Approximately how long would the line have to be?
≈ 164.5 yd



6. At the right is pictured a typical bicycle "diamond" frame, with dimensions given in centimeters. What is the angle between the seat stay and the chain stay?
≈ 79°



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LESSON MASTER 5-3

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective B

In 1–3, evaluate without a calculator. Give an exact answer in radians.

1. $\cos^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{3}$
2. $\text{Arccos}\left(-\frac{1}{2}\right)$ $\frac{2\pi}{3}$
3. $\cos^{-1} 1$ 0

In 4–6, use a calculator to approximate to the nearest hundredth of a degree.

4. $\cos^{-1}(-0.38)$ 112.33°
5. $\cos^{-1}\left(\frac{1}{9}\right)$ 83.62°
6. Arccos 0.999 2.56°

Skills Objective D

In 7 and 8, find θ, where $0 \le \theta \le \pi$, to the nearest hundredth.

7. $\cos\theta = 0.5$ 1.05
8. $2\cos\theta = 0.5$ 1.32

Properties Objective F

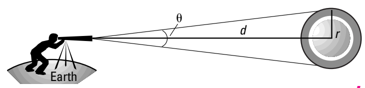
In 9 and 10, the equation for a function is given.
a. State its domain. b. State its range.

9. $f(x) = \cos^{-1} x$
a. $\{x: -1 \le x \le 1\}$
b. $\{y: 0 \le y \le \pi\}$

10. $g(t) = 3\cos^{-1} t$
a. $\{t: -1 \le t \le 1\}$
b. $\{g: 0 \le g \le 3\pi\}$

Uses Objective I

11. As viewed from Earth, any distant astronomical object subtends some angle θ, which depends on the object's radius r and its distance d from Earth.



a. Use the inverse cosine function to write a formula for θ in terms of r and d.
$\theta = 2\cos^{-1}\left(\frac{d}{\sqrt{d^2 + r^2}}\right)$

►

Name

► **LESSON MASTER 5-3** *page 2*

b. The sun has a radius of 432,000 miles and is an average of 92,900,000 miles from Earth. What angle, in minutes, does the sun subtend in the sky?
≈ 32 minutes

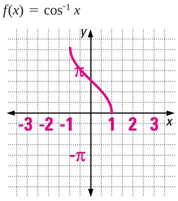
c. The moon has a radius of 1,080 miles and its center is an average of 235,000 miles from Earth's surface. At this distance, what angle (in minutes) does the moon subtend in the sky?
≈ 31 minutes

d. The center of the moon ranges from 217,500 to 248,700 miles from Earth's surface in its elliptical orbit. Explain why when it is farthest from Earth there cannot be a total eclipse of the sun.
The angle subtended by the moon when it is farthest from Earth is about 30 minutes and the apparent size of the moon is not enough to cover the sun.

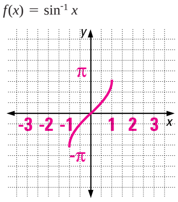
Representations Objective J

In 12 and 13, graph the function on the given set of axes.

12. $f(x) = \cos^{-1} x$



13. $f(x) = \sin^{-1} x$



14. *True or false.* The graph of the inverse cosine function is symmetric to the origin.
False

Name

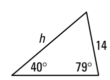
LESSON MASTER 5-4

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective C

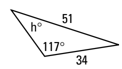
In 1 and 2, use the Law of Sines to find h.

1.



≈ 21.4

2.



≈ 36.4°

In 3 and 4, consider $\triangle ABC$ where $m\angle A = 24°$, $m\angle B = 99°$, and $c = 3.1$.

3. Find the lengths of sides a and b.
$a \approx 1.50$, $b \approx 3.65$

4. Find the area of $\triangle ABC$.
≈ 2.30

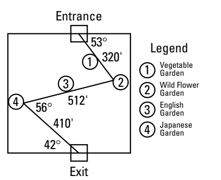
Properties Objective E

5. List three conditions for which the Law of Sines yields a unique solution.
Angle Side Angle; Angle Angle Side; Side Side Angle

6. In $\triangle PRL$, $PR = 3RL$. Find $\frac{\sin L}{\sin P}$.
$\frac{1}{3}$

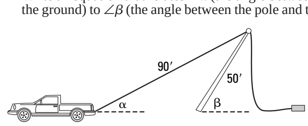
Uses Objective H

7. Suppose a botanical garden is laid out in a large square plot as shown at the right. The path through the garden enters and exits exactly in the middle of opposite sides. What are the dimensions of the garden? Round your answer to the nearest foot.
654 ft, 654 ft



Uses Objective I

8. A 50-foot flagpole is lifted into place with a 90-foot rope as shown. Write an equation that relates $\angle\alpha$ (the angle between the rope and the ground) to $\angle\beta$ (the angle between the pole and the ground).
$90 \sin\alpha = 50 \sin\beta$



Name

LESSON MASTER 5-5

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective B

In 1–3, evaluate without a calculator. Give an exact answer in radians.

1. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{3}$
2. $\text{Arcsin}\left(\frac{\sqrt{2}}{2}\right)$ $\frac{\pi}{4}$
3. $\sin^{-1}(-1)$ $\frac{-\pi}{2}$

In 4–6, use a calculator to approximate to the nearest hundredth of a degree.

4. $\sin^{-1}\left(\frac{3}{4}\right)$ 48.59°
5. $\sin^{-1}\left(-\frac{1}{9}\right)$ -6.38°
6. Arcsin 0.81329 54.42°

Skills Objective D

In 7 and 8, find θ, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, to the nearest hundredth.

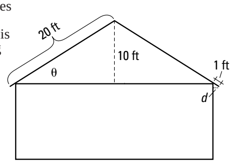
7. $3 \sin \theta = 2$ 0.73
8. $\sin\left(\frac{1}{2}\pi\theta\right) = -0.3120$ -0.20

Properties Objective F

9. *True or false.* The inverse sine and inverse cosine functions have the same domain. True
10. *True or false.* $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \frac{3\pi}{4}$ False

Uses Objective I

11. a. A contractor building a house wishes to calculate the pitch θ of the roof. The contractor knows that the roof is 20 feet long with a 1-foot overhang and a height of 10 feet. Find θ. ≈ 32°



b. The contractor wants to place a window on the side of the house but does not want the overhang to cover the window. What is the shortest distance d that the top of the window can be to the roof? ≈ 0.53 ft

Representations Objective J

12. *True or false.* The graph of the inverse sine function is point symmetric to the origin. True

Name

LESSON MASTER 5-6

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective B

In 1–3, evaluate without a calculator. Give an exact answer in radians.

1. Arctan $\sqrt{3}$ $\frac{\pi}{3}$
2. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\frac{\pi}{6}$
3. $\tan^{-1}\left(\tan\left(\frac{7\pi}{4}\right)\right)$ $\frac{-\pi}{4}$

In 4–6, use a calculator to approximate to the nearest hundredth of a degree.

4. $\tan^{-1}(-0.35)$ -19.29°
5. $\tan^{-1} 50$ 88.85°
6. Arctan 500 89.89°

Skills Objective D

In 7 and 8, -90°< θ < 90°. Find θ to the nearest hundredth.

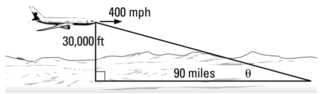
7. $\tan \theta = -5$ -76.89°
8. $\frac{1}{2}\tan(5\theta) = 3$ -55.89°, -19.89°, 16.11°, 52.11°, 88.11°

Properties Objective F

9. What is the domain of the inverse tangent function? all real numbers
10. *True or false.* The inverse sine and inverse tangent functions have the same range. True

Uses Objective I

11. An airplane at a level altitude of 30,000 ft is flying at a velocity of 400 mi/hr due east. At time $t = 0$, the plane is 90 miles due west of an observer on the ground. Write a formula using the inverse tangent function that gives the airplane's angle of inclination θ relative to the observer as a function of the time t in minutes. Remember 1 mile = 5280 feet. $\tan^{-1}\left(\frac{5.68}{90 - \frac{400t}{60}}\right)$



Representations Objective J

12. *True or false.* The graph of the inverse tangent function is symmetric to the y-axis. False
13. State equations for the asymptotes of the graph of the inverse tangent function. $y = \frac{\pi}{2}, y - -\frac{\pi}{2}$

Name

LESSON MASTER 5-7

Questions on SPUR Objectives
See pages 364–367 for objectives.

Skills Objective D

In 1–3, solve, given that 0° ≤ θ ≤ 360°. Give your solution(s) to the nearest hundredth of a degree.

1. $\cos\theta - 0.15 = 0$ 81.37°, 278.63°
2. $4 \sin \theta = \frac{1}{2}$ 7.18°, 172.82°
3. $3 \sin \theta + 2 = -0.5$ 236.44°, 303.56°

In 4 and 5, find the least positive solution in radians.

4. $\frac{1}{\sin\theta} = \frac{7}{6}$ 1.03
5. $\tan^2\theta = -\tan\theta + 6$ 1.12 or -1.25

In 6–8, describe the general solution in radians.

6. $0.2 \cos\left(\frac{1}{2}\theta\right) + 0.2 = 0.4$ 4πk, for all integers k
7. $\sin(5\theta) = 0.75$ ≈ 0.17 + $\frac{2\pi}{5}$ or ≈ 0.4587 + $\frac{2\pi k}{5}$, for all integers k
8. $9\cos^2\theta - 12\cos\theta + 4 = 0$ ≈ 0.84 + 2πk or ≈ 5.44 + 2πk, for all integers k

In 9 and 10, describe the general solution in degrees.

9. $6 \tan(\pi\theta) = 36$ θ = 25.64 + 57.30n, for all integers n
10. $\sin^2\theta - \cos^2\theta = 0$ θ = 45 + 180n, for all integers n

Uses Objective I

11. Suppose a surface wave oscillates according to the equation $h = 1.3\cos\left(\frac{\pi}{6}(t - 4)\right)$, where h is the height of the wave in meters and t is the time in seconds.
   a. Solve this equation for t. $t = \frac{6}{\pi}\cos^{-1}\left(\frac{h}{1.3}\right) + 4$
   b. Find the first two times when the height of the wave is 1 m. ≈ 2.68s and ≈ 5.32s
   c. What is the period of the wave? 12 seconds
   d. Find the general solution for when the height of the wave is 1 m. t ≈ 2.68 + 12n or ≈ 5.32 + 12n, for all positive integers n

Name

LESSON MASTER 6-1

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective A

In 1–4, evaluate without a calculator.

1. $64^{\frac{1}{3}}$ 4
2. $36^{\frac{1}{2}}$ 6
3. $\sqrt[3]{-343}$ -7
4. $\sqrt[4]{16}$ 2

In 5 and 6, rewrite each using a radical sign.

5. $(x^2)^{\frac{1}{4}}$ $\sqrt{x}$
6. $m^{\frac{1}{6}}$ $\sqrt[6]{m}$

In 7–10, rewrite without a radical sign. Assume $w > 0$.

7. $\sqrt[9]{w}$ $w^{\frac{1}{9}}$
8. $\sqrt{3w^2}$ $(3w^3)^{\frac{1}{2}}$
9. $\sqrt[3]{\sqrt{w + 5}}$ $(w + 5)^{\frac{1}{6}}$
10. $\sqrt[4]{16w^2}$ $2w^{\frac{1}{2}}$
11. Order the following from least to greatest: $7^{\frac{1}{3}}, 3^{\frac{1}{7}}, \frac{3}{7}, 3^7, 7^3$. $\frac{3}{7}, 3^{\frac{1}{7}}, 7^{\frac{1}{3}}, 7^3, 3^7$

Properties Objective D

12. *Multiple choice.* For which one of the following is the function g with $g(x) = \sqrt[n]{x}$ not defined? b
   (a) $x \ge 0$, n even, $n > 2$
   (b) $x < 0$, n even, $n > 2$
   (c) $x \ge 0$, n odd, $n > 1$
   (d) $x < 0$, n odd, $n > 1$
13. Give the domain and the range of the rational power function f, where $f(x) = x^{\frac{1}{4}}$. domain: {x: $x \ge 0$} range: {y: $y \ge 0$}

Uses Objective F

14. A regular tetrahedron is a pyramid with four equal edges of length s. Its base is an equilateral triangle with area $B = \frac{\sqrt{3}}{4}s^2$ and whose height $h = \frac{\sqrt{6}}{3}s$.
   a. Give a formula for the volume V of a regular tetrahedron in terms of edge length. Recall that the volume of any pyramid is given by the formula $V = \frac{1}{3}Bh$. $V = \frac{\sqrt{2}}{12}s^3$
   b. Give a formula for the length s of a regular tetrahedron's edge in terms of its volume V. $s = \sqrt[3]{6\sqrt{2}V}$

Home

Name

▶ **LESSON MASTER 6-1** *page 2*

Representations Objective I

15. Graph $f(x) = x^{\frac{1}{4}}$, $g(x) = x^{\frac{1}{3}}$, and $h(x) = x^{\frac{1}{2}}$ on the same window of an automatic grapher.

a. For what, if any, value(s) of x is $f(x) = g(x) = h(x)$? **$x = 0$**

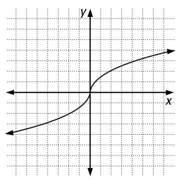
b. For what, if any, value(s) of x is $f(x) > g(x) > h(x)$? **$0 < x < 1$**

c. For what, if any, value(s) of x is $h(x) > g(x) > f(x)$? **$x > 1$**

d. For what, if any, value(s) of x is $g(x) > f(x) > h(x)$? **none**

Representations Objective J

16. The graph at the right is of the function defined by $f(x) = \sqrt[n]{x}$. Is n even or odd? Justify your answer.



odd; the function $f(x) = \sqrt[n]{x}$ is not defined for negative values of x.

Name

LESSON MASTER 6-2

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective A

In 1 and 2, evaluate without a calculator.

1. $16^{\frac{3}{2}}$ **64**

2. $64^{-\frac{2}{3}}$ **$\frac{1}{16}$**

In 3 and 4, rewrite the expression without a radical sign. Assume all variables are positive.

3. $\sqrt[3]{jk^5}$ **$j^{\frac{1}{3}}k^{\frac{5}{3}}$**

4. $\sqrt[4]{a^4b^5}$ **$ab^{\frac{5}{4}}$**

In 5 and 6, rewrite with a radical sign. Assume all variables are positive.

5. $a^{\frac{1}{6}}b^{\frac{7}{6}}$ **$\sqrt[6]{ab^7}$**

6. $c^{-\frac{2}{3}}b^{\frac{3}{4}}$ **$\sqrt[3]{\frac{1}{c^2}} \cdot \sqrt[4]{b^3}$**

Properties Objective D

7. Let f be a rational power function, where $f(x) = x^{-\frac{2}{7}}$.

a. State the domain and the range of f. **domain: $\{x: x > 0\}$ range: $\{y: y > 0\}$**

b. Find an equation for the inverse of f. **$f^{-1}(x) = x^{-\frac{7}{2}}$**

c. State the domain and the range for the inverse of f. **domain: $\{x: x > 0\}$ range: $\{y: y > 0\}$**

d. Is the inverse of f a function? **Yes**

Uses Objective F

8. The weight W of a steel ball bearing varies directly as the cube of the bearing's radius r according to the formula $W = \frac{4}{3}\pi\rho r^3$, where ρ is the density of steel. The surface area A of a bearing varies directly as the square of its radius, because $A = 4\pi r^2$.

a. Express the weight W of a bearing in terms of its surface area A. **$W = \frac{\rho A\sqrt{A}}{6\sqrt{\pi}}$**

b. Express a bearing's surface area A in terms of its weight W. **$A = \left(\frac{6\pi^{\frac{1}{2}}W}{\rho}\right)^{\frac{2}{3}}$**

c. For steel, $\rho \approx 7.8$ g/cm^3. What is the surface area of a bearing weighing 0.26 g? **≈ 0.5 cm^2**

d. What does the exponent in your answer to part b signify?

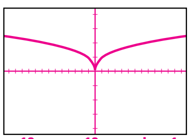
The exponent $\frac{2}{3}$ signifies the change in dimension from surface area to volume.

Name

▶ **LESSON MASTER 6-2** *page 2*

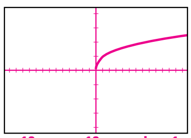
Representations Objective I

9. a. Use an automatic grapher to graph $y = (x^2)^{\frac{1}{6}}$ for $-10 \le x \le 10$.



$-10 \le x \le 10$, x-scale = 1
$-3 \le y \le 3$, y-scale = 1

b. On a new window, graph $y = (x^{\frac{1}{6}})^2$ for $-10 \le x \le 10$.



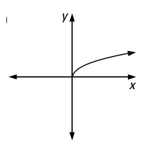
$-10 \le x \le 10$, x-scale = 1
$-3 \le y \le 3$, y-scale = 1

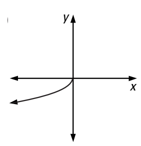
c. Use your results to parts a and b and the Power of a Power Property to explain why negative bases are not used with rational exponents.

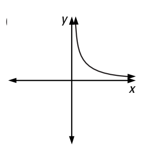
By the Power of a Power Property, $(x^2)^{\frac{1}{6}} = x^{\frac{1}{3}} = (x^{\frac{1}{6}})^2$. But, the graphs in parts a and b are not identical for $x < 0$, so $x^{\frac{1}{3}}$ is defined only for $x \ge 0$.

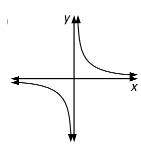
Representations Objective J

10. *Multiple choice.* Which is a graph of the equation $y = x^a$, where $-1 < a < 0$? **c**









Name

LESSON MASTER 6-3

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective C

In 1–6, evaluate without using a calculator.

1. $\log(1 \times 10^7)$ **7**

2. $\log_5 125$ **3**

3. $\log_{13} \frac{1}{169}$ **-2**

4. $\log_8 0.125$ **-1**

Properties Objective D

In 5–7, state the inverse of the function with the given equation.

5. $f(x) = 10 \log x$ **$f^{-1}(x) = 10^{\frac{x}{10}}$**

6. $g(x) = 2^x$ **$g^{-1}(x) = \log_2 x$**

7. $h(x) = \log_5 x$ **$h^{-1}(x) = 5^x$**

Representations Objective I

8. Let f and g be the functions defined by $f(x) = \log x$ and $g(x) = 10^x$, respectively. On separate windows of an automatic grapher, graph $y = f(g(x))$ and $y = g(f(x))$ for $-10 \le x \le 10$.

a. What do your graphs tell you about the relationship between the functions f and g?

f and g are inverses.

b. Explain why the two graphs are different.

g is defined for negative values of x, but f is not.

Representations Objective J

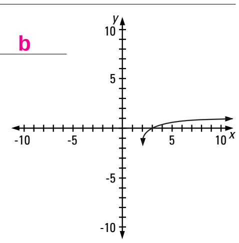
9. *Multiple choice.* Which equation best represents the graph at the right? **b**

(a) $y = \log x$

(b) $y = \log (x - 2)$

(c) $y = \log 2x$

(d) $y = \log_2 x$



Name

LESSON MASTER 6-4

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective C

In 1–3, evaluate without using a calculator.

1. $\ln e^{1.7652}$ **1.7652**
2. $e^{\ln e}$ ***e***
3. $\ln e^{-3}$ **-3**

In 4–6, use a calculator to evaluate to the nearest thousandth.

4. $\ln \pi$ **1.145**
5. $\ln 10$ **2.303**
6. $\ln 0.5$ **-0.693**

Properties Objective D

7. *True or false.* For all $x \geq 1$, $\log_2 x \geq \ln x \geq \log_3 x$. **True**

8. What is the domain of the function h, where $h(x) = \ln\left(\frac{1}{x-1}\right)$? **$\{x: x > 1\}$**

Uses Objective G

9. Two investment plans each have annual interest rates of 8.275%. One plan compounds daily, while the other compounds continuously. If \$5,000 is invested in each plan for a period of 10 years, how much more interest will accrue to the plan that compounds continuously? **\$1.07**

10. The altitude above sea level h (in feet) as a function of barometric pressure p (in lb/in²) can be approximated by the formula $h = -28{,}300 \ln\left(\frac{p}{14.7}\right)$. What is the altitude of Santa Fe, New Mexico, if the average barometric pressure there is about 11.5 lb/in²? **≈ 6948 ft**

Representations Objective I

11. Use an automatic grapher to graph $f(x) = \frac{\ln x}{\ln 10}$ and $g(x) = \log x$.
    a. How do the graphs compare? **The graphs are identical.**
    b. Explain what this means in terms of the relationship between f and g. **The functions are equal.**

Representations Objective J

In 12 and 13, consider the functions f and g, where $f(x) = \ln x$ and $g(x) = \log x$. *True or false.*

12. The graphs of f and g have the same vertical asymptote. **True**

13. The graphs of f and g have the same x-intercept. **True**

65

Name

LESSON MASTER 6-5

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective C

In 1–6, evaluate without using a calculator.

1. $\log_{27} 3$ **$\frac{1}{3}$**
2. $\log (1{,}000{,}000)^{\frac{1}{6}}$ **1**
3. $\log_9 \left(\frac{1}{3}\right)$ **$-\frac{1}{2}$**
4. $\ln (e^{13} \cdot e^{\frac{31}{32}})$ **$13\frac{31}{32}$**
5. $-\frac{1}{3} \log_6 36^{\frac{3}{4}}$ **$-\frac{1}{2}$**
6. $2 \log_{0.5} 8 - 4 \log_{0.5} 2$ **-2**

Properties Objective D

7. What property of logarithms follows from the Zero Exponent Theorem?
   Logarithm of 1 Theorem: $\log_b 1 = 0$ for any nonzero base b

Properties Objective E

In 8–10, use the fact that $\log_{13} 6 \approx 0.6986$ and $\log_{13} 3 \approx 0.4283$ to evaluate the expression.

8. $\log_{13} 2$ **≈ 0.2703**
9. $\log_{13} 1458$ **≈ 2.8401**
10. $\log_{13} \frac{1}{18}$ **≈ 1.1269**

In 11–13, rewrite as a single logarithm.

11. $\ln y^2 + \ln y^3$ **$\ln y^5$**
12. $\ln 2y - \ln 3y$ **$\ln\frac{2}{3}$**
13. $\ln 2^y + \ln 3^y$ **$\ln 6^y$**

14. Estimate 5^{2000}. **$\approx 8.71 \times 10^{1397}$**

15. Recall that $n!$ (n factorial) is the product of the integers from 1 to n. Prove, for any base b, $\sum_{x=1}^{n} \log_b x = \log_b (n!)$.
    By the Logarithm of a Product Theorem,
    $\log_b(n!) = \log_b(1 \cdot 2 \cdot 3 \cdot \ldots \cdot n)$ and
    $\sum_{x=1}^{n} \log_b x = \log_b 1 + \log_b 2 + \log_b 3 + \ldots +$
    $\log_b(n-1) + \log_b n =$
    $\log_b(1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n)$.

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Name

LESSON MASTER 6-6

Questions on SPUR Objectives
See pages 420–423 for objectives.

Skills Objective B

In 1–4, solve to the nearest thousandth.

1. $e^x = 6$ **$x = 1.792$**
2. $7^m = 14$ **$m = 1.356$**
3. $17(4.5)^n = 90$ **$n = 1.108$**
4. $10(0.5)^x = 0.1$ **$x = 6.644$**

Skills Objective C

In 5–8, evaluate to the nearest thousandth.

5. $\log_5 100$ **2.861**
6. $\log_2\left(\frac{2}{7}\right)$ **-1.807**
7. $\log_{\sqrt{3}} 7$ **3.542**
8. $\log_{\pi} e$ **0.874**

Properties Objective E

In 9–10, *true or false*. Assume all variables are positive.

9. $\log_a b \cdot \log_b a = 1$ **True**
10. $\frac{\log_7 x}{\ln x} = \ln 7$ **False**

Uses Objective G

11. A sum of money is deposited in a savings account with an annual interest rate of 3.75%. How many days does it take for the initial deposit to double in value if the compounding occurs in the given manner?
    a. monthly **≈ 6757 days**
    b. continuously **≈ 6747 days**

12. A radioactive isotope has a half-life of 6 days. What percent of the original isotope will be left after 10 weeks? **≈ 0.03%**

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Name

LESSON MASTER 6-7

Questions on SPUR Objectives
See pages 420–423 for objectives.

Properties Objective E

1. If $J = 7e^{5t}$, write $\ln J$ as a linear function of t. **$\ln J = 5t + \ln 7$**

2. If $y = kx^{\frac{5}{2}}$, write $\log y$ as a linear function of $\log x$. **$\log y = \frac{5}{2}\log x + \log k$**

3. A line of best fit for $V^{-\frac{2}{3}}$ in terms of M is $V^{-\frac{2}{3}} = 9.2M + 7.3$. Rewrite the equation to give V as a function of M. **$V = (9.2M + 7.3)^{-\frac{3}{2}}$**

Uses Objective G

4. According to NASA, the number N of Earth-crossing asteroids, that is, those asteroids which have the potential to impact Earth, with a diameter greater than D (in meters) is given by the formula $\log N = b \log D + \log k$, where b and k are constants. There are approximately 150 million Earth-crossing asteroids with a diameter greater than 10 m and 320,000 Earth-crossing asteroids with a diameter greater than 100 m.
   a. Find b and $\log k$. **$b \approx -2.671$; $\log k \approx 10.847$**
   b. Rewrite the formula to give N as a power function in terms of D. **$N \approx k \cdot D^b$**
   c. Estimate the number of Earth-crossing asteroids with diameters between 10 m and 15 m. **≈ 99,200,000**

Uses Objective H

5. The manager of a toy company analyzes the production costs for the company's newest stuffed animal. In the table below are the costs C of producing a given number of units u of the toy.

Units (u)	250	500	750	1000	1250
Production Cost (C)	\$68	\$103	\$150	\$212	\$314

   a. *Multiple choice.* If C is known to be an exponential function of u, which of the following transformations will best linearize the data? **b**
      (a) $(u, C) \rightarrow (\sqrt{u}, C)$ (b) $(u, C) \rightarrow (u, \ln C)$
      (c) $(u, C) \rightarrow (\ln u, \ln C)$ (d) $(u, C) \rightarrow (\log u, C)$
   b. Find the line of best fit using your answer to part a. **$\ln C \approx 0.00151 + 3.86$**
   c. Rewrite your equation in part b to give C as a function of u. **$C \approx 47.45(1.00151)^u$**
   d. To the nearest dollar, how much should it cost to produce 1100 units of the toy? **≈ \$251**

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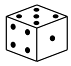
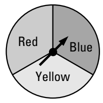
LESSON MASTER 7-1

Questions on SPUR Objectives
See pages 482–485 for objectives.

Skills Objective A

In 1–3, consider an experiment in which a 6-sided die is rolled and a spinner with 3 colors—red, blue, and yellow—is spun.

1. Give the experiment's sample space.
S = {1R, 2R, 3R, 4R, 5R, 6R, 1B, 2B, 3B, 4B, 5B, 6B, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y}

2. List the outcomes in the event "the number rolled is even."
{2R, 2B, 2Y, 4R, 4B, 4Y, 6R, 6B, 6Y}

3. List the outcomes in the event "the number rolled is even and the color spun is yellow."
{2Y, 4Y, 6Y}

Skills Objective B

In 4 and 5, use your results from Exercises 1–3. Find the probability of the given event. Assume the die is fair and the spinner is equally likely to land on any of the three colors.

4. rolling an even number $\frac{1}{2}$

5. rolling an even number and spinning the color yellow $\frac{1}{6}$

Properties Objective E

In 6 and 7, consider an experiment whose sample space S is a set of k equally likely outcomes and E is a subset of S.

6. What is the probability that any given outcome will occur? $\frac{1}{k}$

7. If $P(E) = \frac{1}{3}$, how many outcomes are in E? $\frac{k}{3}$

Uses Objective H

8. Five people—A, B, C, D, and E—apply for two editorial positions at a publishing house. Applicants A and B are left-handed, while C, D, and E are right-handed. Since all applicants are well qualified, they are each equally likely to be hired.

a. Give the sample space of the hired applicants.
S = {AB, AC, AD, AE, BC, BD, BE, CD, CE, DE}

b. Find the probability that one left-handed person and one right-handed person will be hired. $\frac{3}{5}$, or 0.6

Name

LESSON MASTER 7-2

Questions on SPUR Objectives
See pages 482–485 for objectives.

Skills Objective B

In 1–5, consider rolling a single fair die. Let A be the event {1, 2, 3} and B be the event {2, 4, 5, 6}. Suppose $\overline{B}$ is the complement of event B. Find the probability.

1. $P(A)$ $\frac{1}{2}$
2. $P(A \cup B)$ 1
3. $P(A \cap B)$ $\frac{1}{6}$
4. $P(\overline{B})$ $\frac{1}{3}$
5. $P(A \cap \overline{B})$ $\frac{1}{3}$

Properties Objective E

In 6–8, let A and B be two events in a finite sample space. *True or false.*

6. If $P(A) + P(B) = 1$, then A and B must be complementary. False

7. For all A and B, $P(A \cap B) + P(A \cup B) = P(A) + P(B)$. True

8. If A and B are complementary, then A and B must also be mutually exclusive. True

Properties Objective F

In 9–12, *multiple choice.* Consider rolling a fair 6-sided die. Determine whether events A and B are (a) complementary, (b) mutually exclusive but not complementary, or (c) neither mutually exclusive nor complementary.

9. A: rolling a number greater than 2
B: rolling a number less than 2
b

10. A: rolling a number greater than 3
B: rolling a number less than 6
c

11. A: rolling a number 4 or greater
B: rolling a number 4 or less
c

12. A: rolling a number 5 or greater
B: rolling a number less than 5
a

Uses Objective H

13. A pre-election poll suggests that the probability that the Republican candidate will win is 0.42, and the probability that the Democratic candidate will win is 0.47. Find the probability that a third-party candidate will win the election. 0.11

14. A survey is conducted to determine the number of households that recycle in a certain city. It is found that 32% recycle aluminum cans, 47% recycle newspaper, and 28% recycle both aluminum cans and newspaper. What is the probability that a household recycles at least one of these resources? 0.51

15. The estimated probability in the year 2000 that a randomly selected U.S. resident is over the age of 18 is about 74.2% and the probability that the person is under the age of 24 is 35.3%. What is the probability that the person is between the ages of 18 and 24? 0.095

Name

LESSON MASTER 7-3

Questions on SPUR Objectives
See pages 482–485 for objectives.

Skills Objective B

1. What is the probability of flipping 7 fair coins and getting 7 tails? $\frac{1}{128} \approx 0.008$

Skills Objective C

2. Suppose a bag contains 5 balls numbered 1 through 5.

a. In how many ways can you select 5 balls from the bag if you replace the ball after each selection? 3125 ways

b. In how many ways can you select 5 balls from the bag if there is no replacement after selection? 120 ways

c. List one arrangement that you could get from the selection process in part a that you could not get from the selection process in part b. Sample: 3, 4, 2, 3, 5

3. An octal (base 8) number is a number whose digits can be any of the numbers 0 through 7. In each case, determine how many 4-digit octal numbers are possible.

a. The first digit can be 0. 4096 numbers

b. The first digit cannot be 0. 3584 numbers

Skills Objective D

In 4–6, evaluate without using a calculator.

4. 1! 1

5. $\frac{1}{3!}$ $\frac{1}{6}$

6. $\frac{7958!}{7957!}$ 7958

7. Evaluate $\frac{117!}{114!}$. 1,560,780

Uses Objective I

8. The first row of a football team picture includes the eleven starters in the offensive unit, the place kicker, and the coach. The coach stands in the center with six players to his right and six to his left. In how many different ways could the photographer arrange the group with the coach in the center? 479,001,600 ways

9. Each strand of human DNA consists of millions of nucleotides linked together to form a chain. Each nucleotide contains one of four nitrogenous bases—adenine, guanine, thymine, or cytosine. Sequences of these bases determine our genetic code. How many different possible sequences are there for a segment of DNA 100 nucleotides long? $4^{100} \approx 1.607 \times 10^{60}$ sequences

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LESSON MASTER 7-4

Questions on SPUR Objectives
See pages 482–485 for objectives.

Skills Objective C

1. List all the permutations of all the letters M, I, L, and K.

IKLM IMKL KLIM LIKM LMIK MKIL
IKML IMLK KLMI LIMK LMKI MKLI
ILKM KILM KMIL LKIM MILK MLIK
ILMK KIML KMLI LKMI MIKL MLKI

2. How many permutations are there of all the letters in the English alphabet? $\approx 4.0329 \times 10^{26}$

3. How many permutations consisting of 3 letters each can be formed from the letters of GRAPHIC? 210

Skills Objective D

In 4–6, evaluate.

4. ${}_8P_7$ 40,320

5. ${}_{100}P_2$ 9900

6. ${}_{498}P_1$ 498

7. *True or false.* ${}_{10}P_5 = {}_5P_{10}$ False

Skills Objective G

In 8–13, solve.

8. $\frac{x!}{8!} = 90$ $x = 10$

9. $\frac{6}{x!} = \frac{1}{120}$ $x = 6$

10. $\frac{x!}{(x+1)!} = 1$ $x = 0$

11. $\frac{x!}{x} = (x-1)!$ x an integer > 0

12. ${}_nP_8 = 13 \cdot {}_nP_7$ $n = 20$

13. ${}_nP_6 = 90 \cdot {}_nP_4$ $n = 14$

Uses Objective I

14. A researcher conducted an opinion poll in which he asked people to rank their top 5 preferences for mayor from a list of 20 potential candidates. How many such rankings are possible? 1,860,480 rankings

15. How many ways can 120 passengers be seated in an airplane with 150 seats? $\approx 2.154 \times 10^{230}$ ways

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LESSON MASTER 7-5

Questions on SPUR Objectives
See pages 482–485 for objectives.

Properties Objective E

1. Prove: If A and B are independent events in a finite sample space, then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$.
Sample: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
A and B are independent, $P(A \cap B) = P(A)P(B)$;
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$.

2. *True or false.* Mutually exclusive events are never independent. Justify your answer. False:
sample: If at least one event is $\varnothing$, then the events are mutually exclusive and independent;
$A \cap \varnothing = \varnothing$; $P(A \cap \varnothing) = P(\varnothing) = 0$ and
$P(A)P(\varnothing) = 0 \cdot P(A) = 0$.

Properties Objective F

In 5–7, consider the experiment of rolling two fair dice. Determine whether or not the two given events A and B are *independent* or *dependent*. Use the sample space shown on page 427 of your textbook if necessary.

5. A: rolling a sum of 7
B: rolling an even number on the second die — independent

6. A: rolling doubles
B: rolling a 3 on the first die — independent

7. A: rolling doubles
B: rolling a 3 on either die — dependent

Uses Objective H

8. During the 1996–97 season, Michael Jordan made 83.3% of the free throws he attempted. Assume independence of free-throw attempts and find the probability that MJ would
   a. make two of two free throws. ≈ 0.694
   b. miss two of two free throws. ≈ 0.028
   c. make at least one of two free throws. ≈ 0.972

9. Suppose a car manufacturer knows that the probability that a defect will cause an accident is 0.01 and that the probability an accident will be caused by human error is 0.50. If the probability that an accident is caused by human error *and* a defect is 0.05, are the events independent? Justify your answer. No; sample explanation: $P(A) = 0.01$ and $P(B) = 0.5$, so $P(A)P(B) = 0.005$ but $P(A \cap B) = 0.05$.

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LESSON MASTER 7-6

Questions on SPUR Objectives
See pages 482–485 for objectives.

Representations Objective L

1. *Multiple choice.* In which table is P *not* a probability distribution? c

(a)

x	1	2	3
$P(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

(b)

x	1	2	3
$P(x)$	$\frac{1}{2}$	$\frac{1}{2}$	0

(c)

x	1	2	3
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$

(d)

x	1	2	3
$P(x)$	$\frac{3}{8}$	$\frac{7}{24}$	$\frac{1}{3}$

2. A researcher collects the following data about the incubation time of a certain disease.

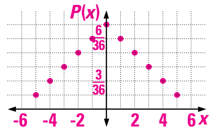
x = Number of days	1	2	3	4	5	6	7
$P(x)$	$\frac{1}{14}$	$\frac{3}{28}$	$\frac{3}{14}$	$\frac{1}{28}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

   a. What is the random variable? incubation time
   b. Find the mean incubation time. 4.5 days

3. Consider the experiment of rolling two fair 6-sided dice.
   a. Construct a probability distribution table in which the value of the random variable is calculated by subtracting the value showing on the second die from the value showing on the first die.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

   b. Graph the distribution in part a as a scatterplot.



   c. Find the expected value of the probability distribution. 0

Name

LESSON MASTER 7-7

Questions on SPUR Objectives
See pages 482–485 for objectives.

Vocabulary

1. What is a Monte Carlo method? Sample: a method of using repeated random trials of an experiment to find relative frequencies and estimate probabilities.

Uses Objective J

2. Suppose a basketball player makes free-throw shots $\frac{5}{6}$ of the time.
   a. Design a simulation using a fair 6-sided die which will estimate the probability that the basketball player will make at least 2 free throws in 3 attempts. Be sure to define a trial.
   Sample: Let rolling a 1–5 be a free throw made and rolling a 6 a free throw missed. A trial is 3 rolls of the die. Calculate the % of trials in which 2 or 3 of the 3 rolls result in 1, 2, 3, 4, or 5.
   b. Use 25 trials to calculate the estimated probability of making at least 2 of 3 free throws. Answers will vary.
   c. The actual probability of making at least 2 free throws in 3 attempts is about 0.93. How could you increase the accuracy of your estimation?
   Sample: Increase the number of trials

3. A conservationist is attempting to repopulate a lake with trout. Each fish released has a 0.4 chance of surviving. Five fish are released at a time.
   a. Use the Table of Random Numbers in the Appendix of your textbook to design a simulation to estimate the probability that all 5 fish in a release will survive. Be sure to define a trial.
   Sample: Let 0, 1, 2, 3 represent fish living and 4, 5, 6, 7, 8, 9 represent fish dying. Choose a random spot in the table and move in some direction. A trial is to read five consecutive digits. Calculate the percent of trials in which all five digits are numbers from 0 to 3.
   b. Use 50 trials to estimate the probability that all 5 fish in a release will survive. (The actual probability is close to 0.01.) Answers will vary.

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LESSON MASTER 7-8

Questions on SPUR Objectives
See pages 482–485 for objectives.

Uses Objective K

1. A college has places for 225 entering freshmen. The dean of admissions has statistics that show that about 32% of those students offered admission will choose to not enroll. To compensate for this, the dean offers admission to 310 students.
   a. Design a simulation using a computer or calculator to estimate the probability that more than 225 students will try to enroll. Sample: Generate 310 random numbers between 0 and 1. Numbers > 0.32 represent students' enrolling; numbers ≤ 0.32 represent students' not enrolling. A trial is 310 numbers. Perform 100 trials, and calculate the percent of trials in which the number of students enrolling is > 225.
   b. Run your simulation and record the estimated probability. Sample: about 2%

2. Estimate the area under the curve $y = x^3$ between $x = 0$ and $x = 1$. Sample: ≈ 0.25

3. Estimate the area under the graph of $y = \cos x$ between 0 and $\frac{\pi}{4}$. Sample: ≈ 0.71

4. A pharmaceutical company reported that 83% of people afflicted with the flu recover within one week when given a certain antibiotic. A doctor is currently treating 63 of his patients with this drug.
   a. Design a simulation using a calculator or computer to estimate the average number of the doctor's patients who will recover within one week of taking the antibiotic.
   Sample: Generate 63 random numbers between 0 and 1. Numbers > 0.17 represent patients' recovering; numbers ≤ 0.17 represent patients' not recovering. A trial is 63 numbers. Perform 50 trials and calculate the average of the numbers from the trials.
   b. Run your simulation and record the estimate. Sample: ≈ 52

Name

LESSON MASTER 8-1

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective A

In 1–6, a sequence is described. a. Identify the formula as *recursive* (R) or *explicit* (E). b. Find the first four terms. c. Find the 10th term.

1. $\begin{cases} r_1 = -4 \\ r_n = -3r_{n-1}, \text{ for all integers } n > 1. \end{cases}$
   a. R b. -4, 12, -36, 108 c. 78, 732

2. $c_n = -n + 7$
   a. E b. 6, 5, 4, 3 c. -3

3. $\begin{cases} b_1 = 0.5 \\ b_n = 2b_{n-1} - 0.25, \text{ for all integers } n \geq 2. \end{cases}$
   a. R b. 0.5, 0.75, 1.25, 2.25 c. 128

4. $t_n = -6n^3 + 27n^2 - 48n + 23$
   a. E b. -4, -13, -40, -121 c. -3757

5. $\begin{cases} j_1 = 3 \\ j_n = (j_{n-1})^2, \text{ for all integers } n \geq 2. \end{cases}$
   a. R b. 3, 9, 81, 6561 c. 3^{512}

6. $k_n = 12 - 3(n - 1)$
   a. E b. 12, 9, 6, 3 c. -15

Skills Objective B

7. Write an explicit formula for the sequence defined in Exercise 1.
   $r_n = -4(-3)^{n-1}$

8. Write a recursive formula for the sequence defined in Exercise 2.
   $\begin{cases} c_1 = 6 \\ c_n = c_{n-1} - 1, \text{ for } n > 1. \end{cases}$

9. Write explicit and recursive formulas for the geometric sequence $\frac{2}{3}, 1, \frac{3}{2}, \frac{9}{4}, \ldots$. Samples are given.
   $g_n = \left(\frac{3}{2}\right)^{n-2}$
   $\begin{cases} g_1 = \frac{2}{3} \\ g_{n-1} = \frac{3}{2}g_{n-1}, \text{ for } n \geq 2. \end{cases}$

Name

▶ **LESSON MASTER 8-1** *page 2*

Properties Objective E

In 10–15, identify the sequences in Exercises 1–6, respectively, as *arithmetic*, *geometric*, or *neither*.

10. geometric 11. arithmetic
12. neither 13. neither
14. neither 15. arithmetic

Uses Objective I

16. Cynthia is to begin training for a long-distance bicycle ride. Her plan is to ride every day, starting with 25-km rides the first week and increasing her distance by 7 km each subsequent week.
    a. What is the distance of Cynthia's rides in the second week of her training? 32 km
    b. Write a recursive formula for the distance of Cynthia's training rides in the *n*th week. $\begin{cases} d_1 = 25 \\ d_n = d_{n-1} + 7, \text{ for } n \geq 2. \end{cases}$
    c. Write an explicit formula for the distance of Cynthia's training rides in the *n*th week. $d_n = 25 + 7(n - 1)$
    d. In what week will the distance of Cynthia's training rides be 102 km? week 12

17. A particular high-powered personal home computer system costs about \$3,000 new. However, due to rapid advances in technology, it depreciates in value by 35% each year.
    a. How much will the computer be worth in its second year? \$1950
    b. Write a recursive formula for the value of the computer in its *n*th year. $\begin{cases} v_1 = 3000 \\ v_n = 0.65v_{n-1}, \text{ for } n > 1. \end{cases}$
    c. Write an explicit formula for the value of the computer in its *n*th year. $v_n = 3000(0.65)^{n-1}$
    d. In what year will the value of the computer first drop below \$500? year 6

Name

LESSON MASTER 8-2

Questions on SPUR Objectives
See pages 551–553 for objectives.

Properties Objective F

In 1–8, decide whether the sequence described is *convergent* or *divergent*. If it is convergent, give the limit.

1. $-4, -2, -\frac{4}{3}, -1, -\frac{4}{5}, \ldots, -\frac{4}{n}, \ldots$ — convergent; 0
2. $k_n = -1.7$ — convergent; -1.7
3. $\begin{cases} a_1 = -5 \\ a_n = a_{n-1} + 5, \text{ for all integers } n \geq 2. \end{cases}$ — divergent
4. $1.8, 1.9, 1.9\overline{3}, 1.95, \ldots, 2 - \frac{1}{5n}, \ldots$ — convergent; 2
5. $t_n = 2 + \left(\frac{1}{100}\right)(n - 1)$ — divergent
6. $g_n = 2\left(\frac{1}{100}\right)^{n-1}$ — convergent; 0
7. $-\frac{4}{3}, -\frac{13}{6}, -\frac{27}{11}, -\frac{23}{9}, \ldots, \frac{-5n^2 - 3n}{2n^2 + 4}, \ldots$ — convergent; $-\frac{5}{2}$
8. $\begin{cases} z_1 = 6 \\ z_n = \left(-\frac{5}{4}\right)z_{n-1}, \text{ for all integers } n > 1. \end{cases}$ — divergent

In 9–12, let $\lim\limits_{n\to\infty} a_n = 2$, $\lim\limits_{n\to\infty} b_n = 7$, and $\lim\limits_{n\to\infty} c_n = 8$. Find the limit.

9. $\lim\limits_{n\to\infty}(a_n - b_n)$ — -5
10. $\lim\limits_{n\to\infty}(b_n \cdot c_n)$ — 56
11. $\lim\limits_{n\to\infty}(4c_n)$ — 32
12. $\lim\limits_{n\to\infty}\left(\frac{a_n b_n}{c_n}\right)$ — $\frac{7}{4}$, or $1\frac{3}{4}$

13. Use an automatic grapher to graph $y = x^{\frac{1}{x}}$.
    a. Give an equation for the horizontal asymptote of the graph of $y = x^{\frac{1}{x}}$. $y = 1$
    b. Find $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$ 1

Name

LESSON MASTER 8-3

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective C

In 1–6, evaluate the arithmetic series.

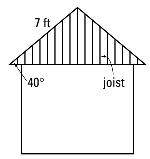
1. $\sum\limits_{n=1}^{6}(2n + 7)$ — 84
2. $\sum\limits_{i=1}^{5}(3i - 6)$ — 15
3. $\sum\limits_{k=1}^{7}\left(-4 - \frac{1}{3}k\right)$ — $-\frac{112}{3}$, or $-37\frac{1}{3}$
4. the sum of the first 100 positive odd integers — 10,000
5. $\sum\limits_{n=1}^{500} 28n$ — 3,507,000
6. $\sum\limits_{i=1}^{200}(16 - 3i)$ — -57,100
7. The sum of the first *k* positive multiples of 7 is 735. Find *k*. — $k = 14$

Uses Objective I

8. Jack Deere made a deal with this father that he would mow the lawn for the entire summer. For the first mowing, he would charge a special introductory rate of \$2, but for each mowing thereafter he would charge 50 cents more than the previous rate. If Jack mowed the lawn a total of 48 times during the summer, how much did he earn in all? \$660

9. To build up endurance, Arnold started an exercise program in which he exercised 30 minutes the first day, 34 minutes the next day, then 38 minutes, 42 minutes, and so on, each day extending his exercise time by 4 minutes. If he continued at this rate, ending at 2 hours, 30 minutes, what was the total time he spent exercising? 2790 min, or 46.5 hr.

10. The roof of a shed is supported by 17 evenly spaced joists, as shown at the right. One side of the roof has a length of 7 ft and makes a 40° angle with the horizontal.



    a. Use Σ-notation to write the series representing the total length of the joists for one side of the roof. $\sum\limits_{n=1}^{8} \frac{7n}{9} \sin 40°$
    b. Find the total length of the joists used for both ends of the shed. ≈ 40.5 ft

Name

LESSON MASTER 8-4

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective C

In 1–6, evaluate the geometric series.

1. $\sum_{n=1}^{5} 12\left(\frac{1}{4}\right)^{n-1}$ $\frac{1023}{64}$, or $15\frac{63}{64}$

2. the sum of the first 8 terms of the sequence $\begin{cases} g_1 = 200 \\ g_n = -3g_{n-1}, \text{ for all integers } n > 1 \end{cases}$ -328,000

3. the sum of the first 20 terms of the geometric sequence with first term -3 and constant ratio $\frac{2}{3}$ $\approx$ -8.997

4. $\sum_{i=1}^{12} 7\left(-\frac{1}{5}\right)^{i-1}$ $\approx$ 5.833

5. $16 + 20 + 25 + \ldots + \frac{3125}{64}$ $\approx$ 180.141

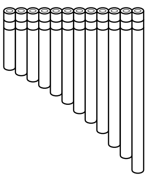
6. $\sum_{n=1}^{10} (x^4)^{n-1}$ $\frac{1 - x^{40}}{1 - x^4}$

7. Find $\sum_{n=20}^{100} \left(\frac{1}{2}\right)^{n-1}$. $\approx 3.815 \times 10^{-6}$

Uses Objective I

8. On her birthday every year, starting at the age of 22 and until retirement, Evelyn plans to deposit $400 into a savings account. Suppose the account will earn 5.5% annual interest.
   a. Using Σ-notation, write the series representing the amount Evelyn will have in the account when she retires at age 65. $\sum_{n=1}^{44} (400(1.055)^{n-1})$
   b. Evaluate the series in part a. $69,429.07
   c. How much of the amount in part b is interest? $51,829.07

9. Fred wants to make a pan flute out of 12 bamboo pipes, similar to the one shown at the right. He wants the shortest pipe to be 8 cm long and each succeeding pipe to be 6% longer than the one before it. If Fred is to get all 12 pieces from a single bamboo stalk, what is the stalk's minimum length? 1.35 meters



Name

LESSON MASTER 8-5

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective G

In 1–6, state whether or not the given infinite geometric series is *convergent* or *divergent*. If the series is convergent, give its sum.

1. $24 + 20 + \frac{50}{3} + \frac{125}{9} + \ldots$ convergent; 144

2. $\sum_{n=1}^{\infty} \left(-\frac{3}{2}\right)^{n-1}$ divergent

3. $325 + \frac{65}{2} + \frac{13}{4} + \frac{13}{40} + \ldots$ convergent; $\frac{3250}{9}$

4. $\sum_{n=1}^{\infty} -6\left(\frac{2}{3}\right)^{n-1}$ convergent; -18

5. $\sum_{n=1}^{\infty} 5(-1)^n$ divergent

6. $\frac{13}{18} - \frac{13}{24} + \frac{13}{32} - \ldots$ convergent; $\frac{26}{63}$

In 7–9, use a computer or calculator to conjecture whether the series is *convergent* or *divergent*. If convergent, give what seems to be its limit.

7. $\sum_{n=1}^{\infty} \frac{1}{n!}$ convergent; $\approx$ 1.718 $(e - 1)$

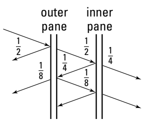
8. $\sum_{i=1}^{\infty} \frac{1}{i^2}$ convergent; $\approx 1.645\left(\frac{\pi^2}{6}\right)$

9. $\sum_{k=1}^{\infty} k^{-\frac{1}{3}}$ divergent

10. Give a convincing argument supporting whether the following series is convergent or divergent: $3 + \frac{3}{2} + 1 + \frac{3}{4} + \frac{3}{5} + \ldots + \frac{3}{n} + \ldots$
Sample: The series is divergent, since it is 3 times the harmonic series which is divergent.

Uses Objective I

11. Consider a window made from two panes of glass, as pictured at the right. If each pane allows 50% of the light to pass through and reflects 50%, what percent of the light hitting the window from the outside is transmitted through to the inside? Remember to consider the light that gets reflected between the two panes as shown. $33\frac{1}{3}\%$



Name

LESSON MASTER 8-6

Questions on SPUR Objectives
See pages 551–553 for objectives.

Vocabulary

In 1–4, evaluate the expression.

1. ${}_{10}C_3$ 120

2. ${}_{10}C_7$ 120

3. $\binom{13}{0}$ 1

4. $\binom{24}{23}$ 24

Uses Objective J

5. Dr. Zweistein has 14 students in a physics class. Over the course of the year, Dr. Zweistein would like to arrange the lab groups so that every student has the opportunity to work with every other student in groups of two. How many different two-person lab groups are there? 91 groups

6. The Rails Club, a group of 25 train fanatics, is to choose four of their members to be on the Board of Directors.
   a. How many different possible boards could the Rails choose? 12,650 bds.
   b. How many different possible boards could the Rails choose if there are to be a Chair, Vice Chair, Treasurer, and Secretary? 303,600 bds.

7. At a burger specialty restaurant, the toppings options are catsup, mayonnaise, mustard, tomatoes, onions, lettuce, mushrooms, Swiss cheese, cheddar cheese, steak sauce, and guacamole. If you can choose between a 4-oz burger or a 6-oz burger and want a mix of 4 toppings, how many different burgers can you order? 660 burgers

8. For a particular lottery, the winning numbers are selected by a machine that randomly chooses 5 table-tennis balls from among 45, numbered 1 to 45. The lottery pays off if you match 5, 4, or 3 of the numbers.
   a. How many different winning number combinations are there? 1,221,759
   b. What is the probability that you will match all 5 of the winning numbers? $\frac{1}{1,221,759}$, or $\approx 8.1849 \times 10^{-7}$
   c. What is the probability that you will match exactly 4 of the 5 winning numbers? $\frac{200}{1,221,759}$, or $\approx 1.6370 \times 10^{-4}$

Name

LESSON MASTER 8-7

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective H

In 1 and 2, use the formula for ${}_nC_r$ to prove the given identity. Samples are given.

1. ${}_{n+1}C_1 - {}_nC_1 = 1$, for all positive integers n.

$${}_{n+1}C_1 - {}_nC_1 = \frac{(n+1)!}{(n+1-1)!1!} - \frac{n!}{(n-1)!1!}$$
$$= \frac{(n+1)!}{n!} - \frac{n!}{n-1!} = n + 1 = 1$$

2. $\frac{\binom{n}{r}}{\binom{n}{n-r}} = 1$, for all nonnegative integers r and n, where $r \le n$.

$$\frac{\binom{n}{r}}{\binom{n}{n-r}} = \frac{\binom{n}{r}}{\frac{n!}{(n-(n-r))!(n-r)!}} = \frac{\binom{n}{r}}{\frac{n!}{r!(n-r)}} = \frac{\binom{n}{r}}{\binom{n}{r}} = 1$$

In 3 and 4, find a value of $k \ne 9$ that will make the statement true.

3. ${}_{17}C_9 = {}_{17}C_k$ $k = 8$

4. ${}_{96}C_k = {}_{96}C_9$ $k = 87$

5. Evaluate $\sum_{r=0}^{15} \binom{15}{r}$. $2^{15} = 32,768$

Representations Objective L

6. Describe the location of the term equal to $\binom{n}{r}$ in Pascal's Triangle.
It is the $(r + 1)$st term in the nth row.

In 7–10, provide the identity involving combinations which accounts for the given property of Pascal's Triangle.

7. Each row is symmetric. ${}_nC_r = {}_nC_{n-r}$

8. The first and last term in each row is 1. ${}_nC_0 = {}_nC_n = 1$

9. The sum of the terms in row 9 is 512. $\sum_{r=0}^{n} {}_nC_r = 2^n$

10. The sum of the third and fourth terms of row 5 is the fourth term of row 6. ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$

11. Use the properties of Pascal's Triangle to complete rows 9 and 10.

row 9:	1	9	36	84	126	126	84	36	9	1	
row 10:	1	10	45	120	210	252	210	120	45	10	1

Home

Name

LESSON MASTER 8-8

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective D

1. Use Σ-notation to write the expansion of $(p+q)^{25}$ as a series.
$\sum_{n=0}^{25} {}_{25}C_n\, p^{25-n}q^n$

2. Rewrite $\sum_{r=0}^{12} {}_{12}C_r(2a)^{12-r}(2b)^r$ in the form $(x+y)^n$.
$(2a+3b)^{12}$

In 3–7, expand each binomial.

3. $(3x-y)^3$
$27x^3 - 27x^2y + 9xy^2 - y^3$

4. $(4a+5b)^4$
$256a^4 + 1280a^3b + 2400a^2b^2 + 2000ab^3 + 625b^4$

5. $\left(\frac{1}{4}f - \frac{1}{3}g\right)^3$
$\frac{1}{64}f^3 - \frac{1}{16}f^2g + \frac{1}{12}fg^2 - \frac{1}{27}g^3$

6. $(1-3p)^8$ $6561p^8 - 17{,}496p^7 + 20{,}412p^6 - 13{,}608p^5 + 5670p^4 - 1512p^3 + 252p^2 - 24p + 1$

7. $(x+x^2)^7$ $x^{14} + 7x^{13} + 21x^{12} + 35x^{11} + 35x^{10} + 21x^9 + 7x^8 + x^7$

8. What is the coefficient of a^2b^5 in the expansion of $(a+b)^7$? 21

9. What is the coefficient of x^4y^7 in the expansion of $(2x-3y)^{11}$? -11,547,360

10. What is the middle term in the expansion of $(1+b)^{24}$? $2{,}704{,}156b^{12}$

11. Without using a calculator, evaluate $\sum_{r=0}^{4} {}_4C_r 1.8^{4-r}1.2^r$. 81

Name

LESSON MASTER 8-9

Questions on SPUR Objectives
See pages 551–553 for objectives.

Skills Objective K

1. Basketball free throws are generally considered independent events. During the 1996–97 NBA regular season, Mark Price of the Golden State Warriors had a free-throw percentage of 0.906. If he were to shoot 10 consecutive free throws, what is the probability that he would make exactly 7 of them? ≈ 0.0499

2. Mr. and Mrs. Brown know that the probability that any child born to them will have blue eyes is $\frac{1}{4}$. If they plan to have 4 children, what is the probability that at least two of the children will have blue eyes? ≈ 0.26

3. In the game Yahtzee®, five fair 6-sided dice are rolled.
   a. What is the probability of rolling exactly three1s? ≈ 0.032
   b. What is the probability of rolling exactly three of any number? ≈ 0.032
   c. What is the probability of rolling three, four, or five of a kind? ≈ 0.0355

4. Sparky the Barbeque Man noted that on the average, 3 out of 5 picnickers prefer hamburgers to hot dogs. Find the probability that of 5 picnickers exactly 3 will want burgers. ≈ 0.3456

5. Suppose 8% of new computers malfunction within the first year. A school purchased 15 computers for its lab.
   a. What is the probability that none of the new computers will malfunction within the first year? ≈ 0.286
   b. What is the probability that more than 2 computers will malfunction within the first year? ≈ 0.113

6. For an upcoming quiz, you have studied enough so that you believe you have an 85% chance of answering each question correctly. Suppose there are 5 questions on the quiz. Table numbers are approximate.
   a. Complete the probability distribution table.

x (number of correct answers)	0	1	2	3	4	5
P(*x*)	.000	.002	.024	.138	.392	.444

   b. What is the probability of your earning an 80% or higher on the quiz? ≈ 0.835
   c. What is the probability of your failing the quiz if failing is less than 60% ≈ 0.027

Name

LESSON MASTER 9-1

Questions on SPUR Objectives
See pages 622–625 for objectives.

Properties Objective F

In 1 and 2, a polynomial expression is given. a. State its degree. b. Give its leading coefficient. c. Give its constant term.

1. $-3t^3 + 4t^5 + 4t - 1$ a. 5 b. 4 c. -1

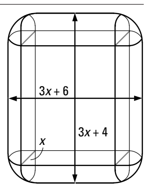
2. $x^5y + x^4y - x^3y^2 + y$ a. 6 b. 1 c. 0

Uses Objective I

3. Jay's company retirement plan has the employee contribute 3% and the employer contribute 2% of the employee's salary at the end of each month to an account which has an annual interest rate r, compounded monthly.
   a. Jay has worked for 7 months and has a monthly salary of S dollars. Write a polynomial expression in x, where x is $1 + \frac{r}{12}$, for the current balance in Jay's retirement account.
   $.05Sx^6 + .05Sx^5 + .05Sx^4 + .05Sx^3 + .05Sx^2 + .05Sx + .05S$
   b. Use Σ-notation to write the expression in part a. $\sum_{i=0}^{6} .05Sx^i$
   c. Give the series in part b as a single fraction. $\frac{.05S(1-x^7)}{1-x}$
   d. Evaluate the expression in part c for a monthly salary of \$3600 and annual interest rate of 3.75%. \$1271.87

Representations Objective J

4. The geometric solid at the right is formed of 4 half-cylinders, 4 quarter-spheres and a rectangular solid It has depth x, height $3x+4$, and width $3x+6$. Use the formulas below to express each quantity as a polynomial in x.



sphere: $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$
cylinder: $V = \pi r^2 h$, $A = 2\pi rh$

   a. the volume of the rectangular solid
   $4x^3 + 20x^2 + 24x$
   b. the volume of the entire geometric solid
   $\left(\frac{7\pi + 24}{6}\right)x^3 + \left(\frac{5\pi + 40}{2}\right)x^2 + 24x$
   c. the surface area of the geometric solid
   $(8 + 5\pi)x^2 + (10\pi + 40)x + 48$

Name

LESSON MASTER 9-2

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective B

1. Consider the polynomial function p with $p(x) = 375x^4 + 400x^3 - 1165x^2 - 966x + 432$.
   a. Complete the table at the right.
   b. Use the table to identify x-interval(s) within which any zeros must lie.
   $-2.0 < x < -1.5$; $-1.5 < x < -1.0$; $0.0 < x < -0.5$; $1.5 < x < 2.0$
   c. Find the greatest zero to the nearest tenth.
   $x = 1.6$

x	*p*(*x*)
-2.5	3964.1875
-2.0	504
-1.5	-191.8125
-1.0	208
-0.5	597.1875
0.0	432
0.5	-268.8125
1.0	-924
1.5	-389.8125
2.0	3040
2.5	11,634.1875

2. Without graphing, find all zeros of the function q with $q(u) = 12u^2 - 13u - 14$.
   $-\frac{2}{3}, \frac{7}{4}$

Properties Objective F

In 3–5, *true or false*. Justify your answer.

3. All polynomial functions have at least one relative extremum.
False; $f(x) = 3x + 2$ increases from left to right.

4. The function of Exercise 2 increases on the interval $-1 \le u \le 4$.
False; $q(-1) = 11$ and $q(0) = -14$, so the function decreases between these two points.

5. The function $f(x) = 4x^6 + 3x^4 + 2x^2 + 1$ is always positive.
True; the first three terms are always non-negative for any value of x, so the sum is positive.

Representations Objective K

In 6 and 7, graph the given equation. a. Estimate any relative extrema to the nearest tenth. b. Estimate all x-intercepts to the nearest tenth. c. Give the y-intercept.

6. $y = 3x^3 - 2x + 5$ a. max.: 5.6; min.: 4.4
   a. ______ b. $x = -1.4$ c. 5

7. $y = 5x^4 - 2x^3 + 7x^2 - x + 1$
   a. min.: 1.0 b. none c. 1

Name

LESSON MASTER 9-3

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective A

In 1–4, determine if y is a polynomial function of x of degree less than 5. If so, find an equation of least degree for y in terms of x.

1.

x	1	2	3	4	5	6	7	8
y	5	29	109	305	701	1405	2549	4289

Yes; $y = x^4 + 3x^2 + 1$

2.

x	1	2	3	4	5	6	7	8
y	4	20	68	166	332	584	940	1418

Yes; $y = 3x^3 - 2x^2 + x + 2$

3.

x	0	2	4	6	8	10
y	12	32	52	24	-100	-368

Yes; $y = -x^3 + 6x^2 + 2x + 12$

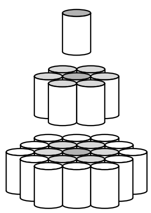
4.

x	1	2	3	4	5	6	7	8
y	2	7	17	37	77	157	317	637

No

Uses Objective I

5. Consider a soup can display in which every layer, except the first, is arranged in a pattern of concentric hexagons, as shown at the right. The first layer has 1 can, the second layer has 7 cans, the third layer has 19 cans, and so on.



a. Give a recursive formula for the number of cans c_n in layer n.

$\begin{cases} c_1 = 1 \\ c_n = c_{n-1} + 6(n-1), \end{cases}$ for $n > 1$

b. Find an explicit polynomial expression for c_n.

$c_n = 3n^2 - 3n + 1$

c. Give a recursive formula for the total number of cans t_n in a display having n layers. (HINT: The sum of the number of cans in a display having $n - 1$ layers and the number of cans in the nth layer. Use the result from part b.)

$\begin{cases} t_1 = 1 \\ t_n = t_{n-1} + 3n^2 - 3n + 1, \end{cases}$ for $n > 1$

d. Find an explicit polynomial expression for t_n.

$t_n = n^3$

Name

LESSON MASTER 9-4

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective C

In 1–4, determine the quotient and the remainder when the first polynomial is divided by the second.

1. $x^3 - 14x^2 + 51x - 54,\ x - 9$ — $x^2 - 5x + 6;\ 0$

2. $y^6 - 8y^4 - 80,\ y - 2$ — $y^5 + 2y^4 - 4y^3 - 8y^2 - 16y - 32;\ -144$

3. $z^4 - z^3 - 9z^2 + 3z + 18,\ 2z^2 - 3$ — $\frac{1}{2}z^2 - \frac{1}{2}z - \frac{15}{4};\ \frac{3}{2}z + \frac{27}{4}$

4. $a^6 + 5a^4 - 21a^3 + 20a - 100,\ a^3 + 4$ — $a^3 + 5a - 25;\ 0$

Properties Objective G

In 5–8, use the Remainder Theorem to find the remainder when the first polynomial is divided by the second.

5. $c^5 - c^4 + c^3 - c^2 + c - 1,\ c - 1$ — 0

6. $x^5 - x^4 + x^3 - x^2 + x - 1,\ x + 1$ — -6

7. $8t^6 + 8t^5 + 8t^4 + 8t^3 + 8t^2 + 8t + 8,\ t - \frac{1}{2}$ — $\frac{127}{8}$

8. $5y^4 + 3y^2 - 1,\ y - \sqrt{2}$ — 25

9. When $2x^2 + x - 5$ is divided by $x - c$, the remainder is -2. Find c. — $c = 1$ or $c = -\frac{3}{2}$

10. When $x^3 + bx^2 - x + 4$ is divided by $x + 3$, the remainder is 7. Find b. — $b = 3$

In 11 and 12, *true or false*. Use the Remainder Theorem to justify your answer.

11. $z^{79} - z$ is divisible by $z + 1$.

True; the remainder is $(-1)^{79} - (-1) = 0$.

12. There exists some polynomial $q(x)$ such that $q(x)(x - 2) = 4x^4 - 2x^2 + 2$.

False; $4(2)^4 - 2(2)^2 + 2 = 58 \neq 0$

Name

LESSON MASTER 9-5

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

1. Find three linear factors of a polynomial $p(x)$, if $p(3) = 0$, $p(2) = 0$, and $p(-5) = 0$. — $x - 3,\ x - 2,\ x + 5$

In 2 and 3, an equation and some of its solutions are given. Find the other solutions.

2. $x^3 + 16x^2 - 23x = 102$; 3 is a solution. — $x = -17;\ x = -2$

3. $120x^4 - 94x^3 + 9x^2 + 6x - 1 = 0$; $-\frac{1}{4}$ and $\frac{1}{2}$ are solutions. — $x = \frac{1}{3};\ x = \frac{1}{5}$

Properties Objective G

4. Let $f(x)$ and $g(x)$ be two polynomials, with x a factor of $f(x)$ and $(x - c)$ a factor of $g(x)$. Use the Factor Theorem to prove that $(x - c)$ is a factor of $f(g(x))$.

Sample: Since $(x - c)$ is a factor of $g(x)$, by the Factor Theorem $g(c) = 0$. Since x is a factor of $f(x)$, $f(0) = 0$. Thus $f(g(c)) = f(0) = 0$. Hence, by the Factor Theorem, $(x - c)$ is a factor of $f(g(x))$.

Representations Objective K

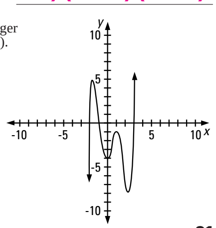
In 5–7, use an automatic grapher to find the zeros of the polynomial function. Use this information to factor the polynomial into linear factors.

5. $f(x) = x^3 - 18x^2 + 71x - 78$ — $(x - 13)(x - 3)(x - 2)$

6. $p(x) = 2x^4 + 9x^3 - 18x^2 - 109x - 84$ — $(x + 1)(x + 3) \cdot (x + 4)(2x - 7)$

7. $q(x) = x^4 - 4x^3 - 13x^2 + 28x + 60$ — $(x - 5)(x - 3)(x + 2)^2$

8. The graph at the right shows the three integer intercepts of a fifth-degree polynomial $f(x)$.



a. What are three linear factors of $f(x)$?

$x + 2,\ x + 1,\ x - 3$

b. Find a polynomial of degree three which is a factor of $f(x)$.

$x^3 - 7x - 6$

Name

LESSON MASTER 9-6

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

In 1–5, factor over the set of polynomials with complex coefficients.

1. $a^2 - 17$ — $(a - \sqrt{17})(a + \sqrt{17})$

2. $b^2 + 16$ — $(b + 4i)(b - 4i)$

3. $c^2 + 2c + 2$ — $(c + 1 + i)(c + 1 - i)$

4. $\frac{1}{2}d^2 + 2d + 3$ — $\left(\frac{1}{2}d + 1 + \frac{\sqrt{2}}{2}i\right)(d + 2 - \sqrt{2}i)$

5. $h^3 + h^2 + 2h$ — $h\left(h + \frac{1}{2} - \frac{\sqrt{7}}{2}i\right)\left(h + \frac{1}{2} + \sqrt{7}i\right)$

Skills Objective E

6. Show that $-\sqrt{5}i$ is a fourth root of 25.

$(-\sqrt{5}i)^4 = (-\sqrt{5})^4 i^4 = 25i^4 = 25 \cdot 1 = 25$

In 7–8, write each expression in $a + bi$ form.

7. Let $z = 7 + 6i$ and w be the complex conjugate of z.

a. w — $7 - 6i$ b. $z + w$ — 14

c. $z - w$ — $12i$ d. zw — 85

e. $\frac{z}{w}$ — $\frac{13}{85} + \frac{84}{85}i$ f. $\frac{w}{z}$ — $\frac{13}{85} - \frac{84}{85}i$

8. Let $z = 5 + 7i$ and $w = 1 - 2i$.

a. $7w + 2z$ — 17 b. zw — $19 - 3i$

c. z^2 — $-24 + 70i$ d. $z - 2w$ — $3 + 11i$

e. $\frac{z}{w}$ — $-\frac{9}{5} + \frac{17}{5}i$ f. $\frac{w}{z + w}$ — $-\frac{4}{61} - \frac{17}{61}i$

In 9–10, give answers in $a + bi$ form.

9. Let $f(z) = z^2 - 6z + 10$. Evaluate each.

a. $f(3 + i)$ — 0 b. $f(3 - 2i)$ — -3

10. Let $S_n = \sum_{k=1}^{n} \left(\frac{i}{2}\right)^k$, where i is $\sqrt{-1}$.

a. Find S_4. — $-\frac{3}{16} + \frac{3}{8}i$ b. Find S_∞. — $-\frac{1}{5} + \frac{2}{5}i$

Home

Name

LESSON MASTER 9-7

Questions on SPUR Objectives
See pages 622–625 for objectives.

Properties Objective F

1. Given $p(z) = (z - \sqrt{7})(z + \sqrt{7})(z - (1 + 2i))^2(z + (-1 + 2i))^2(z - 4)^3$, determine the zero(s) of p with each multiplicity.

a. one $\sqrt{7}, -\sqrt{7}$ **b.** two $1 + 2i, 1 - 2i$ **c.** three 4

Properties Objective H

2. Find all solutions to $x^4 - 2x^3 + 2i^2 - 10x + 25 = 0$, given that one solution is $x = 2 + i$.

$x = 2 + i, 2 - i, -1 + 2i, -1 - 2i$

In 3–5, *true or false*. Justify your answer.

3. The equation $p(x) = x^8 - 1$ has eight complex zeros.

True; Number of Zeros of a Polynomial Theorem

4. Every polynomial function with real coefficients that has a zero $3 + 2i$ also has a zero $3 - 2i$.

True; Conjugate Zeros Theorem

5. It is possible for the graph of a third-degree polynomial with real coefficients to cross the line $y = 4$ exactly twice.

False; $p(x) - 4 = 0$ is a polynomial with real coefficients. By the Conjugate Zeros Theorem, it has an even number of nonreal zeros.

Representations Objective K

In 6–8, a function f is given. a. Graph $f(x)$ to find all the real zeros. b. Factor $f(x)$ to determine all the nonreal zeros.

6. $f(x) = x^3 - 4x^2 + 9x - 10$

a. 2 **b.** $1 + 2i, 1 - 2i$

7. $f(x) = x^4 - 8x^3 + 23x^2 - 30x + 18$

a. 3 **b.** $1 + i, 1 - i$

8. $f(x) = x^5 + 19x^4 + 140x^3 + 506x^2 + 924x + 720$

a. $-4, -5, -6$ **b.** $-2 + \sqrt{2}i, -2 - \sqrt{2}i$

Name

LESSON MASTER 9-8

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

1. Find the four fourth roots of 64. $2\sqrt{2}, -2\sqrt{2}, 2\sqrt{2}i, -2\sqrt{2}i$

2. Find the three cube roots of -1. $-1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

In 3–10, factor the given polynomial completely over the set of polynomials with integer coefficients.

3. $9 - 16x^2$ $(3 + 4x)(3 - 4x)$

4. $125n^3 - 27m^3$ $(5n - 3m)(25n^2 + 15nm + 9m^2)$

5. $x^6 - 512$ $(x^2 - 8)(x^4 + 8x^2 + 64)$

6. $u^8 - v^8$ $(u + v)(u - v)(u^2 + v^2)(u^4 + v^4)$

7. $343x^3y^3 + 1$ $(7xy + 1)(49x^2y^2 - 7xy + 1)$

8. $t^6 - 729$ $(t + 3)(t - 3)(t^2 + 3t + 9)(t^2 - 3t + 9)$

9. $m^5 + 216m^2$ $m^2(m + 6)(m^2 - 6m + 36)$

10. $32w^3 - 4$ $4(2w - 1)(4w^2 + 2w + 1)$

In 11–15, factor the given polynomial completely over the set of polynomials with rational coefficients.

11. $a^5b^5 + c^5$

$(ab + c)(a^4b^4 - a^3b^3c + a^2b^2c^2 - abc^3 + c^4)$

12. $c^7 + 128$

$(c + 2)(c^6 - 2c^5 + 4c^4 - 8c^3 + 16c^2 - 32c + 64)$

13. $-r^5 - t^{10}$

$(r + t^2)(-r^4 + r^3t^2 - r^2t^4 + rt^6 - t^8)$

14. $\frac{d^9}{512} + 1$

$\frac{1}{512}(d + 2)(d^2 - 2d + 4)(d^6 - 8d^3 + 64)$

15. $g^6 - g$

$g(g - 1)(g^4 + g^3 + g^2 + g + 1)$

Name

LESSON MASTER 9-9

Questions on SPUR Objectives
See pages 622–625 for objectives.

Skills Objective D

In 1–4, factor the expression completely over the set of polynomials with integer coefficients.

1. $4r^3 - 6r^2 - 6r + 9$ $(2r - 3)(2r^2 - 3)$

2. $3s^5 - 12s^3 - s^2 + 4$ $(s + 2)(s - 2)(3s^3 - 1)$

3. $2x^3 + 12x^2 - 5x - 30$ $(x + 6)(2x^2 - 5)$

4. $u^4 + 5u^3 - u - 5$ $(u - 1)(u + 5)(u^2 + u + 1)$

In 5–8, find all solutions.

5. $y^3 + 8y^2 - 3y = 24$ $y = -8, y = -\sqrt{3}, y = \sqrt{3}$

6. $27m^3 - 18m^2 = 12m - 8$ $m = -\frac{2}{3}, m = \frac{2}{3}$

7. $20x^3 - 4x^2 - 25x = -5$ $x = \frac{1}{5}, x = \frac{\sqrt{5}}{2}, x = -\frac{\sqrt{5}}{2}$

8. $n^6 - n^4 = n^2 - 1$ $n = -1, n = 1, n = -i, n = i$

In 9–14, factor the expression completely over the set of polynomials with integer coefficients.

9. $6x^2 + 3xy + 2x + y$ $(3x + 1)(2x + y)$

10. $3a^4 - 5a^2b - 2b^2$ $(3a^2 + b)(a^2 - 2b)$

11. $x^3w^2 - x^3z^2 - y^3w^2 + y^3z^2$ $(w + z)(w - z)(x - y)(x^2 + xy + y^2)$

12. $a^4 + a^2d + a^2b + bd$ $(a^2 + d)(a^2 + b)$

13. $12x^5 + 4x^4y - 3x^3 - x^2y$ $x^2(2x + 1)(2x - 1)(3x + y)$

14. $2n^3 + n^2m + 2nm + m^2 + 6n + 3m$ $(2n + m)(n^2 + m + 3)$

In 15–17, solve for x.

15. $4x^2 + 11xy - 3y^2 = 0$ $x = \frac{y}{4}, x = -3y$

16. $24x^2 - 18xw = 20x - 15w$ $x = \frac{5}{6}, x = \frac{3w}{4}$

17. $3x^3 - 3x^2 - 2ax^2 + 2ax = 0$ $x = 0, x = 1, x = \frac{2a}{3}$

Name

LESSON MASTER 10-1

Questions on SPUR Objectives
See pages 692–695 for objectives.

Properties Objective C

1. Consider the binomial distribution function B with $n = 10$ and $p = 0.83$.

a. Give a formula for $B(k)$. $B(k) = {}_{10}C_k \cdot 0.83^k \cdot 0.17^{10-k}$

b. Give the function's domain. $0 \le k \le 10$ for integers k

c. Give the distribution's mode. 9

d. Give the function's maximum. ≈ 0.32

e. Describe how the domain and maximum value change if $n = 100$ and $p = 0.83$.

Domain increases; maximum value decreases.

2. Show that for a binomial distribution B with a fixed number of trials n and a fixed probability $p = 0.5$, $B(k) = B(n - k)$.

$B(k) = {}_nC_k \cdot 0.5^k \cdot 0.5^{n-k} = {}_nC_k \cdot 0.5^n$; $B(n - k) = {}_nC_{n-k} \cdot 0.5^{n-k} \cdot 0.5^{n-(n-k)} = {}_nC_{n-k} \cdot 0.5^n$. Since ${}_nC_k = {}_nC_{n-k}$, $B(k) = B(n - k)$.

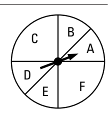
Uses Objective E

3. Suppose the Food and Drug Administration is testing a new prescription medication which the manufacturer claims has a success rate of 60%. Assume this success rate and find the probability that at least 60% of the subjects given the medication will respond positively for the given number of people.

a. 10 people $\approx 63.3\%$ **b.** 20 people $\approx 59.6\%$

Representations Objective I

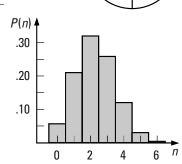
4. Consider the spinner pictured at the right. Sectors A, B, D, and E have central angles of 45°, while sectors C and F have central angles of 90°.



a. If the spinner is spun once, what is the probability that it will land in either sector A or sector C? 0.375

b. Suppose the spinner is spun 6 times. Construct a histogram for the probability distribution P, where $P(n)$ is the probability that the spinner lands in sector A or sector C n times.

Bar heights: 0.06, 0.21, 0.32, 0.26, 0.12, 0.03, 0.003



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LESSON MASTER 10-2

Questions on SPUR Objectives
See pages 692–695 for objectives.

Skills Objective A

In 1–4, a binomial experiment is described.
a. Find the mean number of successes.
b. Find the standard deviation for the number of successes.

1. The number of trials is 30 and the probability of success on each trial is 0.7. a. 21 b. ≈ 2.5
2. The number of trials is 100 and the probability of failure on each trial is 0.85. a. 15 b. ≈ 3.57
3. A student guesses randomly on a test with 50 true-false questions. a. 25 b. ≈ 3.54
4. Two fair dice are tossed 180 times. A success is tossing a 7. a. 30 b. 5

Uses Objective E

5. A manufacturer of spark plugs has estimated that the probability that a spark plug will be defective is 0.0125. A trucking company recently bought ten gross (1440) of these spark plugs for their fleet.
   a. About how many spark plugs should the trucking company expect to be defective? **18 spark plugs**
   b. What is the probability that the trucking company will find exactly the expected number of defective spark plugs? **≈ 0.094**
6. Suppose you flip a quarter 16 times and count the times it lands heads up.
   a. What is the expected number of heads? **8 heads**
   b. What is the probability that the number of heads is no more than one standard deviation from the expected number? **≈ 0.79**
7. Suppose the following experiment is conducted: A die with s sides marked 1 through s is tossed n times and the number of times a 1 is tossed is recorded. After many repetitions of the experiment, it is found that the number of 1s tossed has a mean of 33 and a standard deviation of 5.5.
   a. If the die is assumed to be fair, what is the most likely number of sides it has? **12 sides**
   b. If the die is assumed to be fair, how many times n was it tossed in each experiment? **396 times**

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LESSON MASTER 10-3

Questions on SPUR Objectives
See pages 692–695 for objectives.

Skills Objective F **Sample answers are given.**

1. Roland and Diane are playing a board game with a 6-sided die. Diane boasts that she has a lucky way of tossing the die which will give her a 6 more often than normal. Roland does not believe her, so he asks Diane to toss the die 12 times. She does so and gets five 6s.
   a. State a null and an alternative hypothesis for testing Diane's claim.
   H_0: Diane has no effect on the outcomes.
   H_1: Diane has an effect on the outcomes.
   b. Can your null hypothesis be rejected at the 0.05 significance level? Justify your answer.
   Yes; the probability of tossing five or more 6s in 12 times is ≈ 0.036 ≤ 0.05. So the null hypothesis can be rejected at the 0.05 level.
   c. Roland, still doubting Diane's luck, does a test to see if the die is biased. He tosses it 12 times and gets three 6s. Test the claim that the die is biased at the 0.05 significance level. Be sure to clearly state your hypotheses.
   H_0: The die is unbiased toward 6s. H_1: The die is biased toward 6s. The probability of tossing three or more 6s in 12 times is ≈ 0.32 > 0.05. So the null hypothesis cannot be rejected.
2. A thermometer manufacturer claims that at least 95% of its thermometers are accurate to within 0.1°C.
   a. State a null and an alternative hypothesis for testing the manufacturer's claim. **H_0: At least 95% of the thermometers are accurate. H_1: Fewer than 95% of the thermometers are accurate.**
   b. Suppose that of 20 thermometers randomly selected and checked for accuracy, five are found to give temperature readings which are off by more than 0.1°C. Test the manufacturer's claim at the 0.01 significance level.
   The probability that five or more thermometers are off by more than 0.1°C is ≈ 0.0026 < 0.01. So, the claim can be rejected at the 0.01 level.

Name

LESSON MASTER 10-4

Questions on SPUR Objectives
See pages 692–695 for objectives.

Vocabulary

1. What are the *inflection points* of the graph of the sine function? **$(n\pi, 0)$, for integers n**

Properties Objective D

2. Under what condition(s) does the graph of a binomial distribution approach that of a normal curve? **The graph approaches a normal curve as the number of trials, n, increases.**

In 3–7, *true or false*.

3. The parent and standard normal functions have the same domain. **True**
4. The parent and standard normal functions have the same range. **False**
5. The areas under the parent and standard normal curves are the same. **False**
6. The parent and standard normal curves have the same inflection points. **False**
7. The scale-change transformation $S: (x, y) \to \left(\sqrt{2\pi}x, \frac{y}{\sqrt{2}}\right)$ maps the parent normal curve onto the standard normal curve. **False**

In 8–11, let $f(x) = e^{-x^2}$ and $g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$.

8. $f(0)$ **1**
9. $g(0)$ **$\frac{1}{\sqrt{2\pi}} \approx 0.40$**
10. $\lim_{x\to\infty} f(x)$ **0**
11. $\lim_{z\to\infty} g(z)$ **0**
12. What is the median of a standard normal distribution? **0**
13. Consider the graph of the normal distribution function given by $h(x) = \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{3}}$.
   a. Find the graph's inflection points. **$\left(\frac{\sqrt{6}}{2}, \frac{1}{\sqrt{e\pi}}\right), \left(-\frac{\sqrt{6}}{2}, \frac{1}{\sqrt{e\pi}}\right)$**
   b. Find the area between the graph and the x-axis. **$\sqrt{3}$**

Name

LESSON MASTER 10-5

Questions on SPUR Objectives
See pages 692–695 for objectives.

Skills Objective B

Use the Standard Normal Distribution Table in Appendix D.

1. a. Find the probability that a standard normal random variable z will take on a value between 0.05 and -1.02. **0.366**
   b. Determine the area under the standard normal curve from $z = -1.02$ to $z = 0.05$. **0.366**
2. About what percent of the data in a standard normal distribution are within 1.2 standard deviations of the mean? **≈ 77%**

In 3–6, evaluate the given probability.

3. $P(z > 1.5)$ **0.668**
4. $P(z < -2.24)$ **0.0125**
5. $P(|z| \le 1)$ **0.6826**
6. $P(-1.35 < z < 1.12)$ **0.7801**

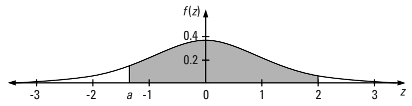
Properties Objective D

In 7–9, let z be a random variable with a standard normal distribution and let a be some constant. *True or false.*

7. $P(z > a) = 1 - P(z < a)$ **True**
8. $P(z = a) = 0$ **True**
9. $P(|z| < a) = P(z < a) - P(z > a)$ **True**

Representations Objective J

10. Refer to the standard normal curve graphed below.



   a. Let the area of the shaded region for $z < 0$ be 0.398. Find a. **-1.27**
   b. What is the probability that a standard normal random variable will be in the shaded region of the graph? **0.8752**

Name

LESSON MASTER 10-6

Questions on SPUR Objectives
See pages 692–695 for objectives.

Properties Objective D

1. Explain why a binomial distribution with mean 16 and standard deviation 1.8 should not be approximated by a normal distribution.

Since $\mu = np = 16$ and $\sigma = \sqrt{npq} = 1.8$, $q \approx 0.20$. So $p \approx 0.8$. Thus $n \approx 20$ and $nq \approx 4 < 5$. So nq is not great enough for the binomial distribution to be approximated by a normal distribution.

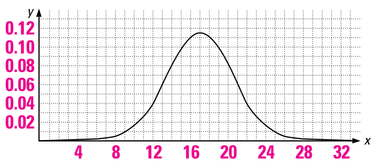
Uses Objective E

2. Suppose a global computer network is able to transmit digital data at speeds of 1.5 Mbps (megabits per second). Assume this is the mean speed and that the speed is normally distributed with a standard deviation of 0.2 Mbps. If someone needs to send a data file of 400 Mb to an office in Japan, what is the probability that the data transmission will take less than 5 minutes? 0.2033

3. A flask is divided in half by a permeable membrane which allows atoms of helium to pass freely from either side of the flask to the other side. The flask is filled with approximately 10^6 helium atoms. Assuming that there is an equal probability that a helium atom is on either side of the membrane, find the probability that at any moment either side of the flask will contain more than 50.1% of the helium atoms. ≈ 0.0228

Representations Objective J

4. Consider a normal probability distribution with mean 17 and standard deviation 3.5.
   a. Find an equation to model this distribution. $\frac{1}{3.5\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-17}{3.5}\right)^2}$
   b. Draw a graph of the distribution.



   c. What is the area under this graph between $x = 13.5$ and $x = 20.5$? ≈ 0.68

Name

LESSON MASTER 10-7

Questions on SPUR Objectives
See pages 692–695 for objectives.

Uses Objective F

1. Suppose a car manufacturer claimed its new economy car has an average highway gas mileage of 37 mpg with a standard deviation of 5 mpg. After receiving numerous complaints, a consumer group decided to test a random sample of 40 such vehicles. For the sample, they found the mean gas mileage to be 35 mpg. At the 0.01 significance level, test whether the average gas mileage is less than the manufacturer claims. Be sure to state your hypotheses clearly.

H_0: The highway mileage is 37 mpg. H_1: The highway mileage is not 37 mpg. $\mu^{\bar{x}} = 35$ and $\sigma^{\bar{x}} = \frac{5}{\sqrt{40}} \approx 0.791$, so $\frac{\bar{x} - \mu}{\sigma} \approx -2.53$. $P(z \le -2.53) \approx 0.0057 < 1$, so H_0 can be rejected.

Uses Objective G

2. In Ohio in 1995–1996 the scores of the seniors taking the mathematics section of the Scholastic Aptitude Test (SAT-M) had a mean of 535 and a standard deviation of 104. What is the probability that a random sample of 40 of these students will have a mean SAT-M score less than 520? ≈ 0.1814

3. a. Consider an experiment in which a fair six-sided die is tossed 400 times. Describe the distribution of the outcomes. Give the distribution's mean and standard deviation. (Recall $\mu = \sum_{i=1}^{n} x_i \cdot P(x)$ and $\sigma^2 = \sum_{i=1}^{n} (x_i^2 \cdot P(x_i)) - \mu^2$.)

   The distribution is uniform with a constant probability of $\frac{1}{6}$. The mean is 3.5; the standard deviation is ≈ 1.708.

   b. Suppose you repeat the experiment of part a many times. Describe the distribution of the means of these experiments. Give the distribution's mean and standard deviation.

   The distribution of sample means is normal with a mean of 3.5 and a standard deviation of $\approx \frac{1.708}{\sqrt{400}} \approx 0.0854$.

Name

LESSON MASTER 10-8

Questions on SPUR Objectives
See pages 692–695 for objectives.

Uses Objective H

1. A military expedition using a GPS (Global Positioning System) receiver finds their altitude to be 1233 ± 28 m. If the margin of error represents the 95% confidence interval for the receiver, what is the probability that the expedition is above 1261 m? 0.025

2. Suppose an astronomer has made many measurements of the distance to a distant star cluster and finds the data to be normally distributed with a mean of 624 kiloparsecs and a standard deviation of 35 kiloparsecs. (1 kiloparsec = 3.08×10^{19} meters)
   a. What is the 90% confidence interval for the distance to the cluster? 566 kpc $\le \mu \le$ 682 kpc
   b. What is the 99% confidence interval for the distance to the cluster? 534 kpc $\le \mu \le$ 714 kpc

3. A research team studying the dietary habits of American adult females charts the daily sodium intake of 100 randomly selected women over the course of 2 months. For their sample group, they find the daily sodium intake to have a mean of 2,850 mg with a standard deviation of 450 mg. Because the sample size is large, the researchers feel the standard deviation of the sample represents the standard deviation of the entire population of American women.
   a. What should the research team report as the 95% confidence interval for the mean daily sodium intake of American women? 2762 mg $\le \mu \le$ 2938 mg
   b. If the research team had wanted to report the mean sodium intake with a 95% confidence interval of 36 mg, what should their sample size have been? 2401

4. Standardized tests are called "standardized" because they are given to a large number of students whose scores serve as the standard scores for others. Initially, standardized tests must be given trials. Suppose a standardized test with a total score of 40 is sampled with 200 randomly selected students from a population, and there is a mean of 16.7 and a standard deviation of 6.3 for the sample.
   a. Give the 90% confidence interval for the population mean. $15.97 \le \mu \le 17.43$
   b. To reduce this confidence interval to one half its size, what size sample would be needed? 806

Name

LESSON MASTER 11-1

Questions on SPUR Objectives
See pages 749–751 for objectives.

Skills Objective A

In 1–4, find the product, if possible.

$A = \begin{bmatrix} 7 & 6 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ 2 & 7 \\ 8 & 5 \end{bmatrix}$ $C = [x \;\; y]$

1. AB not possible
2. BA $\begin{bmatrix} -4 & -4 \\ 35 & 26 \\ 71 & 58 \end{bmatrix}$
3. CA $\begin{bmatrix} 7x + 3y & 6x + 2y \end{bmatrix}$
4. A^2 $\begin{bmatrix} 67 & 54 \\ 27 & 22 \end{bmatrix}$

Properties Objective D

In 5–8, A is a 2×4 matrix, B is 4×3, and C is 3×4. Determine the dimensions of the indicated product matrix, if the product can be formed.

5. BC 4×4
6. CB 3×3
7. $A(BC)$ 2×4
8. $A(CB)$ not possible

Uses Objective F

In 9 and 10, use the production matrix P and the cost matrix C shown here.

$$P = \begin{array}{cc} \text{Motors} & \text{Rotors} \end{array} \quad \begin{bmatrix} 10{,}000 & 15{,}000 \\ 5{,}000 & 7{,}000 \\ 6{,}000 & 6{,}000 \end{bmatrix} \begin{array}{l} \text{Factory 1} \\ \text{Factory 2} \\ \text{Factory 3} \end{array} \qquad C = \begin{array}{cc} \text{Cost to Produce} & \text{Cost to Consumer} \end{array} \quad \begin{bmatrix} 50 & 150 \\ 5 & 10 \end{bmatrix} \begin{array}{l} \text{Motors} \\ \text{Rotors} \end{array}$$

9. Calculate PC; tell what the product matrix represents. $\begin{bmatrix} 575{,}000 & 1{,}650{,}000 \\ 285{,}000 & 820{,}000 \\ 330{,}000 & 960{,}000 \end{bmatrix}$

   1st col.: production costs; 2nd col.: customer costs.

10. Find the profits made by each factory before marketing and other costs, assuming that all items produced are sold.

    Factory 1 1,075,000 Factory 2 535,000 Factory 3 630,000

Home

Name

LESSON MASTER 11-2

Questions on SPUR Objectives See pages 749–751 for objectives.

Representations Objective G

1. a. Find the matrix product $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$. $\begin{bmatrix}x\\-y\end{bmatrix}$

b. What transformation does the 2 × 2 matrix in part **a** represent? reflection over x-axis

2. a. Find the matrix product $\begin{bmatrix}0.5 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$. $\begin{bmatrix}0.5x\\2y\end{bmatrix}$

b. What transformation does the 2 × 2 matrix in part **a** represent? vertical stretch of 2; horizontal contraction of 0.5

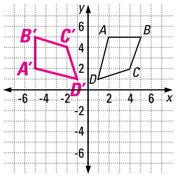
3. Write a matrix representing the size-change transformation of magnitude $\frac{1}{4}$, center (0, 0). $\begin{bmatrix}\frac{1}{4} & 0\\0 & \frac{1}{4}\end{bmatrix}$

Representations Objective I

4. a. Write the matrix representing $\triangle PQR$, where $P = (1, 7)$, $Q = (-3, 6)$, and $R = (-1, -8)$. $\begin{bmatrix}1 & -3 & -1\\7 & 6 & 8\end{bmatrix}$

b. Write the matrix for the image of $\triangle PQR$ under r_{y-axis}. $\begin{bmatrix}-1 & 3 & 1\\2 & 6 & -8\end{bmatrix}$

5. a. Write a matrix M representing the polygon $ABCD$ graphed at the right. $\begin{bmatrix}2 & 5 & 4 & 1\\5 & 5 & 2 & 1\end{bmatrix}$



b. Write a matrix for the image $A'B'C'D'$ of polygon $ABCD$ under the transformation $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$. $\begin{bmatrix}-5 & -5 & -2 & -1\\2 & 5 & 4 & 1\end{bmatrix}$

c. Draw the image $A'B'C'D'$ on the grid at the right.

Name

LESSON MASTER 11-3

Questions on SPUR Objectives See pages 749–751 for objectives.

Properties Objective D

1. *True or false.* The matrix $\begin{bmatrix}2 & 2\\3 & 3\end{bmatrix}$ can represent the transformation $S: (x, y) \to (2x, 3y)$. Justify your answer. False; $S(1, 0) = (2, 0)$ and $S(0, 1) = (0, 3)$. So by the Matrix Basis Theorem S is represented by $\begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}$.

Representations Objective G

2. Suppose $T(1, 0) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and $T(0, 1) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

a. Find a 2 × 2 matrix that represents the transformation T. $\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

b. Find $T\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. (0, 1)

Representations Objective H

In 3 and 4, two transformations are given. a. Represent the composite transformation as a product of two matrices and compute the matrix product. b. Describe the composite transformation.

3. a rotation of 270° clockwise, followed by a reflection over the y-axis.

a. $\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ b. reflection over line $y = x$

4. a reflection over the y-axis, followed by a rotation 270° clockwise

a. $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$ b. reflection over line $y = -x$

Representations Objective I

In 5 and 6, let $\begin{bmatrix}9 & -6\\-5 & 2\end{bmatrix}$ represent the transformation t, $\begin{bmatrix}8 & 9\\3 & 2\end{bmatrix}$ represent the transformation u, and $\begin{bmatrix}4 & 0 & 2\\2 & 1 & 0\end{bmatrix}$ represent $\triangle ABC$.

5. Calculate $t \circ u(\triangle ABC)$. $\begin{bmatrix}354 & 69 & 108\\-218 & -41 & -68\end{bmatrix}$

6. Calculate $u \circ t(\triangle ABC)$. $\begin{bmatrix}48 & -30 & 54\\40 & -14 & 34\end{bmatrix}$

Name

LESSON MASTER 11-4

Questions on SPUR Objectives See pages 749–751 for objectives.

Representations Objective G

1. Give the matrix for R_{40} with elements to 4 decimal places. $\begin{bmatrix}0.7660 & -0.6428\\0.6428 & 0.7660\end{bmatrix}$

2. Give the matrix for $R_{2\pi/3}$ with exact values. $\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

3. What transformation does $\begin{bmatrix}-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\end{bmatrix}$ represent? rotation of $135° \pm 360°$

Representations Objective H

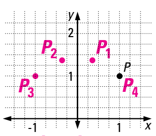
4. Find a matrix that represents a rotation of magnitude θ followed by a rotation of magnitude θ. $\begin{bmatrix}\cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta\\2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta\end{bmatrix}$

5. a. Write a matrix A that represents the composite of a rotation 30° counterclockwise, followed by a reflection over the y-axis. $\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$

b. Compute A^2. What transformation is represented? $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$; identity matrix

Representations Objective I

6. Let $P = (1, 1)$. Find the exact coordinates and plot each point.



a. $P_1 = R_{30}(P)$ $\left(\frac{\sqrt{3} - 1}{2}, \frac{\sqrt{3} + 1}{2}\right)$

b. $P_2 = r_y(P_1)$ $\left(\frac{1 - \sqrt{3}}{2}, \frac{\sqrt{3} + 1}{2}\right)$

c. $P_3 = R_{30}(P_2)$ (-1, 1)

d. $P_4 = r_y(P_3)$ (1, 1)

e. How does the answer to part d relate to A^2 of Exercise 5? $P_4 = r_y \circ R_{30} \circ r_y \circ R_{30}(P)$; A is the composite matrix for $r_y \circ R_{30}$. $P_4 = A^2(P)$. A^2 is the identity matrix so $P_4 = (1, 1)$

Name

LESSON MASTER 11-5

Questions on SPUR Objectives See pages 749–751 for objectives.

Properties Objective E

In 1–4, give the exact values.

1. sin 15° $\frac{\sqrt{6} - \sqrt{2}}{4}$

2. cos 15° $\frac{\sqrt{6} + \sqrt{2}}{4}$

3. tan $\frac{5\pi}{12}$ $2 + \sqrt{3}$

4. cos 165° $\frac{-(\sqrt{6} + \sqrt{2})}{4}$

In 5–7, simplify without using a calculator.

5. cos 71° cos 109° − sin 71° sin 109° $\cos 180° = -1$

6. sin 28° cos 32° + cos 28° sin 32° $\sin 60° = \frac{\sqrt{3}}{2}$

7. $\cos\left(\frac{3\pi}{5}\right)\cos\left(\frac{7\pi}{20}\right) + \sin\left(\frac{3\pi}{5}\right)\sin\left(\frac{7\pi}{20}\right)$ $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

8. Find the exact matrix for R_{75}. $\begin{bmatrix}\frac{\sqrt{6} - \sqrt{2}}{4} & \frac{-\sqrt{6} - \sqrt{2}}{4}\\\frac{\sqrt{6} + \sqrt{2}}{4} & \frac{\sqrt{6} - \sqrt{2}}{4}\end{bmatrix}$

In 9–12, A and B are acute angles with $\sin A = 0.8$ and $\cos B = 0.3$. Determine the value.

9. cos (A + B) -0.5832

10. cos (A − B) 0.9432

11. sin (A + B) 0.8124

12. sin (A − B) -0.3324

In 13–18, simplify the expression.

13. sin (90° + θ) cos θ

14. cos (90° + θ) -sin θ

15. $\sin\left(\frac{3\pi}{2} - \theta\right)$ -cos θ

16. $\cos\left(\frac{3\pi}{2} - \theta\right)$ -sin θ

17. sin (45° + θ) $\frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)$

18. cos (45° + θ) $\frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)$

Name

LESSON MASTER 11-6

Questions on SPUR Objectives
See pages 749–751 for objectives.

Properties Objective E

In 1–4, evaluate the expression without using a calculator.

1. $2 \sin 15° \cos 15°$ — $\frac{1}{2}$
2. $1 - 2 \sin^2 22°30'$ — $\frac{\sqrt{2}}{2}$
3. $4 \cos^2\left(\frac{2\pi}{3}\right) - 4 \sin^2\left(\frac{2\pi}{3}\right)$ — -2
4. $8 \sin\left(\frac{2\pi}{3}\right) \cos\left(\frac{2\pi}{3}\right)$ — $-2\sqrt{3}$

In 5–8, use the appropriate Double Angle Formula to simplify the expression.

5. $\cos^2 37° - \sin^2 37°$ — $\cos 74°$
6. $2 \sin 25° \cos 25°$ — $\sin 50°$
7. $1 - 2 \sin^2\left(\frac{5\pi}{12}\right)$ — $\cos \frac{5\pi}{6}$
8. $2 \cos^2 35° - 1$ — $\cos 70°$
9. If $\angle A$ is an acute angle and $\cos A = \frac{1}{5}$, find the exact value of $\cos 2A$. — $-\frac{23}{25}$
10. Express $\sin 2\theta$ as a function of $\sin \theta$, for $0 \le \theta \le \frac{\pi}{2}$. — $2 \sin \theta(1 - \sin^2 \theta)^{\frac{1}{2}}$
11. Express $\sin 3\theta$ as a function of $\sin \theta$ and $\cos \theta$. — $\sin \theta(4 \cos^2 \theta - 1)$

Name

LESSON MASTER 11-7

Questions on SPUR Objectives
See pages 749–751 for objectives.

Skills Objective B

1. Consider the following system of equations: $\begin{cases} 3x + y = 22 \\ 4x - 2y = 6 \end{cases}$

   a. Represent the system by a matrix equation.
   $\begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 6 \end{bmatrix}$

   b. Find the inverse of the coefficient matrix from part **a**.
   $\begin{bmatrix} \frac{1}{5} & \frac{1}{10} \\ \frac{2}{5} & -\frac{3}{10} \end{bmatrix}$

   c. Multiply both sides of the matrix equation in part **a** on the left by the answer in part **b** to find the solution to the system of equations.
   $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

2. Solve the system $\begin{cases} 2x + 3y - 2z = 0 \\ x - 2y - z = -14 \\ 2x + 2y - 3z = 2 \end{cases}$ given that

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & -2 & -1 \\ 2 & 2 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{8}{7} & \frac{5}{7} & -1 \\ \frac{1}{7} & \frac{-2}{7} & 0 \\ \frac{6}{7} & \frac{2}{7} & -1 \end{bmatrix}.$$

   $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 4 \\ -6 \end{bmatrix}$

Skills Objective C

In 3–6, find the inverse of the given matrix, if it exists.

3. $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ — $\begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & -\frac{2}{7} \end{bmatrix}$

4. $\begin{bmatrix} -7 & -2 \\ 5 & 1 \end{bmatrix}$ — $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{5}{3} & -\frac{2}{3} \end{bmatrix}$

5. $\begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix}$ — $\begin{bmatrix} \cos 45° & -\sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix}$

6. $\begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix}$ — Does not exist

Properties Objective D

7. Prove that $\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}^{-1} = \begin{bmatrix} x^{-1} & 0 \\ 0 & y^{-1} \end{bmatrix}$ for $x \neq 0$ and $y \neq 0$.

$$\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \frac{1}{xy}\begin{bmatrix} y & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} \frac{y}{xy} & 0 \\ \frac{0}{} & \frac{x}{xy} \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 \\ 0 & y^{-1} \end{bmatrix}$$

8. Prove that any matrix of the form $\begin{bmatrix} x & y \\ ax & ay \end{bmatrix}$ has no inverse. The determinant $ayx - ayx = 0$, so the matrix has no inverse.

Name

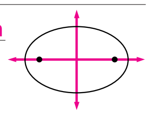
LESSON MASTER 12-1

Questions on SPUR Objectives
See pages 802–803 for objectives.

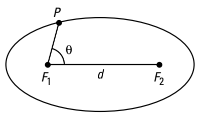
Properties Objective D

1. a. At the right is an ellipse with its two foci. Measure to find the ellipse's focal constant to the nearest millimeter. — 31 mm

   b. Draw all the ellipse's symmetry lines.



2. Consider the ellipse pictured at the right, with foci F_1 and F_2, where $F_1F_2 = d$, and focal constant $k > d$. Suppose P is a point on the ellipse.



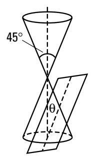
   a. Use the Law of Cosines to prove that $PF_1 = \frac{k^2 - d^2}{2k - 2d \cos \theta}$.

Let $PF_1 = a$. Then $PF_2 = k - a$. By the Law of Cosines, $a^2 + d^2 - 2ad \cos \theta = (k - a)^2 = k^2 - 2ak + a^2$. So, $2ak - 2ad \cos \theta = k^2 - d^2$; $a(2k - 2d \cos \theta) = k^2 - d^2$; $a = \frac{(k^2 - d^2)}{(2k - 2d \cos \theta)}$.

   b. What are the maximum and minimum values for PF_1? maximum $(\theta = 0°)$: $\frac{k + d}{2}$; minimum $(\theta = 180°)$: $\frac{k - d}{2}$

Properties Objective E

In 3–7, consider a cone generated by two lines which intersect at an angle of 45°, as pictured at the right. Determine the conic section formed by the intersection of this cone with a plane not passing through the cone's vertex, if the smallest angle θ between the plane and the cone's axis has the given measure.



3. 45° — parabola
4. 75° — ellipse
5. 23° — hyperbola
6. 90° — ellipse or circle
7. 0° — hyperbola

Name

LESSON MASTER 12-2

Questions on SPUR Objectives
See pages 802–803 for objectives.

Skills Objective A

1. Find an equation for the ellipse with foci (0, 3) and (0, -3) and focal constant 10. — $\frac{x^2}{16} + \frac{y^2}{25} = 1$
2. Find an equation for the ellipse with center at the origin, horizontal axis with length 16, and vertical axis with length 4. — $\frac{x^2}{64} + \frac{y^2}{4} = 1$

Uses Objective F

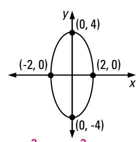
3. The comet Temple-Tuttle has an elliptical orbit with the sun as one focus. The major axis of this ellipse has length 20.66 a.u. (astronomical units) and the minor axis has length 8.79 a.u.

   a. Find Temple-Tuttle's perihelion distance, that is, its least distance to the sun, in a.u. — 0.98 a.u.

   b. Find Temple-Tuttle's aphelion distance, that is, its greatest distance from the sun, in a.u. — 19.68 a.u.

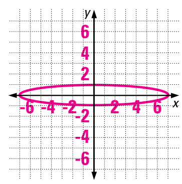
Representations Objective G

4. Give an equation for this ellipse.



$\frac{x^2}{4} + \frac{y^2}{16} = 1$

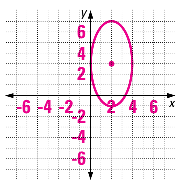
5. Graph $x^2 + 49y^2 = 49$.



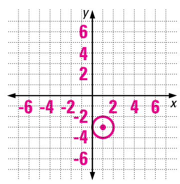
Representations Objective H

In 6 and 7, graph the ellipse with the given equation.

6. $\frac{(x - 2)^2}{4} + \frac{(y - 3)^2}{16} = 1$



7. $(x - 1)^2 + (y + 3)^2 = 1$



Name

LESSON MASTER 12-3 Questions on SPUR Objectives
See pages 802–803 for objectives.

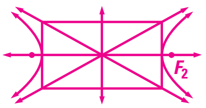
Skills Objective A

1. Find an equation for the hyperbola with foci (5, 0) and (-5, 0) and focal constant 8. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

2. Give equations for the asymptotes of the hyperbola $\frac{x^2}{36} - \frac{y^2}{9} = 1$. $y = \frac{1}{2}x,\ y = -\frac{1}{2}x$

Properties Objective D

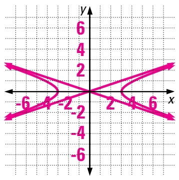
3. a. Draw a hyperbola in which the distance between the foci is 42 mm and the focal constant is 36 mm.
   b. Draw the hyperbola's lines of symmetry.



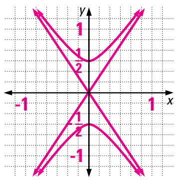
Representations Objective G

In 4 and 5, graph the equation. Include all asymptotes.

4. $\frac{x^2}{9} - y^2 = 1$



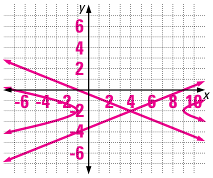
5. $4y^2 - 9x^2 = 1$



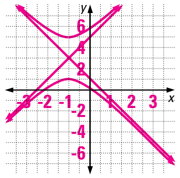
Representations Objective H

In 6 and 7, graph the equation. Include all asymptotes.

6. $\frac{(x-4)^2}{25} - \frac{(y+2)^2}{4} = 1$



7. $(y-3)^2 - 4(x+1)^2 = 1$



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Name

LESSON MASTER 12-4 Questions on SPUR Objectives
See pages 802–803 for objectives.

Skills Objective B

In 1–6, find an equation for the image of the given figure under the given rotation about the origin.

1. $\frac{x^2}{36} - y^2 = 1;\ R_{45°}$ $-35x^2 + 74xy - 35y^2 = 72$

2. $x^2 + \frac{y^2}{9} = 1;\ R_{30°}$ $7x^2 + 4\sqrt{3}xy + 3y^2 = 9$

3. $\frac{x^2}{3} - \frac{y^2}{7} = 0.8;\ R_{\pi}$ $\frac{x^2}{3} - \frac{y^2}{7} = 0.8$

4. $y = 2x^2 + 3x + 6;\ R_{90°}$ $x = -2y^2 - 3y - 6$

5. $-15x^2 - 34\sqrt{3}xy + 19y^2 = 96;\ R_{\frac{\pi}{3}}$ $9x^2 - 8y^2 = 24$

6. $x^2 + y^2 = 20;\ R_{\theta}$ $x^2 + y^2 = 20$

7. Find an equation for the image of the parabola $y = (x+2)^2 - 5$ under a rotation of 45° about the point (-2, -5).
$(x+2)^2 + 2(x+2)(y+5) + \sqrt{2}(x+2) +$
$(y+5)^2 - \sqrt{2}(y+5) = 0$

8. Find an equation for the image of the hyperbola $\frac{y^2}{25} - \frac{x^2}{16} = 1$ under a rotation of $\frac{\pi}{2}$ about the point (1, 0).
$\frac{(x-1)^2}{25} - \frac{(y+1)^2}{16} = 1$

Properties Objective D

In 9–11, tell whether or not the hyperbola is a rectangular hyperbola. Justify your answer.

9. $\frac{x^2}{4} - \frac{y^2}{16} = 1$ No; asymptotes $y = 2x$ and $y = -2x$ not ⊥

10. $3xy = -5$ Yes; asymptotes $y = 0$ and $x = 0$ ⊥

11. $\frac{y^2}{3} - \frac{x^2}{3} = 1$ Yes; asymptotes $y = x$ and $y = -x$ ⊥

In 12 and 13, *true or false*.

12. All hyperbolas are similar. False

13. All rectangular hyperbolas are similar. True

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Name

LESSON MASTER 12-5 Questions on SPUR Objectives
See pages 802–803 for objectives.

Skills Objective C

In 1 and 2, rewrite the equation in the general form $Ax^2 + Bxy + Cy^2 + Dx + Gy + F = 0$. Then give values of A, B, C, D, E, and F for the equation.

1. $\frac{x^2}{9} + \frac{y^2}{64} = 1$ $x^2 + 9y^2 - 576 = 0$
$A = 64, B = 0, C = 9, D = 0, E = 0, F = -576$

2. $\frac{(x-3)^2}{9} - y^2 = 4$ $x^2 - 9y^2 - 6x - 27 = 0$
$A = 1, B = 0, C = -9, D = -6, E = 0, F = -27$

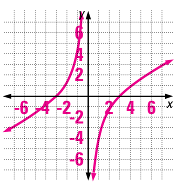
Properties Objective E

3. Where must a plane intersect a cone to form a degenerate conic section? the cone's vertex

4. What geometric figure(s) can be a degenerate hyperbola? two intersecting lines

Representations Objective H

5. Graph $x^2 - 2xy = 9$ by solving for y.



Representations Objective I

In 6–10, describe the graph of the relation represented by the given equation.

6. $x^2 + y^2 + 2x - 6y + 10 = 0$ degenerate ellipse or point

7. $x^2 - 6xy + 9y^2 + 8x - 6y + 1 = 0$ parabola

8. $21x^2 + 10\sqrt{3}xy + 31y^2 - 144 = 0$ ellipse

9. $7x^2 - 14xy + 2y^2 + 6x - 3y - 5 = 0$ hyperbola

10. $x^2 - y^2 + 2y - 1 = 0$ hyperbola or two intersecting lines

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Name

LESSON MASTER 13-1 Questions on SPUR Objectives
See pages 863–865 for objectives.

Skills Objective A

In 1–4, give exact values.

1. $\csc \frac{\pi}{4}$ $\sqrt{2}$
2. $\cot \frac{\pi}{2}$ 0
3. $\sec(-240°)$ -2
4. $\csc 75°$ $\sqrt{6} - \sqrt{2}$

In 5–8, evaluate to the nearest hundredth.

5. $\cot 4$ 0.86
6. $\sec 70°$ 2.92
7. $\csc(-35°)$ -1.74
8. $(\csc 154°)^{-1}$ 0.44

In 9–12, let $\sin\theta = 0.34$ where $0 \le \theta \le \frac{\pi}{2}$. Evaluate to the nearest hundredth.

9. $\csc\theta$ 2.94
10. $\sec\theta$ 1.06
11. $\cot\theta$ 2.77
12. $\sec(\pi + \theta)$ -1.06

Properties Objective E

In 13 and 14, tell if the function with the given equation is *even*, *odd*, or *neither*.

13. $f(\theta) = \sec\theta$ even
14. $g(\theta) = \csc\theta$ odd

In 15–18, *true or false*. Let f be the tangent function and g be the cotangent function.

15. f and g have the same domain. False

16. f and g have the same range. True

17. f and g have the same period. True

18. The graphs of $y = f(x)$ and $y = g(x)$ have the same asymptotes. False

19. Identify all points of discontinuity of the graph of $y = \csc x$.
$x = \pm n\pi$, where n is an integer.

20. Find all values of x such that $\cot x = \tan x$.
$x = \frac{\pi}{4} \pm \frac{n\pi}{2}$, where n is an integer.

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