# Chapter 7: Sampling Distributions

## Section 7.2
## Sample Proportions

The Practice of Statistics, 4th edition – For AP*
STARNES, YATES, MOORE

# Chapter 7
# Sampling Distributions

- 7.1 What is a Sampling Distribution?
- **7.2 Sample Proportions**
- 7.3 Sample Means

# Section 7.2
# Sample Proportions

**Learning Objectives**

After this section, you should be able to…

- ✓ FIND the mean and standard deviation of the sampling distribution of a sample proportion
- ✓ DETERMINE whether or not it is appropriate to use the Normal approximation to calculate probabilities involving the sample proportion
- ✓ CALCULATE probabilities involving the sample proportion
- ✓ EVALUATE a claim about a population proportion using the sampling distribution of the sample proportion

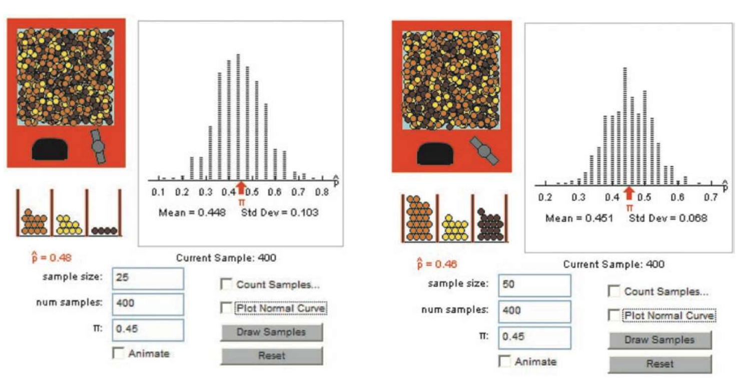
## The Sampling Distribution of $\hat{p}$

Sample Proportions

How good is the statistic $\hat{p}$ as an estimate of the parameter $p$? The sampling distribution of $\hat{p}$ answers this question.

Consider the approximate sampling distributions generated by a simulation in which SRSs of *Reese's Pieces* are drawn from a population whose proportion of orange candies is 0.45.

What do you notice about the shape, center, and spread of each?



## The Sampling Distribution of $\hat{p}$

**Shape**: In some cases, the sampling distribution of $\hat{p}$ can be approximated by a Normal curve. This seems to depend on both the sample size $n$ and the population proportion $p$.

**Center**: The mean of the distribution is $\mu_{\hat{p}} = p$. This makes sense because the sample proportion $\hat{p}$ is an unbiased estimator of $p$.

$\mu_{\hat{p}} = p$

$\hat{p}$ is an unbiased estimator or $p$

**Spread**: For a specific value of $p$, the standard deviation $\sigma_{\hat{p}}$ gets smaller as $n$ gets larger. The value of $\sigma_{\hat{p}}$ depends on both $n$ and $p$.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

As sample size increases, the spread decreases.

Sample Proportions

## The Sampling Distribution of $\hat{p}$

We can summarize the facts about the sampling distribution of $\hat{p}$ as follows:

**Sampling Distribution of a Sample Proportion**

Choose an SRS of size $n$ from a population of size $N$ with proportion $p$ of successes. Let $\hat{p}$ be the sample proportion of successes. Then:

The **mean** of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$

The **standard deviation** of the sampling distribution of $\hat{p}$ is

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

as long as the *10% condition* is satisfied: $n \le (1/10)N$.

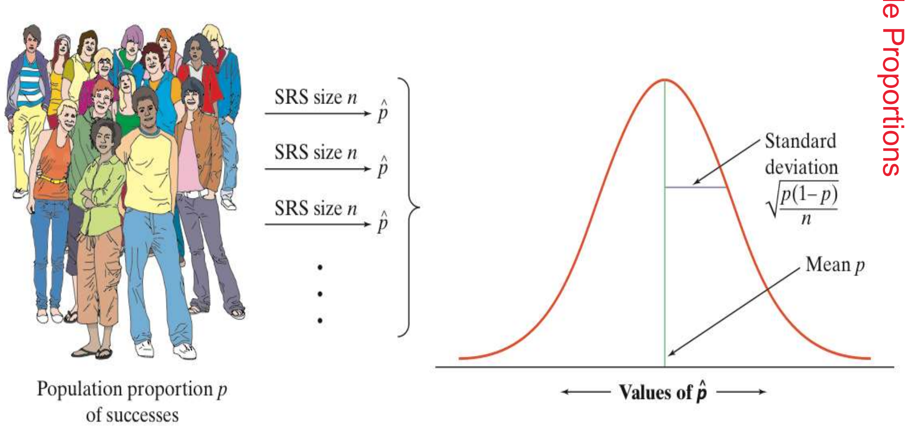
As *n* increases, the sampling distribution becomes **approximately Normal**. Before you perform Normal calculations, check that the *Normal condition* is satisfied: $np \ge 10$ and $n(1-p) \ge 10$.

Sample Proportions

## The Sampling Distribution of $\hat{p}$

Sample Proportions

We can summarize the facts about the sampling distribution of $\hat{p}$ as follows:




## Using the Normal Approximation for $\hat{p}$

Sample Proportions

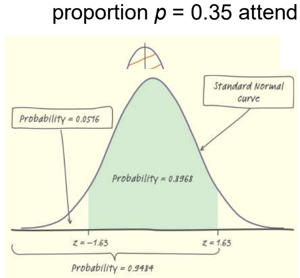
Inference about a population proportion $p$ is based on the sampling distribution of $\hat{p}$. When the sample size is large enough for $np$ and $n(1-p)$ to both be at least 10 (the Normal condition), the sampling distribution of $\hat{p}$ is approximately Normal.

STEP 4

A polling organization asks an SRS of 1500 first-year college students how far away their home is. Suppose that 35% of all first-year students actually attend college within 50 miles of home. What is the probability that the random sample of 1500 students will give a result within 2 percentage points of this true value?

**STATE**: We want to find the probability that the sample proportion falls between 0.33 and 0.37 (within 2 percentage points, or 0.02, of 0.35).

**PLAN**: We have an SRS of size $n$ = 1500 drawn from a population in which the proportion $p$ = 0.35 attend college within 50 miles of home.




$$\mu_{\hat{p}} = 0.35 \qquad \sigma_{\hat{p}} = \sqrt{\frac{(0.35)(0.65)}{1500}} = 0.0123$$

**DO**: Since $np$ = 1500(0.35) = 525 and $n(1 - p)$ = 1500(0.65)=975 are both greater than 10, we'll standardize and then use Table A to find the desired probability.

$$z = \frac{0.33 - 0.35}{0.123} = -1.63 \qquad z = \frac{0.37 - 0.35}{0.123} = 1.63$$

$$P(0.33 \le \hat{p} \le 0.37) = P(-1.63 \le Z \le 1.63) = 0.9484 - 0.0516 = 0.8968$$

**CONCLUDE**: About 90% of all SRSs of size 1500 will give a result within 2 percentage points of the truth about the population.

# Section 9.2
# Sample Proportions

## Summary

In this section, we learned that…

- ✓ When we want information about the population proportion $p$ of successes, we often take an SRS and use the sample proportion $\hat{p}$ to estimate the unknown parameter $p$. The **sampling distribution** of $\hat{p}$ describes how the statistic varies in all possible samples from the population.
- ✓ The **mean** of the sampling distribution of $\hat{p}$ is equal to the population proportion $p$. That is, $\hat{p}$ is an unbiased estimator of $p$.
- ✓ The **standard deviation** of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ for an SRS of size $n$. This formula can be used if the population is at least 10 times as large as the sample (the 10% condition). The standard deviation of $\hat{p}$ gets smaller as the sample size $n$ gets larger.
- ✓ When the sample size $n$ is larger, the sampling distribution of $\hat{p}$ is close to a Normal distribution with mean $p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- ✓ In practice, use this Normal approximation when both $np \geq 10$ and $n(1 - p) \geq 10$ (the Normal condition).