
(b) State: Our parameters of interest are $p_{1}=$ true proportion of young adults who use Twitter and $p_{2}=$ true proportion of older adults who use Twitter. We want to estimate the difference $p_{1}-p_{2}$ at a $90 \%$ confidence level. Plan: We should use a two-sample $z$ interval for $p_{1}-p_{2}$ if the conditions are met. Random: The data come from two independent random samples. $10 \%$ : $n_{1}=316$ is less than $10 \%$ of all young adults and $n_{2}=532$ is less than $10 \%$ of all older adults.
Large Counts: $n_{1} \hat{p}_{1}=316(0.26)=82.16 \approx 82, n_{1}\left(1-\hat{p}_{1}\right)=316(0.74)=233.84 \approx 234$, $n_{2} \hat{p}_{2}=532(0.14)=74.48 \approx 74$, and $n_{2}\left(1-\hat{p}_{2}\right)=532(0.86)=457.52 \approx 458$ are all at least 10 .
Do: The $90 \%$ confidence interval is $(0.26-0.14) \pm 1.645 \sqrt{\frac{0.26(1-0.26)}{316}+\frac{0.14(1-0.14)}{532}}=$ $0.12 \pm 1.645(0.0289)=0.12 \pm 0.048=(0.072,0.168)$ Using technology: $(0.073,0.168)$. Conclude: We are $90 \%$ confident that the interval from 0.073 to 0.168 captures the true difference in the proportions of young adults and older adults who use Twitter.
10.11 (a) State: Our parameters of interest are $p_{1}=$ true proportion of young men who live in their parents' home and $p_{2}=$ true proportion of young women who live in their parents' home. We want to estimate the difference $p_{1}-p_{2}$ at a $99 \%$ confidence level. Plan: We should use a two-sample $z$ interval for $p_{1}-p_{2}$ if the conditions are met. Random: Even though the data came from a single random sample, it is reasonable to consider the two samples independent because knowing the response of a male shouldn't help us predict the response of a female. $10 \%: n_{1}=$ 2253 is less than $10 \%$ of the population of young men and $n_{2}=2629$ is less than $10 \%$ of the population of young women. Large Counts: The number of successes and failures in both groups are at least 10 (Young men: 986 successes, 1267 failures. Young women: 923 successes, 1706 failures). Do: From the data we find that $n_{1}=2253, \hat{p}_{1}=\frac{986}{2253}=0.438, n_{2}=2629$, and $\hat{p}_{2}=\frac{923}{2629}=0.351$. The $99 \%$ confidence interval is
$(0.438-0.351) \pm 2.576 \sqrt{\frac{0.438(0.562)}{2253}+\frac{0.351(0.649)}{2629}}=0.087 \pm 0.036=(0.051,0.123)$.
Conclude: We are $99 \%$ confident that the interval from 0.051 to 0.123 captures the true difference in the proportions of young men and young women who live in their parents' home. (b) Because the interval does not contain 0 , there is convincing evidence that the true proportion of young men who live at their parents' home is different than the true proportion of young women who live in their parents' home.
10.15 State: We want to perform a test at the $\alpha=0.05$ significance level of the hypotheses stated in Exercise 13. Plan: We should use a two-sample $z$ test for $p_{1}-p_{2}$ if the conditions are met. Random: The data come from independent random samples. $10 \%$ : $n_{1}=800$ is less than $10 \%$ of all teens and $n_{2}=400$ is less than $10 \%$ of all young adults. Large Counts: The number of successes and failures in both groups are at least 10 (Teens: 632 successes and 168 failures. Young adults: 268 successes and 132 failures.) Do: The proportions of those owning iPods or MP3 players in each group are $\hat{p}_{1}=\frac{632}{800}=0.79$ and $\hat{p}_{2}=\frac{268}{400}=0.67$. The pooled proportion is $\hat{p}_{C}=\frac{632+268}{800+400}=\frac{900}{1200}=0.75$. The test statistic is $z=\frac{(0.79-0.67)-0}{\sqrt{\frac{(0.75)(0.25)}{800}+\frac{(0.75)(0.25)}{400}}}=4.53$ and the $P$-value is $2 P(Z \geq 4.53) \approx 0$. Conclude: Because the $P$-value of close to 0 is less than $\alpha=$ 0.05 , we reject $H_{0}$. There is convincing evidence that the true proportions of teens who would say that they own an iPod or MP3 player is different than the true proportions of young adults who would say that they own an iPod or MP3 player.
10.16 (a) State: We want to perform a test at the $\alpha=0.05$ significance level of the hypotheses stated in Exercise 14. Plan: We should use a two-sample $z$ test for $p_{1}-p_{2}$ if the conditions are met. Random: The data come from independent random samples. $10 \%: n_{1}=1679$ is less than $10 \%$ of high school freshmen in Illinois and $n_{2}=1366$ is less than $10 \%$ of high school seniors in Illinois. Large Counts: The number of successes and failures in both groups are at least 10 (Freshmen: 34 successes, 1645 failures. Seniors: 24 successes, 1342 failures). Do: The proportions of those using anabolic steroids in each group are $\hat{p}_{1}=\frac{34}{1679}=0.0203$ and $\hat{p}_{2}=\frac{24}{1366}=0.0176$. The pooled proportion is $\hat{p}_{C}=\frac{34+24}{1679+1366}=\frac{58}{3045}=0.0190$. The test statistic is $z=\frac{(0.0203-0.0176)-0}{\sqrt{\frac{(0.019)(0.981)}{1679}+\frac{(0.019)(0.981)}{1366}}}=0.54$ and the $P$-value is
$2 P(Z \geq 0.54)=2(0.2946)=0.5892$. Conclude: Because the $P$-value of 0.5892 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that there is a difference in the true proportions of all Illinois freshmen and seniors who have used anabolic steroids.

## Unît 6 Homework

Homework \#2:
Chapter 10 \#'s, 33, 39b\&c, 40b\&c, 43, 49, 53
10.33 The Random condition is met because these are two independent random samples. The $10 \%$ condition is met because 20 is less than $10 \%$ of all males at the school and 20 is less than $10 \%$ of all females at the school. The Normal/Large Sample condition is not met for these data. There are fewer than 30 observations in each group and the stemplot for Males shows several outliers.
(b) State: Our parameters of interest are $\mu_{1}=$ the true mean summer earnings of male students and $\mu_{2}=$ the true mean summer earnings of female students. We want to estimate the difference $\mu_{1}-\mu_{2}$ at a $90 \%$ confidence level. Plan: We should use a two-sample $t$ interval for $\mu_{1}-\mu_{2}$ if the conditions are met. Random: Even though the data came from a single random sample, it is reasonable to consider the two samples independent because knowing the response of a male shouldn't help us predict the response of a female. $10 \%: n_{1}=675$ is less than $10 \%$ of male students at a large university and $n_{2}=621$ is less than $10 \%$ of female students at a large university. Normal/Large Sample: $n_{1}=675 \geq 30$ and $n_{2}=621 \geq 30$. Do: The conservative degrees of freedom is $621-1=620$. Using Table B and df $=100$, our $90 \%$ confidence interval is

$$
(1884.52-1360.39) \pm 1.660 \sqrt{\frac{(1368.37)^{2}}{675}+\frac{(1037.46)^{2}}{621}}=524.13 \pm 111.45=(412.68,635.58)
$$

Using technology: $(413.62,634.64)$ with $\mathrm{df}=1249.21$. Conclude: We are $90 \%$ confident that the interval from $\$ 413.62$ to $\$ 634.64$ captures the true difference in mean summer earnings of male students and female students at this large university.
(c) It we took many random samples of 675 males and 621 females from this university and each time constructed a $90 \%$ confidence interval in this same way, about $90 \%$ of the resulting intervals would capture the true difference in mean earnings for males and females.
10.40 (a) The use of two-sample $t$ procedures is still justified because both sample sizes are large ( $92 \geq 30$ and $86 \geq 30$ ).
(b) State: Our parameters of interest are $\mu_{1}=$ the true mean reliability rating of Anglo customers and $\mu_{2}=$ the true mean reliability rating of Hispanic customers. We want to estimate the difference $\mu_{1}-\mu_{2}$ at a $95 \%$ confidence level. Plan: We should use a two-sample $t$ interval for $\mu_{1}-\mu_{2}$ if the conditions are met. Random: Even though the data came from a single random sample, it is reasonable to consider the two samples independent because knowing the response of an Anglo customer shouldn't help us predict the response of a Hispanic customer. 10\%: $n_{1}=$ 92 is less than $10 \%$ of Anglo customers and $n_{2}=86$ is less than $10 \%$ of Hispanic customers.
Normal/Large Sample: $n_{1}=92 \geq 30$ and $n_{2}=86 \geq 30$. Do: The conservative degrees of freedom is $86-1=85$. Using Table B and $\mathrm{df}=80$, the $95 \%$ confidence interval is
$(6.37-5.91) \pm 1.990 \sqrt{\frac{(0.60)^{2}}{92}+\frac{(0.93)^{2}}{86}}=0.46 \pm 0.235=(0.225,0.695)$. Using technology:
$(0.226,0.694)$ with $\mathrm{df}=143.69$. Conclude: We are $95 \%$ confident that the interval from 0.226 to 0.694 captures the true difference in mean reliability rating for Anglos and Hispanics.
(c) If we took many random samples of 92 Anglos and 86 Hispanics and each time constructed a $95 \%$ confidence interval in this same way, about $95 \%$ of the resulting intervals would capture the true difference in mean reliability rating for Anglos and Hispanics.
10.43 State: We want to perform a test at the $\alpha=0.05$ significance level of $H_{0}: \mu_{1}-\mu_{2}=0$ versus $H_{a}: \mu_{1}-\mu_{2} \neq 0$ where $\mu_{1}$ is the true mean number of words spoken per day by female students and $\mu_{2}$ is the true mean number of words spoken per day by male students. Plan: We should use a two-sample $t$ test if the conditions are met. Random: These data come from independent random samples. $10 \%: n_{1}=56$ is less than $10 \%$ of females at a large university and $n_{2}=56$ is less than $10 \%$ of males at a large university. Normal/Large Sample: $n_{1}=56 \geq 30$ and $n_{2}=56 \geq 30$. Do: The test statistic is $t=\frac{(16177-16569)-0}{\sqrt{\frac{(7520)^{2}}{56}+\frac{(9108)^{2}}{56}}}=-0.248$. The conservative
degrees of freedom is $56-1=55$. Using Table B and $\mathrm{df}=50$, the $P$-value is greater than $2(0.25)$ $=0.50$. Using technology: $t=-0.248, \mathrm{df}=106.20, P$-value $=0.8043$. Conclude: Because the $P$ value of 0.8043 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that the true mean number of words spoken per day by female students is different than the true mean number of words spoken per day by male students at this university.
10.49 (a) State: We want to perform a test at the $\alpha=0.05$ significance level of $H_{0}: \mu_{1}-\mu_{2}=10$ versus $H_{a}: \mu_{1}-\mu_{2}>10$ where $\mu_{1}$ is the true mean cholesterol reduction for people like the ones in the study when using the new drug and $\mu_{2}$ is the true mean cholesterol reduction for people like the ones in the study when using the current drug. Plan: We should use a two-sample $t$ test if the conditions are met. Random: The data come from two groups in a randomized experiment. Normal/Large Sample: Both samples had less than 30 observations, but we are told that graphs of the data show no strong skewness or outliers. Thus, a two-sample $t$ procedure is appropriate. Do: The test statistic is $t=\frac{(68.7-54.1)-10}{\sqrt{\frac{(13.3)^{2}}{15}+\frac{(11.93)^{2}}{14}}}=0.982$. Using the
conservative degrees of freedom $(14-1=13)$, the $P$-value is between 0.15 and 0.20 . Using technology: $t=0.982, \mathrm{df}=26.96, P$-value $=0.1675$ (Note: To perform this test on the calculator, use $\bar{x}_{1}=68.7-10=58.7$ ). Conclude: Because the $P$-value of 0.1675 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that the true mean cholesterol reduction is more than $10 \mathrm{mg} / \mathrm{dl}$ greater for the new drug than for the current drug.
(b) Because we failed to reject the null hypothesis, we could have committed a Type II error. It is possible that the difference in mean cholesterol reduction is more than $10 \mathrm{mg} / \mathrm{dl}$ greater for the new drug than the current drug, but we didn't find convincing evidence that it was.
10.53 (a) Two-sample $t$ test. The data are being produced using two distinct groups of cars in a randomized experiment.
(b) Paired $t$ test. This is a matched pairs experimental design where both treatments are applied to each subject.
(c) Two-sample $t$ test. Even though there are before and after measurements for each woman in the experiment, the data are being produced using two distinct groups of women.

11.7 State: We want to perform a test at the $\alpha=0.05$ significance level of $H_{0}$ : Nuthatches do not prefer particular types of trees when searching for seeds and insects versus $H_{a}$ : Nuthatches do prefer particular types of trees when searching for seeds and insects; Or,
$H_{0}: p_{\text {firs }}=0.54, p_{\text {pines }}=0.40, p_{\text {other }}=0.06$ versus $H_{a}:$ At least two of the $p_{i}^{\prime} \mathrm{s}$ is incorrect. Plan:
We should use a chi-square test for goodness of fit if the conditions are met. Random: The data come from a random sample. $10 \%: n=156$ is less than $10 \%$ of all nuthatches. Large Counts:
The expected counts in each category are all at least 5 (firs: $156(0.54)=84.24$, pines:
$156(0.40)=62.4$, and other: $156(0.06)=9.36$ ) Do: The test statistic is
$\chi^{2}=\frac{(70-84.24)^{2}}{84.24}+\frac{(79-62.4)^{2}}{62.4}+\frac{(7-9.36)^{2}}{9.36}=7.418$. With $\mathrm{df}=3-1=2$, the $P$-value is
between 0.02 and 0.025 . Using technology: $P$-value $=0.0245$. Conclude: Because the $P$-value of 0.0245 is less than $\alpha=0.05$, we reject $H_{0}$. There is convincing evidence that nuthatches prefer particular types of trees when they are searching for seeds and insects.
11.15 State: We want to perform a test of $H_{0}$ : All 12 astrological signs are equally likely versus $H_{a}$ : All 12 astrological signs are not equally likely. Or, $H_{0}: p_{i}=\frac{1}{12}$ versus $H_{a}$ : At least two of the $p_{i}$ 's is incorrect, where each $p_{i}$ is the true proportion of people with each astrological sign. We will use $\alpha=0.05$. Plan: We should use a chi-square test for goodness of fit if the conditions are met. Random: The data come from a random sample. $10 \%: n=4344$ is less than $10 \%$ of all people in the United States. Large Counts: The expected counts in each category are all at least 5 (because the total sample size is 4344 and there are 12 months, the expected count is $4344\left(\frac{1}{12}\right)=362$ for each month). Do: The test statistic is
$\chi^{2}=\frac{(321-362)^{2}}{362}+\frac{(360-362)^{2}}{362}+\ldots+\frac{(355-362)^{2}}{362}=19.76$. With $\mathrm{df}=12-1=11$, the $P$-value
is between 0.025 and 0.05 . Using technology: $P$-value $=0.0487$. Conclude: Because the $P$-value of 0.0487 is less than $\alpha=0.05$, we reject $H_{0}$. There is convincing evidence that the 12 astrological signs are not equally likely.
Follow-up analysis: The breakdown of the chi-square statistic is: $\chi^{2}=4.64+0.01+0.07+$ $0.40+1.22+4.42+2.49+3.01+2.65+0.18+0.54+0.14$. From this we see that the largest contributors to the statistic are Aries and Virgo. There are fewer Aries (321-362=-41) and more Virgos (402-362 = 40) than we would expect.
11.16 State: We want to perform a test at the $\alpha=0.05$ significance level of $H_{0}$ : Froot Loops contain an equal proportion of each flavor versus $H_{a}$ : Froot Loops do not contain an equal proportion of each flavor. Or, $H_{0}: p_{i}=\frac{1}{6}$ versus $H_{a}$ : At least two of the $p_{i}$ 's is incorrect, where each $p_{i}$ is the true proportion of Froot Loops of each flavor. Plan: We should use a chi-square test for goodness of fit if the conditions are met. Random: The data come from a random sample. $10 \%$ : $n=120$ is less than $10 \%$ of all Froot Loops. Large Counts: The expected counts in each category are all at least 5 (because the total sample size is 120 , the expected count in each category is $120\left(\frac{1}{6}\right)=20$ ). Do: The test statistic is $\chi^{2}=\frac{(28-20)^{2}}{20}+\frac{(21-20)^{2}}{20}+\frac{(16-20)^{2}}{20}+\frac{(25-20)^{2}}{20}+\frac{(14-20)^{2}}{20}+\frac{(16-20)^{2}}{20}=7.9$. With $\mathrm{df}=6-1=5$, the $P$-value is between 0.15 and 0.20 . Using technology: $P$-value $=0.1618$. Conclude: Because the $P$-value of 0.1618 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that Froot Loops do not contain an equal proportion of each flavor. Follow-up analysis: Not necessary.
11.17 State: We want to perform a test at the $\alpha=0.05$ significance level of $H_{0}$ : Mendel's $3: 1$ genetic model is correct versus $H_{a}$ : Mendel's $3: 1$ genetic model is not correct. Or, $H_{0}: p_{\text {smooth }}=0.75, p_{\text {wrinkled }}=0.25$ versus $H_{a}:$ Both of the proportions are incorrect. Plan: We should use a chi-square test for goodness of fit if the conditions are met. We are told to assume the conditions for inference are met. Do: The test statistic is
$\chi^{2}=\frac{(423-417)^{2}}{417}+\frac{(133-139)^{2}}{139}=0.3453$. With $\mathrm{df}=2-1=1$, the $P$-value is greater than 0.25 . Using technology: $P$-value $=0.5568$. Conclude: Because the $P$-value of 0.5568 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that Mendel's 3:1 genetic model is not correct.

11.34 (a) The conditional distributions for each drug are given in the table below along with a graph.

| Relapsed? | Desipramine | Lithium | Placebo |
| :---: | :---: | :---: | :---: |
| Yes | $10 / 24=0.417$ | $18 / 24=0.75$ | $20 / 24=0.833$ |
| No | $14 / 24=0.583$ | $6 / 24=0.25$ | $4 / 24=0.167$ |



Patients who used desipramine had the lowest relapse rate, followed by lithium and then placebo.
(b) State: We want to perform a test of $H_{0}$ : There is no difference in the true proportion of cocaine addicts like these who relapse when using desipramine, lithium, or placebo versus $H_{a}$ : There is a difference in the true proportion of cocaine addicts like these who relapse when using desipramine, lithium, or placebo at the $\alpha=0.05$ level. Plan: We should use a chi-square test for homogeneity if the conditions are met. Random: The data came from 3 groups in a randomized experiment. Large Counts: The expected counts (shown below) are all at least 5.

|  | Desipramine | Lithium | Placebo |
| :---: | :---: | :---: | :---: |
| Yes | 16 | 16 | 16 |
| No | 8 | 8 | 8 |

Do: The test statistic is $\chi^{2}=\frac{(10-16)^{2}}{16}+\ldots+\frac{(4-8)^{2}}{8}=10.5$. With $\mathrm{df}=(2-1)(3-1)=2$, the $P$ value is between 0.005 and 0.01 . Using technology: $P$-value $=0.0052$. Conclude: Because the $P$-value of 0.0052 is less than $\alpha=0.05$, we reject $H_{0}$. We have convincing evidence that there is a difference in the true proportion of cocaine addicts like these who relapse when using desipramine, lithium, or placebo.

11.45 State: We want to perform a test of $H_{0}$ : There is no association between education level and opinion about a handgun ban in the adult population versus $H_{a}$ : There is an association between education level and opinion about a handgun ban in the adult population at the $\alpha=0.05$ level. Plan: We should use a chi-square test for independence if the conditions are met. Random: The data came from a random sample. $10 \%: n=1201$ is less than $10 \%$ of all adults. Large Counts: All expected counts are at least 5 (see table below).

|  | Less than high <br> school | High school <br> grad | Some <br> college | College <br> grad | Postgraduate <br> degree |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Yes | 46.94 | 86.19 | 187.36 | 94.29 | 71.22 |
| No | 69.06 | 126.81 | 275.64 | 138.71 | 104.78 |

Do: The test statistic is $\chi^{2}=\frac{(58-46.94)^{2}}{46.94}+\ldots+\frac{(99-104.78)^{2}}{104.78}=8.525 . \quad$ With $\mathrm{df}=$ $(2-1)(5-1)=4$, the $P$-value is between 0.05 and 0.10 . Using technology: $\chi^{2}=8.525, \mathrm{df}=4$, $P$-value $=0.0741$. Conclude: Because the $P$-value of 0.0741 is greater than $\alpha=0.05$, we fail to reject $H_{0}$. We do not have convincing evidence that there is an association between educational level and opinion about a handgun ban in the adult population.



