Chapter 8: Binomial and Geometric Distribution

Section 8.2
Geometric Distributions

The Practice of Statistics, 4th edition – For AP*

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Section 8.2
Geometric Distribution

Learning Objectives

After this section, you should be able to…

- ✓ CALCULATE probabilities involving geometric random variables

Geometric Settings

In a binomial setting, the number of trials n is fixed and the binomial random variable X counts the number of successes. In other situations, the goal is to repeat a chance behavior *until a success occurs*. These situations are called **geometric settings.**

Definition:

A **geometric setting** arises when we perform independent trials of the same chance process and record the number of trials until a particular outcome occurs. The four conditions for a geometric setting are

B • **B**inary? The possible outcomes of each trial can be classified as "success" or "failure."

I • **I**ndependent? Trials must be independent; that is, knowing the result of one trial must not have any effect on the result of any other trial.

T • **T**rials? The goal is to count the number of trials until the first success occurs.

S • **S**uccess? On each trial, the probability p of success must be the same.

Geometric Random Variable

In a geometric setting, if we define the random variable Y to be the number of trials needed to get the first success, then Y is called a **geometric random variable**. The probability distribution of Y is called a **geometric distribution.**

Definition:

The number of trials Y that it takes to get a success in a geometric setting is a **geometric random variable.** The probability distribution of Y is a **geometric distribution** with parameter p, the probability of a success on any trial. The possible values of Y are 1, 2, 3, ….

***Note*: Like binomial random variables, it is important to be able to distinguish situations in which the geometric distribution does and doesn't apply!**

Example: The Birthday Game

Read the activity on page 398. The random variable of interest in this game is Y = the number of guesses it takes to correctly identify the birth day of one of your teacher's friends. What is the probability the first student guesses correctly? The second? Third? What is the probability the k^{th} student guesses corrrectly?

Verify that *Y* is a geometric random variable.

B: Success = correct guess, Failure = incorrect guess
I: The result of one student's guess has no effect on the result of any other guess.
T: We're counting the number of guesses up to and including the first correct guess.
S: On each trial, the probability of a correct guess is 1/7.

Calculate *P*(*Y* = 1), *P*(*Y* = 2), *P*(*Y* = 3), and *P*(*Y* = k)

$$P(Y=1)=1/7$$
$$P(Y=2)=(6/7)(1/7)=0.1224$$
$$P(Y=3)=(6/7)(6/7)(1/7)=0.1050$$

Notice the pattern?

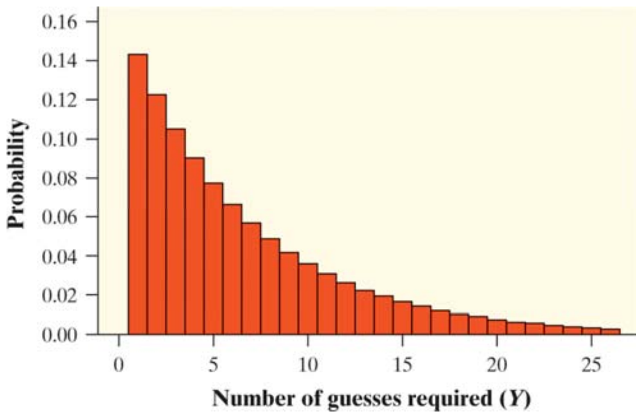
Geometric Probability

If Y has the geometric distribution with probability p of success on each trial, the possible values of Y are 1, 2, 3, … . If k is any one of these values,

$$P(Y=k)=(1-p)^{k-1}p$$

Mean of a Geometric Distribution

The table below shows part of the probability distribution of Y. We can't show the entire distribution because the number of trials it takes to get the first success could be an incredibly large number.



y_i	1	2	3	4	5	6	…
p_i	0.143	0.122	0.105	0.090	0.077	0.066	

Shape: The heavily right-skewed shape is characteristic of any geometric distribution. That's because the most likely value is 1.

Center: The mean of Y is $\mu_Y = 7$. We'd expect it to take 7 guesses to get our first success.

Spread: The standard deviation of Y is $\sigma_Y = 6.48$. If the class played the Birth Day game many times, the number of homework problems the students receive would differ from 7 by an average of 6.48.

If Y is a geometric random variable with probability p of success on each trial, then its **mean** (expected value) is $E(Y) = \mu_Y = 1/p$.

$$\sigma_Y = \sqrt{\frac{(1-p)}{p^2}}$$

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Summary

In this section, we learned that…

- ✓ A **geometric setting** consists of repeated trials of the same chance process in which each trial results in a success or a failure; trials are independent; each trial has the same probability *p* of success; and the goal is to count the number of trials until the first success occurs. If *Y* = the number of trials required to obtain the first success, then *Y* is a **geometric random variable**. Its probability distribution is called a **geometric distribution**.

- ✓ If *Y* has the geometric distribution with probability of success *p*, the possible values of *Y* are the positive integers 1, 2, 3, ... The **geometric probability** that *Y* takes any value is

$$P(Y = k) = (1 - p)^{k-1} p$$

- ✓ The **mean** (expected value) of a geometric random variable *Y* is 1/p.

Looking Ahead…

Homework…

Chapter 8: #'s 37, 39, 40, 44, 45