Chapter 6: Probability: What are the Chances?

Section 6.1
Randomness and Probability

The Practice of Statistics, 4th edition – For AP*

STARNES, YATES, MOORE

Section 6.1
Randomness and Probability

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE the idea of probability
- ✓ DESCRIBE myths about randomness
- ✓ DESIGN and PERFORM simulations

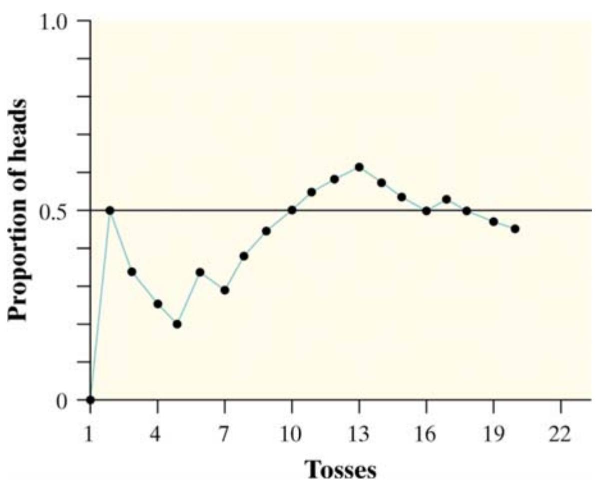
The Idea of Probability

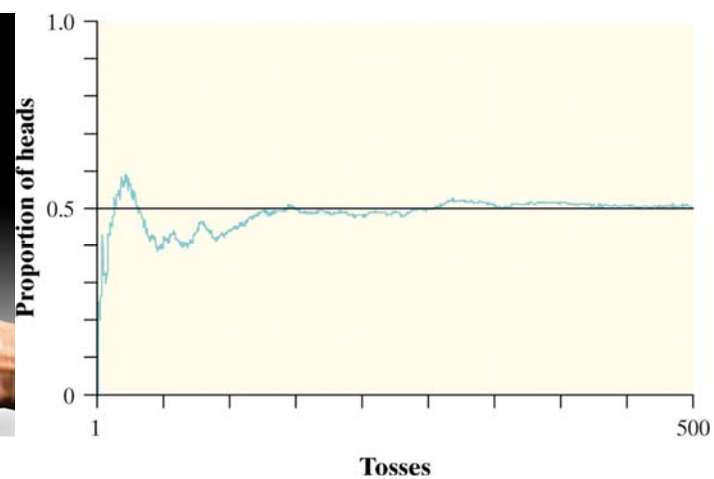
Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run.

The **law of large numbers** says that if we observe more and more repetitions of any chance process, the proportion of times that a specific outcome occurs approaches a single value.

Definition:

The **probability** of any outcome of a chance process is a number between 0 (never occurs) and 1(always occurs) that describes the proportion of times the outcome would occur in a very long series of repetitions.





Section 6.1
Randomness and Probability

Summary

In this section, we learned that…

- ✓ A chance process has outcomes that we cannot predict but have a regular distribution in many distributions.
- ✓ The **law of large numbers** says the proportion of times that a particular outcome occurs in many repetitions will approach a single number.
- ✓ The long-term relative frequency of a chance outcome is its **probability** between 0 (never occurs) and 1 (always occurs).
- ✓ Short-run regularity and the law of averages are myths of probability.

Chapter 6: Probability: What are the Chances?

Section 6.2
Probability Rules

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Section 6.2
Probability Rules

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE chance behavior with a probability model
- ✓ DEFINE and APPLY basic rules of probability

Probability Models

In Section 6.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

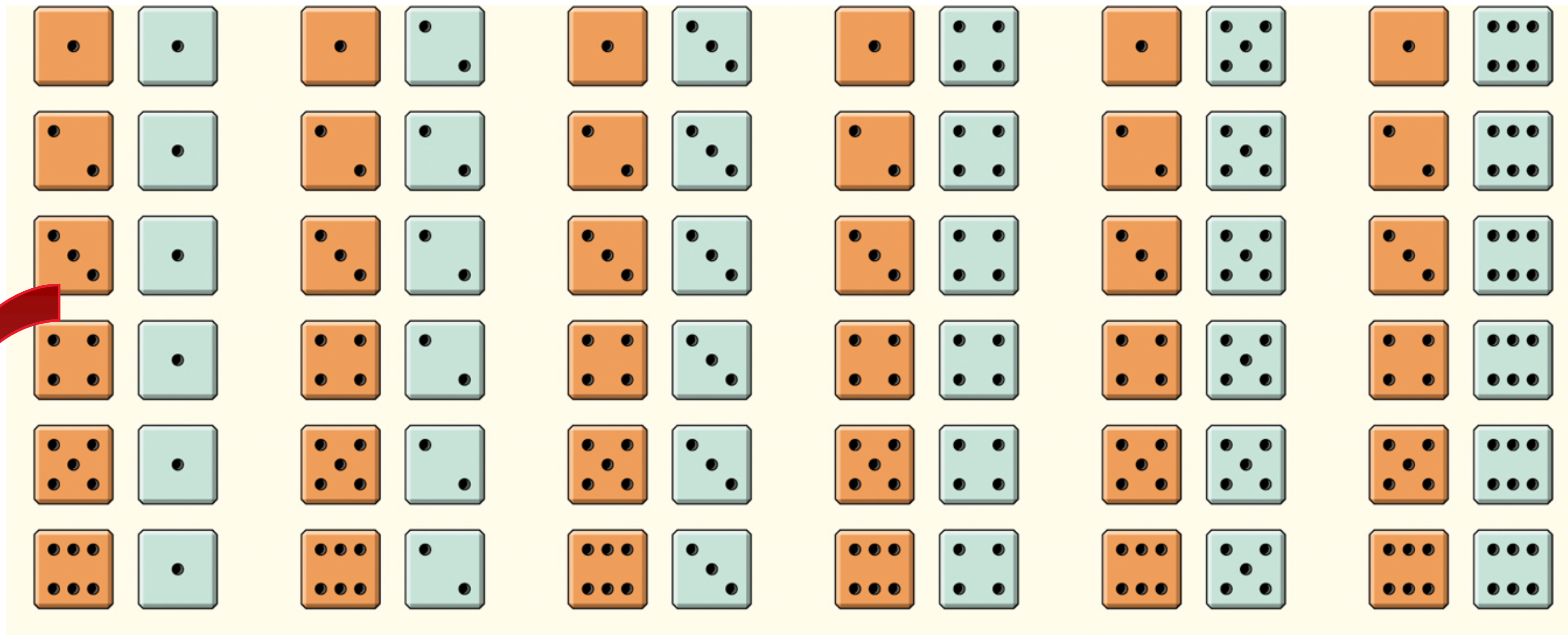
Definition:

The **sample space *S*** of a chance process is the set of all possible outcomes.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome.

Example: Roll the Dice

Give a probability model for the chance process of rolling two fair, six-sided dice – one that's red and one that's green.



Sample Space 36 Outcomes

Since the dice are fair, each outcome is equally likely. Each outcome has probability 1/36.

Probability Models

Probability models allow us to find the probability of any collection of outcomes.

Definition:

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*).

In the dice-rolling example, suppose we define event *A* as "sum is 5."

There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Suppose event *B* is defined as "sum is not 5." What is P(*B*)? *P*(*B*) = 1 – 4/36 = 32/36

Basic Rules of Probability

All probability models must obey the following rules:

- **The probability of any event is a number between 0 and 1.**
- **All possible outcomes together must have probabilities whose sum is 1.**
- **If all outcomes in the sample space are equally likely, the probability that event *A* occurs can be found using the formula**

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **The probability that an event does not occur is 1 minus the probability that the event does occur.**
- **If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.**

Definition:

Two events are **mutually exclusive (disjoint)** if they have no outcomes in common and so can never occur together.

Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

$$P(S) = 1.$$

- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B).$$

Example: Distance Learning

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12

(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and
0.57 + 0.17 + 0.14 + 0.12 = 1

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

***P*(not 18 to 23 years) = 1 – *P*(18 to 23 years)**
= 1 – 0.57 = 0.43

Section 6.2
Probability Rules

Summary

In this section, we learned that…

- ✓ A **probability model** describes chance behavior by listing the possible outcomes in the **sample space *S*** and giving the probability that each outcome occurs.
- ✓ An **event** is a subset of the possible outcomes in a chance process.
- ✓ For any event A, $0 \le P(A) \le 1$
- ✓ $P(S) = 1$, where S = the sample space
- ✓ If all outcomes in S are equally likely, $P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$
- ✓ $P(A^C) = 1 - P(A)$, where A^C is the **complement** of event A; that is, the event that A does not happen.

Section 6.2
Probability Rules

Summary

In this section, we learned that…

- ✓ Events A and B are **mutually exclusive (disjoint)** if they have no outcomes in common. If A and B are disjoint, $P(A \text{ or } B) = P(A) + P(B)$.

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Homework…

Chapter 6, #'s: 2, 17, 19, 21-23, 25, 26, 31, 32, 33