7

Random Variables

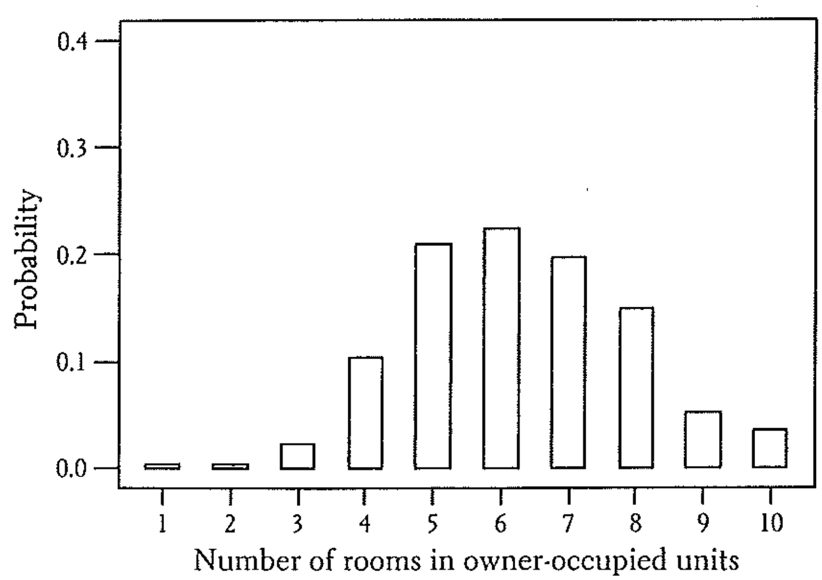
7.1 (a) $P(\text{less than } 3) = P(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$. (b)–(c) Answers vary.

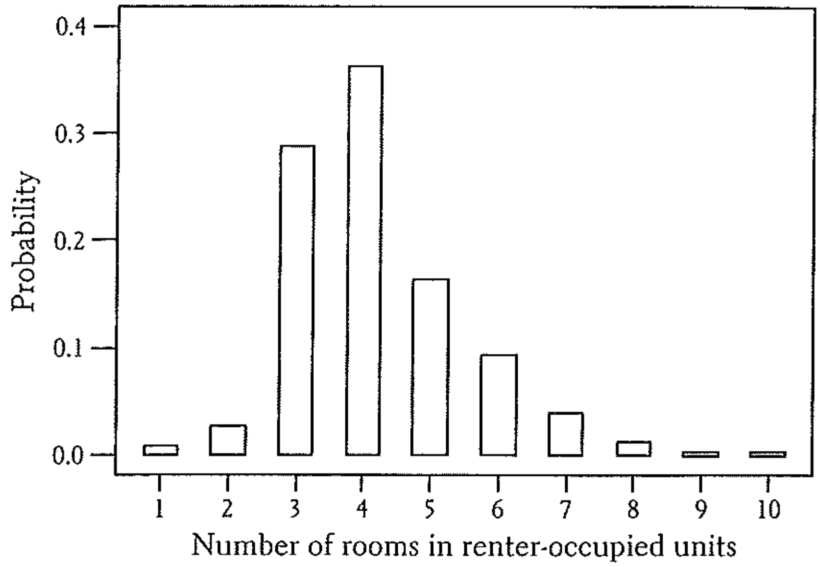
7.2 (a) BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG. Each has probability 1/8. (b) Three of the eight arrangements have two (and only two) girls, so $P(X = 2) = 3/8 = 0.375$. (c) See table.

Value of X	0	1	2	3
Probability	1/8	3/8	3/8	1/8

7.3 (a) 1%. (b) All probabilities are between 0 and 1; the probabilities add to 1. (c) $P(X \leq 3) = 0.48 + 0.38 + 0.08 = 1 - 0.01 - 0.05 = 0.94$. (d) $P(X < 3) = 0.48 + 0.38 = 0.86$. (e) Write either $X \geq 4$ or $X > 3$. The probability is $0.05 + 0.01 = 0.06$. (f) Read two random digits from Table B. Here is the correspondence: 01 to 48 ⇔ Class 1, 49 to 86 ⇔ Class 2, 87 to 94 ⇔ Class 3, 95 to 99 ⇔ Class 4, and 00 ⇔ Class 5. Repeatedly generate 2 digit random numbers. The proportion of numbers in the range 01 to 94 will be an estimate of the required probability.

7.4





The rooms distribution is skewed to the right for renters and roughly symmetric for owners. This suggests that renter-occupied units tend, on the whole, to have fewer rooms than owner-occupied units.

7.5 (a) $\{X \geq 5\}$. $P(X \geq 5) = P(X = 5) + P(X = 6) + \ldots + P(X = 10) = 0.868$.

(b) $\{X > 5\}$ = the event that the unit has more than five rooms. $P(X > 5) = P(X = 6) + P(X = 7) + \ldots + P(X = 10) = 0.658$.

(c) A discrete random variable has a countable number of values, each of which has a distinct probability ($P(X = x)$). $P(X \geq 5)$ and $P(X > 5)$ are different because the first event contains the value $X = 5$ and the second does not.

7.6 (a) $P(0 \leq X \leq 0.4) = 0.4$.

(b) $P(0.4 \leq X \leq 1) = 0.6$.

(c) $P(0.3 \leq X \leq 0.5) = 0.2$.

(d) $P(0.3 < X < 0.5) = 0.2$.

(e) $P(0.226 \leq X \leq 0.713) = 0.713 - 0.226 = 0.487$.

(f) A continuous distribution assigns probability 0 to every individual outcome. In this case, the probabilities in (c) and (d) are the same because the events differ by 2 individual values, 0.3 and 0.5, each of which has probability 0.

7.7 (a) $P(X \leq 0.49) = 0.49$.

(b) $P(X \geq 0.27) = 0.73$.

(c) $P(0.27 < X < 1.27) = P(0.27 < X < 1) = 0.73$.

(d) $P(0.1 \leq X \leq 0.2 \text{ or } 0.8 \leq X \leq 0.9) = 0.1 + 0.1 = 0.2$.

(e) $P(\text{not } [0.3 \leq X \leq 0.8]) = 1 - 0.5 = 0.5$.

(f) $P(X = 0.5) = 0$.

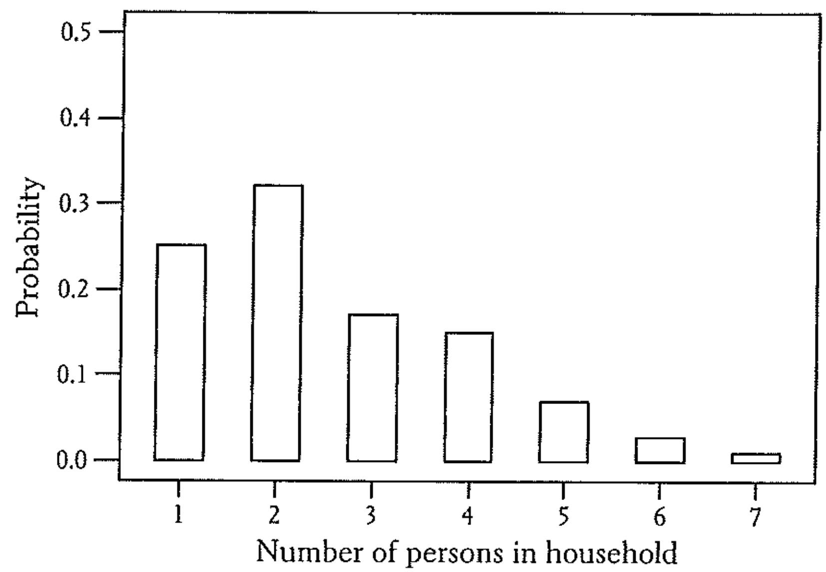
7.8 (a) $P(\hat{p} \geq 0.45) = P\left(Z \geq \frac{0.45 - 0.4}{0.023}\right) = P(Z \geq 2.17) = 0.0150$.

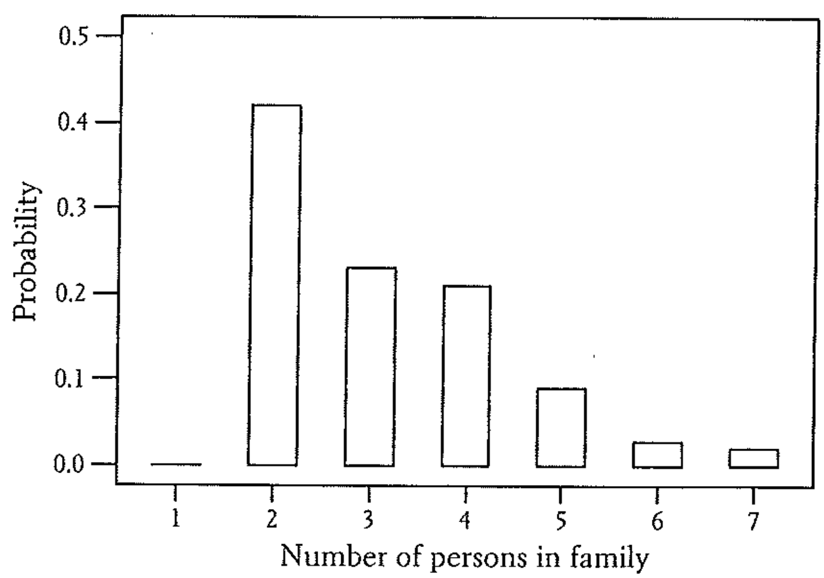
(b) $P(\hat{p} < 0.35) = P(Z < -2.17) = 0.0150$.

(c) $P(0.35 \leq \hat{p} \leq 0.45) = P(-2.17 \leq Z \leq 2.17) = 0.9700$.

7.9 For a *sample* simulation of 400 observations from the N(0.4, 0.023) distribution, there were 0 observations less than 0.25, so the relative frequency is $0/400 = 0$. The actual probability that $\hat{p} < 0.25$ is $P(Z < -6.52) \approx 3.5 \times 10^{-11}$, essentially 0.

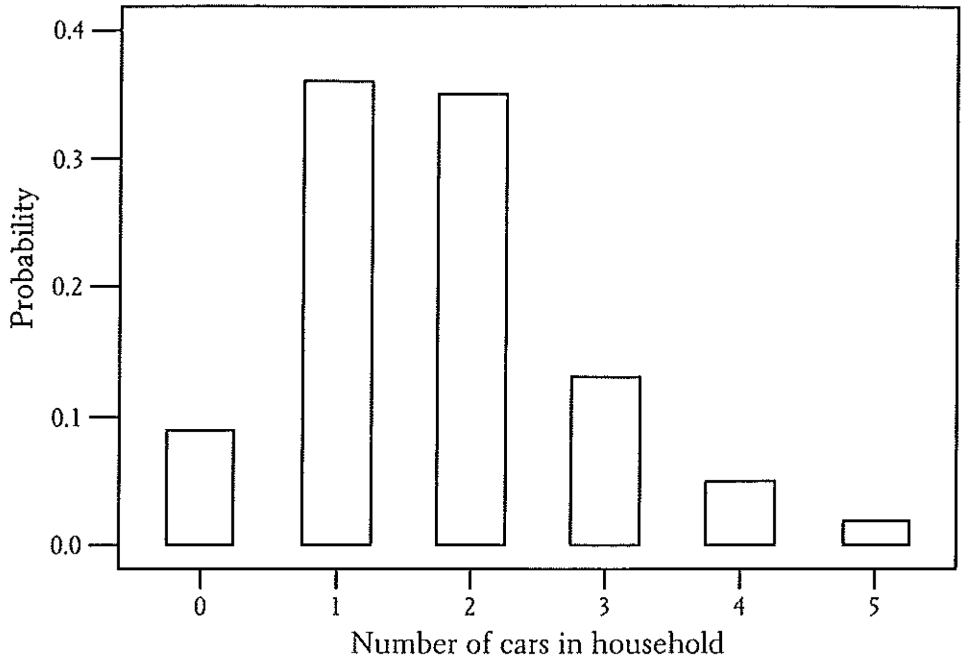
7.10 (a) Both sets of probabilities sum to 1. (b) Both distributions are skewed to the right; however, the event $\{X = 1\}$ has a higher probability in the household distribution. This reflects the fact that a family must consist of two or more persons. Also, the events $\{X = 3\}$ and $\{X = 4\}$ have slightly higher probabilities in the family distribution, which may reflect the fact that families are more likely than households to have children living in the dwelling unit.





7.11 (a) $\{Y > 1\}$. $P(Y > 1) = P(Y = 2) + P(Y = 3) + \ldots + P(Y = 7) = 0.75$. Or, $P(Y > 1) = 1 - P(Y = 1) = 1 - .25 = .75$. (b) $P(2 < Y \leq 4) = P(Y = 3) + P(Y = 4) = 0.32$. (c) $P(Y \neq 2) = 1 - P(Y = 2) = 0.68$.

7.12 (a) The probabilities sum to 1.



(b) $\{X \geq 1\}$ = the event that the household owns at least one car. $P(X \geq 1\} = P(X = 1) + P(X = 2) + \ldots + P(X = 5) = 0.91$.

(c) $P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) = 0.20$. 20% of households own more cars than a two-car garage can hold.

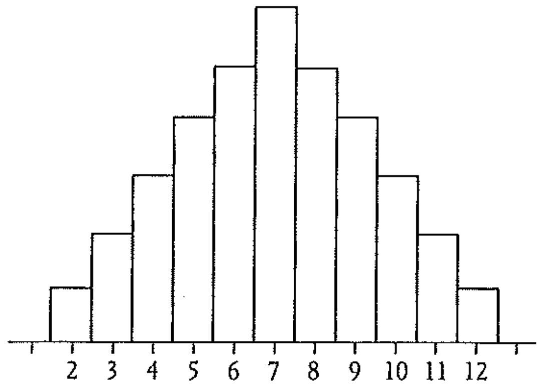
7.13 (a) The 36 possible pairs of "up faces" are

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(b) Each pair must have probability 1/36.

(c) Let x = sum of up faces. Then

Sum	Outcomes	Probability
$x = 2$	(1, 1)	$p = 1/36$
$x = 3$	(1, 2) (2, 1)	$p = 2/36$
$x = 4$	(1, 3) (2, 2) (3, 1)	$p = 3/36$
$x = 5$	(1, 4) (2, 3) (3, 2) (4, 1)	$p = 4/36$
$x = 6$	(1, 5) (2, 4) (3, 3) (4, 2) (5, 1)	$p = 5/36$
$x = 7$	(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)	$p = 6/36$
$x = 8$	(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)	$p = 5/36$
$x = 9$	(3, 6) (4, 5) (5, 4) (6, 3)	$p = 4/36$
$x = 10$	(4, 6) (5, 5) (6, 4)	$p = 3/36$
$x = 11$	(5, 6) (6, 5)	$p = 2/36$
$x = 12$	(6, 6)	$p = 1/36$



(d) $P(7 \text{ or } 11) = 6/36 + 2/36 = 8/36$ or $2/9$. (e) P (any sum other than 7) $= 1 - P(7) = 1 - 6/36 = 30/36 = 5/6$ by the complement rule.

7.14 Here is a table of the possible observations of Y that can occur when we roll one standard die and one "weird" die. As in Problem 7.13, there are 36 possible pairs of faces; however, a number of the pairs are identical to each other.

	1	2	3	4	5	6
0	1	2	3	4	5	6
0	1	2	3	4	5	6
0	1	2	3	4	5	6
6	7	8	9	10	11	12
6	7	8	9	10	11	12
6	7	8	9	10	11	12

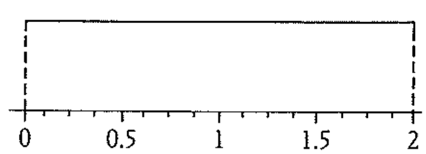
The possible values of Y are 1, 2, 3, 4, ..., 12. Each value of Y has probability $\frac{3}{36} = \frac{1}{12}$.

7.15 (a) 75.2%. (b) All probabilities are between 0 and 1; the probabilities add to 1. (c) $P(X \geq 6) = 1 - 0.010 - 0.007 = 0.983$. (d) $P(X > 6) = 1 - 0.010 - 0.007 - 0.007 = 0.976$. (e) Either $X \geq 9$ or $X > 8$. The probability is $0.068 + 0.070 + 0.041 + 0.752 = 0.931$.

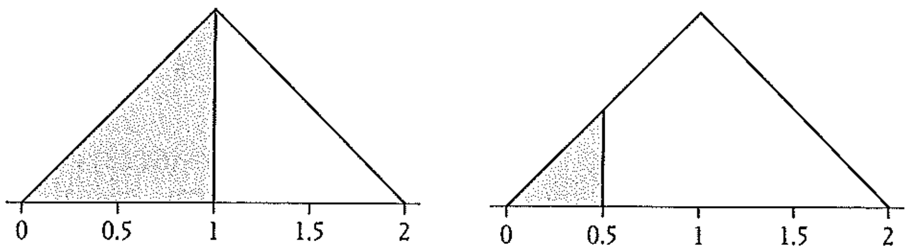
7.16 (a) $(0.6)(0.6)(0.4) = 0.144$. (b) The possible combinations are SSS, SSO, SOS, OSS, SOO, OSO, OOS, OOO (S = support, O = oppose). $P(SSS) = 0.6^3 = 0.216$, $P(SSO) = P(SOS) = P(OSS) = (0.6^2)(0.4) = 0.144$, $P(SOO) = P(OSO) = P(OOS) = (0.6)(0.4^2) = 0.096$, and $P(OOO) = 0.4^3 = 0.064$. (c) The distribution is given in the table: The probabilities are found by adding the probabilities from (b), noting that (e.g.) $P(X = 1) = P(SSO \text{ or } SOS \text{ or } OSS)$. (d) Write either $X \geq 2$ or $X > 1$. The probability is $0.288 + 0.064 = 0.352$.

Value of X	0	1	2	3
Probability	0.216	0.432	0.288	0.064

7.17 (a) The height should be $\frac{1}{2}$, since the area under the curve must be 1. The density curve is below. (b) $P(y \leq 1) = \frac{1}{2}$, (c) $P(0.5 < y < 1.3) = 0.4$. (d) $P(y \geq 0.8) = 0.6$.



7.18 (a) The area of a triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$. (b) $P(Y < 1) = 0.5$. (c) $P(Y < 0.5) = 0.125$.



7.19 The resulting histogram should *approximately* resemble the triangular density curve of Figure 7.8, with any deviations or irregularities depending upon the specific random numbers generated.

7.20 (a) $P(\hat{p} \geq 0.16) = P(Z \geq \frac{0.16 - 0.15}{0.0092}) = P(Z \geq 1.09) = 0.1379$. (b) $P(0.14 \leq \hat{p} \leq 0.16) = P(-1.09 \leq Z \leq 1.09) = 0.7242$.

7.21 In this case, we will simulate 500 observations from the N(.15, .0092) distribution. The required TI-83 commands are as follows:

ClrList L_1
randNorm (.15, .0092, 500) $\rightarrow L_1$
sortA(L_1)

Scrolling through the 500 simulated observations, we can determine the relative frequency of observations that are at least .16 by using the complement rule. For a sample simulation, there are 435 observations less than .16, thus the desired relative frequency is $1 - 435/500 = 65/500 = .13$. The actual probability that $p \geq .16$ is .1385. 500 observations yield a reasonably close approximation.

7.22 $\mu = (0)(0.10) + (1)(0.15) + (2)(0.30) + (3)(0.30) + (4)(0.15) = 2.25$.

7.23 Owner-occupied units: $\mu = (1)(.003) + (2)(.002) + (3)(.023) + (4)(.104) + (5)(.210) + (6)(.224) + (7)(.197) + (8)(.149) + (9)(.053) + (10)(.035) = 6.284$.

Renter-occupied units: $\mu = (1)(.008) + (2)(.027) + (3)(.287) + (4)(.363) + (5)(.164) + (6)(.093) + (7)(.039) + (8)(.013) + (9)(.003) + (10)(.003) = 4.187$.

The larger value of μ for owner-occupied units reflects the fact that the owner distribution was symmetric, rather than skewed to the right, as was the case with the renter distribution. The "center" of the owner distribution is roughly at the central peak class, 6, whereas the "center" of the renter distribution is roughly at the class 4.

7.24 (a) If your number is *abc*, then of the 1000 three-digit numbers, there are six—*abc, acb, bac, bca, cab, cba*—for which you will win the box. Therefore, we win nothing with probability $\frac{994}{1000} = 0.994$ and win \$83.33 with probability $\frac{6}{1000} = 0.006$.

(b) The expected payoff on a \$1 bet is $\mu = (\$0)(0.994) + (\$83.33)(0.006) = \$0.50$.

(c) The casino keeps 50 cents from each dollar bet in the long run, since the expected payoff = 50 cents.

7.25 (a) The payoff is either \$0 or \$3; see table on next page. (b) For each \$1 bet, $\mu_x = (\$0)(0.75) + (\$3)(0.25) = \$0.75$. (c) The casino makes 25 cents for every dollar bet (in the long run).

Value of X	0	3
Probability	0.75	0.25

7.26 In 7.22, we had $\mu = 2.25$, so $\sigma_x^2 = (0 - 2.25)^2(0.10) + (1 - 2.25)^2(0.15) + (2 - 2.25)^2(0.30) + (3 - 2.25)^2(0.30) + (4 - 2.25)^2(0.15) = 1.3875$, and $\sigma_x = \sqrt{1.3875} = 1.178$.

7.27 Mean:

Household size: $\mu = (1)(.25)+(2)(.32)+(3)(.17)+(4)(.15)+(5)(.07)+(6)(.03)+(7)(.01) = 2.6$.

Family size: $\mu = (1)(0)+(2)(.42)+(3)(.23)+(4)(.21)+(5)(.09)+(6)(.03)+(7)(.02) = 3.14$.

Standard deviation:

Household size: $\sigma^2 = (1 - 2.6)^2(.25) + (2 - 2.6)^2(.32) + (3 - 2.6)^2(.17) + (4 - 2.6)^2(.15) + (5 - 2.6)^2(.07) + (6 - 2.6)^2(.03) + (7 - 2.6)^2(.01) = 2.02$, and $\sigma = \sqrt{2.02} = 1.421$.

Family size: $\sigma^2 = (1 - 3.14)^2(0) + (2 - 3.14)^2(.42) + (3 - 3.14)^2(.23) + (4 - 3.14)^2(.21) + (5 - 3.14)^2(.09) + (6 - 3.14)^2(.03) + (7 - 3.14)^2(.02) = 1.5604$, and $\sigma = \sqrt{1.5604} = 1.249$.

The family distribution has a slightly larger mean than the household distribution, reflecting the fact that the family distribution assigns more "weight" (probability) to the values 3 and 4. The two standard deviations are roughly equivalent; the household standard deviation may be larger because of the fact that it assigns a nonzero probability to the value 1.

7.28 We would expect the owner distribution to have a wider spread than the renter distribution. The central "peak" of the owner distribution is more spread out than the left-hand "peak" of the renter distribution, and as a result the average distance between a value and the mean is slightly larger in the owner case.

Owner-occupied units: $\sigma^2 = (1 - 6.284)^2(.003) + (2 - 6.284)^2(.002) + (3 - 6.284)^2(.023) + (4 - 6.284)^2(.104) + (5 - 6.284)^2(.210) + (6 - 6.284)^2(.224) + (7 - 6.284)^2(.197) + (8 - 6.284)^2(.149) + (9 - 6.284)^2(.053) + (10 - 6.284)^2(.035) = 2.689344$, and $\sigma = \sqrt{2.689344} = 1.64$.

Renter-occupied units: $\mu = (1 - 4.187)^2(.008) + (2 - 4.187)^2(.027) + (3 - 4.187)^2(.287) + (4 - 4.187)^2(.363) + (5 - 4.187)^2(.164) + (6 - 4.187)^2(.093) + (7 - 4.187)^2(.039) + (8 - 4.187)^2(.013) + (9 - 4.187)^2(.003) + (10 - 4.187)^2(.003) = 1.710031$, and $\sigma = \sqrt{1.710031} = 1.308$.

7.29 (a) $\mu_x = (0)(0.03) + (1)(0.16) + (2)(0.30) + (3)(0.23) + (4)(0.17) + (5)(0.11) = 2.68$. $\sigma_x^2 = (0 - 2.68)^2(0.03) + (1 - 2.68)^2(0.16) + (2 - 2.68)^2(0.30) + (3 - 2.68)^2(0.23) + (4 - 2.68)^2(0.17) + (5 - 2.68)^2(0.11) = 1.7176$, and $\sigma_x = \sqrt{1.7176} = 1.3106$.

(b) To simulate (say) 500 observations of x, using the T1-83, we will first simulate 500 random integers between 1 and 100 by using the command:

randInt(1,100,500) $\rightarrow$ L_1

The command sortA(L_1) sorts these random observations in increasing order. We now identify 500 observations of x as follows:

Integers	1 to 3	correspond to	$x = 0$
	4 to 19		$x = 1$
	20 to 49		$x = 2$
	50 to 72		$x = 3$
	73 to 89		$x = 4$
	90 to 100		$x = 5$

For a sample run of the simulation, we obtained

12	observations of	$x = 0$
86		$x = 1$
155		$x = 2$
118		$x = 3$
75		$x = 4$
54		$x = 5$

These data yield a sample mean and standard deviation of $\bar{x} = 2.64$, $s = 1.292$, very close to μ and σ.

7.30 The graph for $x_{max} = 10$ displays visible variation for the first ten values of x, whereas the graph for $x_{max} = 100$ gets closer and closer to $\mu = 64.5$ as x increases. This illustrates that the larger the sample size (represented by the integers 1, 2, 3, ... in L_1), the closer the sample means x get to the population mean $\mu = 64.5$. (In other words, this exercise illustrates the law of large numbers in a graphical manner.)

7.31 Below is the probability distribution for L, the length of the longest run of heads or tails. P(You win) $= P$(run of 1 or 2) $= \frac{89}{512} = 0.1738$, so the expected outcome is $\mu = (\$2)(0.1738) + (-\$1)(0.8262) = -\$0.4785$. On the average, you will lose about 48 cents each time you play. (Simulated results should be close to this exact result; how close depends on how many trials are used.)

Value of L	1	2	3	4	5	6	7	8	9	10
Probability	$\frac{1}{512}$	$\frac{88}{512}$	$\frac{185}{512}$	$\frac{127}{512}$	$\frac{63}{512}$	$\frac{28}{512}$	$\frac{12}{512}$	$\frac{5}{512}$	$\frac{2}{512}$	$\frac{1}{512}$

7.32 (a) The wheel is not affected by its past outcomes—it has no memory; outcomes are independent. So on any one spin, black and red remain equally likely.

(b) Removing a card changes the composition of the remaining deck, so successive draws are not independent. If you hold 5 red cards, the deck now contains 5 fewer red cards, so your chance of another red decreases.

7.33 No: Assuming all "at-bat"s are independent of each other, the 35% figure only applies to the "long run" of the season, not to "short runs."

7.34 (a) Independent: Weather conditions a year apart should be independent. (b) Not independent: Weather patterns tend to persist for several days; today's weather tells us something about tomorrow's. (c) Not independent: The two locations are very close together, and would likely have similar weather conditions.

7.35 (a) Dependent: since the cards are being drawn from the deck without replacement, the nature of the third card (and thus the value of Y) will depend upon the nature of the first two cards that were drawn (which determine the value of X).

(b) Independent: X relates to the outcome of the first roll, Y to the outcome of the second roll, and individual dice rolls are independent (the dice have no memory).

7.36 The total mean is $40 + 5 + 25 = 70$ minutes.

7.37 (a) The total mean is $11 + 20 = 31$ seconds. (b) No: Changing the standard deviations does not affect the means. (c) No: The total mean does not depend on dependence or independence of the two variables.

7.38 Assuming that the two times are independent, the total variance is $\sigma_{\text{total}}^2 = \sigma_{\text{pos}}^2 + \sigma_{\text{att}}^2 = 2^2 + 4^2 = 20$, so $\sigma_{\text{total}} = \sqrt{20} = 4.472$ seconds. Assuming that the two times are dependent with correlation 0.3, the total variance is $\sigma_{\text{total}}^2 = \sigma_{\text{pos}}^2 + \sigma_{\text{att}}^2 + 2\rho\sigma_{\text{pos}}\sigma_{\text{att}} = 2^2 + 4^2 + 2(0.3)(2)(4) = 24.8$, so $\sigma_{\text{total}} = \sqrt{24.8} = 4.98$ seconds. The positive correlation of 0.3 indicates that the two times have some tendency to either increase together or decrease together, which increases the variability of their sum.

7.39 Since the two times are independent, the total variance is $\sigma_{\text{total}}^2 = \sigma_{\text{first}}^2 + \sigma_{\text{second}}^2 = 2^2 + 1^2 = 5$, so $\sigma_{\text{total}} = \sqrt{5} = 2.236$ minutes.

7.40 (a) $\sigma_Y^2 = (300 - 445)^2(0.4) + (500 - 445)^2(0.5) + (750 - 455)^2(0.1) = 19.225$ and $\sigma_Y = 138.65$ units. (b) $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 = 7{,}800{,}000 + 19{,}225 = 7{,}819{,}225$, so $\sigma_{X+Y} = 2796.29$ units. (c) $\sigma_Z^2 = \sigma_{2000X}^2 + \sigma_{3500Y}^2 = (2000)^2\sigma_X^2 + (3500)^2\sigma_Y^2$, so $\sigma_Z = \$5{,}606{,}738$.

7.41 If F and L are their respective scores, then $F - L$ has a $N(0, \sqrt{2^2 + 2^2}) = N(0, 2\sqrt{2})$ distribution, so $P(|F - L| > 5) = P(|Z| > 1.7678) = 0.0771$ (table value: 0.0768).

7.42 (a) Let X = the value of the stock after two days. The possible combinations of gains and losses on two days are presented in the table below, together with the calculation of the corresponding values of X.

1st day	2nd day	Value of X
Gain 30%	Gain 30%	1000 + (.3)(1000) = 1300
		1300 + (.3)(1300) = 1690
Gain 30%	Lose 25%	1000 + (.3)(1000) = 1300
		1300 − (.25)(1300) = 975
Lose 25%	Gain 30%	1000 − (.25)(1000) = 750
		750 + (.3)(750) = 975
Lose 25%	Lose 25%	1000 − (.25)(1000) = 750
		750 − (.25)(750) = 562.50

Since the returns on the two days are independent and P(gain 30%) = P(lose 25%) = 0.5, the probability of each of these combinations is (.5)(.5) = .25. The probability distribution of X is therefore

x	1690	975	562.5
$P(X = x)$	0.25	0.5	0.25

The probability that the stock is worth more than \$1000 = $P(X = 1690) = 0.25$.
(b) $\mu = (1690)(.25) + (975)(.5) + (562.5)(.25) = 1050.625$, or approximately \$1051.

7.43 The probability distribution of digits under Benford's Law reflects *long-term* behavior. For each digit v, $P(V = v) \approx$ the long-term relative frequency of v, with the accuracy of the approximation improving as the number of observations increases. We would expect a large number of items to reflect Benford's Law more accurately than a small number of items.

7.44 (a) First die: $\mu = (1)(\frac{1}{6}) + (3)(\frac{1}{6}) + (4)(\frac{1}{6}) + (5)(\frac{1}{6}) + (6)(\frac{1}{6}) + (8)(\frac{1}{6}) = 4.5$.
Second die: $\mu = (1)(\frac{1}{6}) + (2)(^1/_3) + (3)(^1/_3) + (4)(\frac{1}{6}) = 2.5$.

(b) The table on the facing page gives the distribution of X = sum of spots for the two dice. Each of the 36 observations in the table has probability $\frac{1}{36}$.

	1	3	4	5	6	8
1	2	4	5	6	7	9
2	3	5	6	7	8	10
2	3	5	6	7	8	10
3	4	6	7	8	9	11
3	4	6	7	8	9	11
4	5	7	8	9	10	12

The probability distribution of X is:

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

(c) $\mu = (2)(\frac{1}{36}) + (3)(\frac{1}{18}) + (4)(\frac{1}{12}) + (5)(\frac{1}{9}) + (6)(\frac{5}{36}) + (7)(\frac{1}{6}) + (8)(\frac{5}{36}) + (9)(\frac{1}{9}) + (10)(\frac{1}{12}) + (11)(\frac{1}{18}) + (12)(\frac{1}{36}) = 7$.
Using addition rule for means: μ = mean from first die + mean from second die = 4.5 + 2.5 = 7.

7.45 (a) Randomly selected students would presumably be unrelated. (b) $\mu_{f-m} = \mu_f - \mu_m = 120 - 105 = 15$. $\sigma_{f-m}{}^2 = \sigma_f^2 + \sigma_m^2 = 28^2 + 35^2 = 2009$, so $\sigma_{f-m} = 44.82$. (c) Knowing only the mean and standard deviation, we cannot find that probability (unless we assume that the distribution is normal). Many different distributions can have the same mean and standard deviation.

7.46 (a) $\mu_x = 550°$ Celsius; $\sigma_x^2 = 32.5$, so $\sigma_x = 5.701°$C. (b) Mean: 0°C; standard deviation: 5.701°C. (c) $\mu_y = \frac{9}{5}\mu_x + 32 = 1022°$F, and $\sigma_y = \frac{9}{5}\sigma_x = 10.26°$F.

7.47 Read two-digit random numbers. Establish the correspondence 01 to 10 ⇔ 540°, 11 to 35 ⇔ 545°, 36 to 65 ⇔ 550°, 66 to 90 ⇔ 555°, and 91 to 99, 00 ⇔ 560°. Repeat many times, and record the corresponding temperatures. Average the temperatures to approximate μ; find the standard deviations of the temperatures to approximate σ.

7.48 (a) The machine that makes the caps and the machine that applies the torque are not the same. (b) T (torque) is $N(7, 0.9)$ and S (cap strength) is $N(10, 1.2)$, so $T - S$ is $N(-3, \sqrt{0.9^2 + 1.2^2}) = N(-3, 1.5)$. Then $P(T > S) = P(T - S > 0) = P(Z > 2) = 0.0228$.

7.49 (a) Yes: This is always true; it does not depend on independence. (b) No: It is not reasonable to believe that X and Y are independent.

7.50 (a) $R_1 + R_2$ is normal with mean 100+250 = 350Ω and s.d. $\sqrt{2.5^2 + 2.8^2} \doteq 3.7537\Omega$. (b) $P(345 \le R_1 + R_2 \le 355) = P(-1.3320 \le Z \le 1.3320) = 0.8172$ (table value: 0.8164).

7.51 The monthly return on a portfolio of 80% Magellan and 20% Japan can be written as 0.8W + 0.2Y. The mean return is $\mu_{0.8W+0.2Y} = 0.8\mu_W + 0.2\mu_Y = (0.8)(1.14) + (0.2)(1.59) = 1.23\%$, which is higher than μ_W. The variance of return is $\sigma^2_{0.8W+0.2Y} = (0.8)^2\sigma_W^2 + (0.2)^2\sigma_Y^2 + 2\rho_{WY}(0.8\sigma_W)(0.2\sigma_Y) = (0.64)(4.64)^2 + (0.04)(6.75)^2 + 2(0.54)(0.8)(4.64)(0.2)(6.75) = 21.01354$. The standard deviation of return is $\sigma_{0.8W+0.2Y} = \sqrt{21.01354} = 4.584\%$, which is lower than σ_W.

7.52 Assuming that $\rho_{WY} = 0$, the variance of return is $\sigma^2_{0.8W+0.2Y} = (0.8)^2\sigma_W^2 + (0.2)^2\sigma_Y^2 = (0.64)(4.64)^2 + (0.04)(6.75)^2 = 15.601444$, and the standard deviation is $\sigma_{0.8W+0.2Y} = \sqrt{15.601444} = 3.95\%$. This is smaller than the result 4.584% from Problem 7.51. The mean return is not affected by the zero correlation, since it only depends upon the means of the individual returns.

7.53 The monthly return on a portfolio of 60% Magellan, 20% Real Estate, and 20% Japan can be expressed as $0.6W + 0.2X + 0.2Y$. The mean return is $\mu_{0.6W+0.2X+0.2Y} = 0.6\mu_W + 0.2\mu_X + 0.2\mu_Y = (0.6)(1.14) + (0.2)(0.16) + (0.2)(1.59) = 1.034\%$. The variance of return is $\sigma^2_{0.6W+0.2X+0.2Y} = (0.6)^2\sigma^2_W + (0.2)^2\sigma^2_X + (0.2)^2\sigma^2_Y + 2\rho_{WX}(0.6\sigma_W)(0.2\sigma_X) + 2\rho_{WY}(0.6\sigma_W)\ (0.2\sigma_Y) + 2\rho_{XY}(0.2\sigma_X)(0.2\sigma_Y) = (0.36)(4.64)^2 + (0.04)(3.61)^2 + (0.04)(6.75)^2 + 2(0.19)(0.6)(4.64)(0.2)(3.61) + 2(0.54)(0.6)(4.64)(0.2)(6.75) + 2(-0.17)(0.2)(3.61)(0.2)(6.75) = 14.58593224$. The standard deviation of return is $\sqrt{14.58593224} = 3.82\%$.

7.54 (a) Writing (x, y), where x is Ann's choice and y is Bob's choice, the sample space has 16 elements:

(A,A)	(A,B)	(A,C)	(A,D)	(B,A)	(B,B)	(B,C)	(B,D)
0	2	−3	0	−2	0	0	3
(C,A)	(C,B)	(C,C)	(C,D)	(D,A)	(D,B)	(D,C)	(D,D)
3	0	0	−4	0	−3	4	0

(b) The value of X is written below each entry in the table.
(c) Below.
(d) The mean is 0, so the game is fair. The variance is 4.75, so $\sigma_X \doteq 2.1794$.

Value of X	−4	−3	−2	0	2	3	4
Probability	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{8}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

7.55 The missing probability is 0.99058 (so that the sum is 1). This gives mean earnings $\mu_x =$ \$303.3525.

7.56 The mean μ of the company's "winnings" (premiums) and their "losses" (insurance claims) is positive. Even though the company will lose a large amount of money on a small number of policyholders who die, it will gain a small amount on the majority. The law of large numbers says that the average "winnings" minus "losses" should be close to μ, and overall the company will almost certainly show a profit.

7.57 $\sigma^2_X = 94{,}236{,}826.64$, so that $\sigma_X = \$9707.57$.

7.58 (a) $\mu_Z = \frac{1}{2}\mu_T = \mu_X = \$303.3525$. $\sigma_Z = \sqrt{\frac{1}{4}\sigma^2_X + \frac{1}{4}\sigma^2_Y} = \sqrt{\frac{1}{2}\sigma^2_X} = \$6864.29$. (b) With this new definition of Z: $\mu_Z = \mu_X = \$303.3525$ (unchanged). $\sigma_Z = \sqrt{\frac{1}{4}\sigma^2_X} = \frac{1}{2}\sigma_X = \$4853.78$ (smaller by a factor of $1/\sqrt{2}$).

7.59 $X - Y$ is $N(0, \sqrt{0.3^2 + 0.3^2}) \doteq N(0, 0.4243)$, so $P(|X - Y| \geq 0.8) = P(|Z| \geq 1.8856) = 1 - P(|Z| \leq 1.8856) = 0.0593$ (table value: 0.0588).

7.60 (a) $\mu_X = (1)(0.1) + (1.5)(0.2) + (2)(0.4) + (4)(0.2) + (10)(0.1) = 3$ million dollars. $\sigma^2_X = (4)(0.1) + (2.25)(0.2) + (1)(0.4) + (1)(0.2) + (49)(0.1) = 503.375$. so $\sigma_X \doteq 22.436$ million dollars.
(b) $\mu_Y = 0.9\mu_X - 0.2 = 2.5$ million dollars, and $\sigma_Y = 0.9\sigma_X \doteq 20.192$ million dollars.

7.61 (a) $\mu_{Y-X} = \mu_Y - \mu_X = 2.001 - 2.000 = 0.001$ g. $\sigma_{Y-X}{}^2 = \sigma_Y{}^2 + \sigma_X{}^2 = 0.002^2 + 0.001^2 = 0.000005$, so $\sigma_{Y-X} = 0.002236$ g.
(b) $\mu_Z = \frac{1}{2}\mu_X + \frac{1}{2}\mu_Y = 2.0005$ g. $\sigma^2_Z = \frac{1}{4}\sigma^2_X + \frac{1}{4}\sigma^2_Y = 0.00000125$, so $\sigma_Z = 0.001118$ g. Z is slightly more variable than Y, since $\sigma_Y < \sigma_Z$.

7.62 (a) To do one repetition, start at any point in Table B and begin reading digits. As in Example 5.24, let the digits 0, 1, 2, 3, 4 = girl and 5, 6, 7, 8, 9 = boy, and read a string of digits until a "0 to 4" (girl) appears or until four consecutive "5 to 9"s (boys) have appeared, whichever comes first. Then let the observation of x = number of children for this repetition = the number of digits in the string you have read. Repeat this procedure 25 times to obtain your 25 observations.

(b) The possible outcomes and their corresponding values of x = number of children are as follows:

	Outcome	
$x = 1$	G	(first child is a girl)
$x = 2$	BG	(second child is a girl)
$x = 3$	BBG	(third child is a girl)
$x = 4$	BBBG,BBBB	(four children)

Using the facts that births are independent, the fact that B and G are equally likely to occur on any one birth, and the multiplication rule for independent events, we find that

$$\begin{aligned}
P(x = 1) &= 1/2 \\
P(x = 2) &= (1/2)(1/2) = 1/4 \\
P(x = 3) &= (1/2)(1/2)(1/2) = 1/8 \\
P(x = 4) &= (1/2)(1/2)(1/2)(1/2) + (1/2)(1/2)(1/2)(1/2) \\
&= 1/16 + 1/16 = 1/8
\end{aligned}$$

The probability distribution of x is therefore:

x_i	1	2	3	4
p_i	1/2	1/4	1/8	1/8

(c) $$\begin{aligned}
\mu_x &= \Sigma x_i p_i \\
&= (1)(1/2) + (2)(1/4) + (3)(1/8) + (4)(1/8) \\
&= 1/2 + 1/2 + 3/8 + 1/2 \\
&= 1.875
\end{aligned}$$

7.63 (a) A single random digit simulates each toss, with (say) odd = heads and even = tails. The first round is two digits, with two odds a win; if you don't win, look at two more digits, again with two odds a win.

(b) The probability of winning is $\frac{1}{4} + (\frac{3}{4})(\frac{1}{4}) = \frac{7}{16}$, so the expected value is $(\$1)(\frac{7}{16}) + (-\$1)(\frac{9}{16}) = -\frac{2}{16} = -\$0.125$.

7.64 $\mu_X = (\mu - \sigma)(0.5) + (\mu + \sigma)(0.5) = \mu$, and $\sigma_X = \sigma$ since $\sigma_X^2 = [\mu - (\mu - \sigma)]^2(0.5) = [\mu - (\mu + \sigma)]^2(0.5) = \sigma^2(0.5) + \sigma^2(0.5) = \sigma^2$.

7.65 By the general addition rule for variances, $\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y = \sigma^2_X + \sigma^2_Y + 2(1)\sigma_X\sigma_Y = \sigma^2{}_X + \sigma^2_Y + 2\sigma_X\sigma_Y = (\sigma_X + \sigma_Y)^2$. Taking square roots yields $\sigma_{X+Y} = \sigma_X + \sigma_Y$.

7.66 (a) Two standard deviations: $d_1 = 2(0.002) = 0.004$ and $d_2 = 2(0.001) = 0.002$. (b) $\sigma_{X+Y+Z} = \sqrt{0.002^2 + 0.001^2 + 0.001^2} \doteq 0.002449$, so $d \doteq 0.005$—considerably *less* than $d_1 + 2d_2 = 0.008$. The engineer was incorrect.

7.67 We want to find a, b such that $\mu_Y = a + b\mu_X = 0$ and $\sigma^2_Y = b^2\sigma^2_X = 1$. Substituting $\mu_X = 1400$ and $\sigma_X = 20$, we have $a + b(1400) = a + 1400b = 0$ and $b^2(20)^2 = 400b^2 = 1$. Solving the second equation for b yields $b = \frac{1}{20}$. Substituing this value into the first equation and solving for a yields $a = -1400(\frac{1}{20}) = -70$.

7.68 (a) The possible values of X are 3, 4, 5, 6, ..., 18 (all positive integers between 3 and 18).
(b) We get a sum of 5 if and only if either one die shows 3 and the other two dice show l's or two dice show 2's and the third shows 1. Each of these arrangements can occur in three ways, thus the event $\{X = 5\}$ contains 6 outcomes. The total number of possible outcomes when three dice are rolled is $6 \times 6 \times 6 = 216$. $P(X = 5) = \frac{6}{216} = \frac{1}{36}$.
(c) For each X_i, the mean $\mu_i = 3.5$ and the variance $\sigma_i^2 = 2.917$. The mean of the sum $X = \mu_1 + \mu_2 + \mu_3 = 3(3.5) = 10.5$. The X_i are independent, so the variance of the sum $X = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 3(2.917) = 8.751$, and the standard deviation of $X = \sqrt{8.751} = 2.958$.