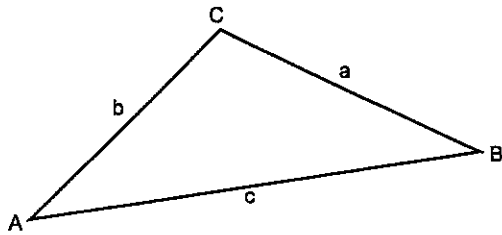


Law of Sines

Given the triangle ABC with lengths a, b, c and angles $A, B,$ and C then,



1. $\frac{\sin A}{a} = \frac{\sin B}{b}$
2. $\frac{\sin C}{c} = \frac{\sin B}{b}$
3. $\frac{\sin A}{a} = \frac{\sin C}{c}$

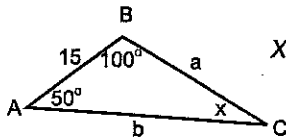
Note: a is the length of the side opposite angle A .

Solve for the all missing sides and angles in each triangle. Round sides to nearest tenth and angles to nearest degree.

Example:

$$100 + 50 + x = 180$$

$$x = 30$$



$$\frac{\sin 50}{a} = \frac{\sin 30}{15}$$

$$\frac{\sin 100}{b} = \frac{\sin 30}{15}$$

$$15 \sin 50 = a \sin 30$$

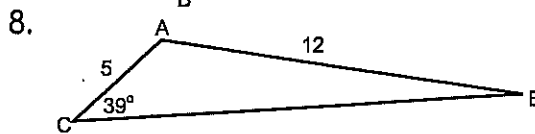
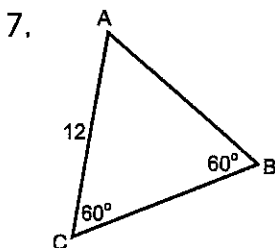
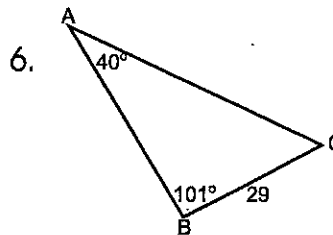
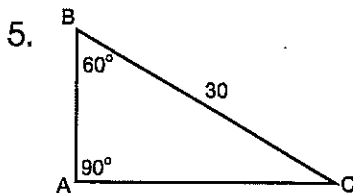
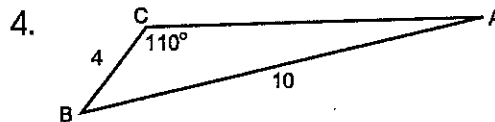
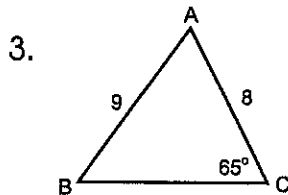
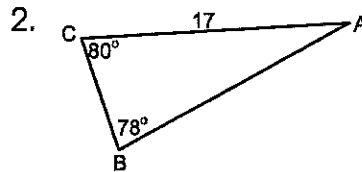
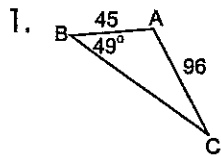
$$15 \sin 100 = b \sin 30$$

$$a = \frac{15 \sin 50}{\sin 30} = 22.981 \quad b = \frac{15 \sin 100}{\sin 30} = 29.50$$

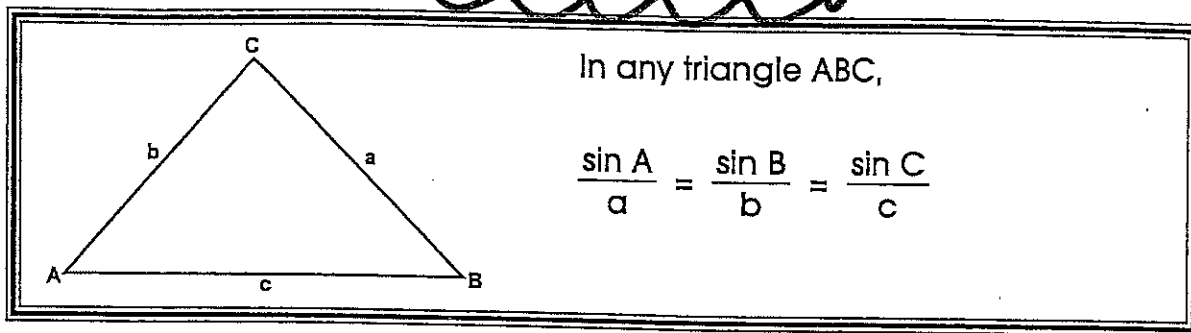
$$a = 22.9$$

$$b = 29.5$$

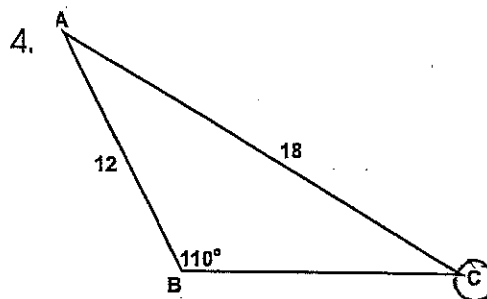
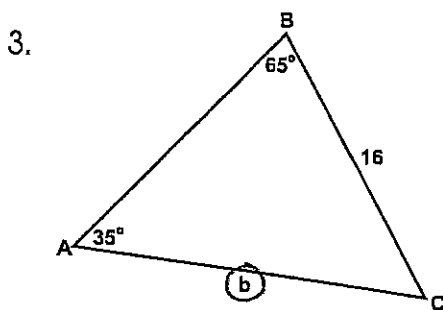
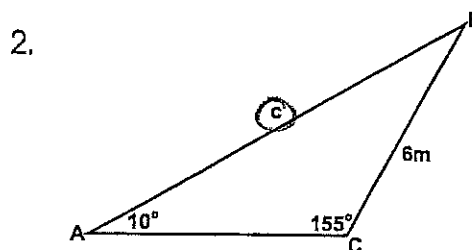
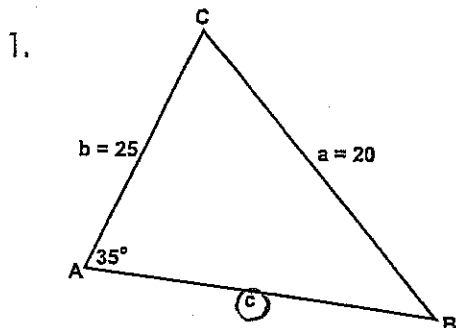
Note: The law of sines works for all triangles.



Laws of Sines



Use the law of sines to state an equation to find the missing part \bigcirc

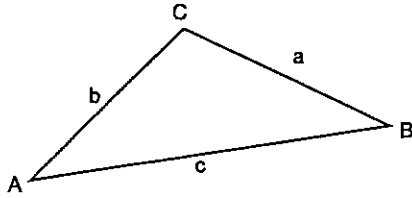


Find the indicated part of $\triangle ABC$. Round angles to the nearest ~~_____~~ ^{degree}, and lengths to the nearest tenth.

5. $c = 10, \angle A = 48^\circ, \angle C = 63^\circ, a = \underline{\hspace{2cm}}$
6. $a = 20, b = 15, \angle A = 40^\circ, \angle B = \underline{\hspace{2cm}}$
7. $a = 40, b = 50, \angle A = 37^\circ, \angle B = \underline{\hspace{2cm}}$
8. $a = 11, c = 15, \angle A = 40^\circ, \angle C = \underline{\hspace{2cm}}$
9. $c = 30, \angle A = 42^\circ, \angle C = 98^\circ, a = \underline{\hspace{2cm}}$
10. $a = 1.5, b = 2.0, \angle B = 35^\circ, \angle A = \underline{\hspace{2cm}}$
11. $a = 16, \angle A = 35^\circ, \angle C = 65^\circ, c = \underline{\hspace{2cm}}$
12. $b = 18, c = 32, \angle C = 100^\circ, \angle B = \underline{\hspace{2cm}}$

Law of Cosines

Given the triangle ABC with lengths a, b, c and angles $A, B,$ and C then,



$$1. a^2 = b^2 + c^2 - 2bc \cos A$$

$$2. b^2 = a^2 + c^2 - 2ac \cos B$$

$$3. c^2 = a^2 + b^2 - 2ab \cos C$$

Note: a is the length of the side opposite angle A .

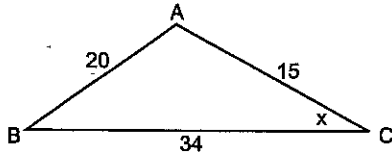
The law of cosines is significantly more difficult to use than that of the law of sines. Consider two cases: The first is solving for side a of a triangle, the second is solving for angle A .

$$1. a^2 = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$2. A = \cos^{-1} \left(\frac{a^2 - b^2 - c^2}{2bc} \right)$$

Solve for x on each triangle. Round length to nearest tenth and angle to the nearest degree.

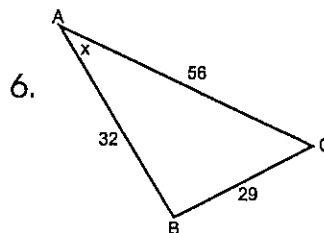
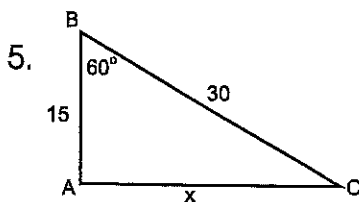
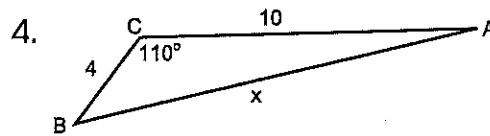
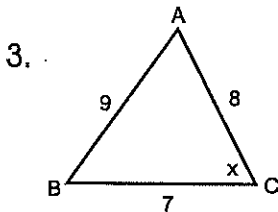
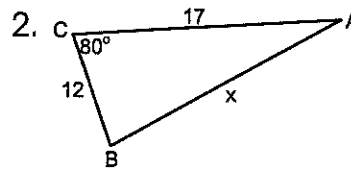
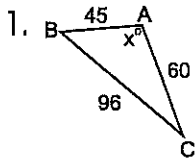
Example:



$$A = \cos^{-1} \left(\frac{34^2}{2} - \frac{15^2 - 20^2}{(15)(20)} \right) = \cos^{-1} \left(\frac{531}{600} \right) = \cos^{-1} (0.885) = 152.25$$

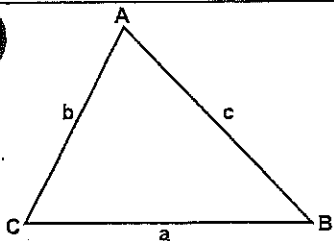
$$A = 152^\circ$$

Note: The law of cosines works for all triangles.



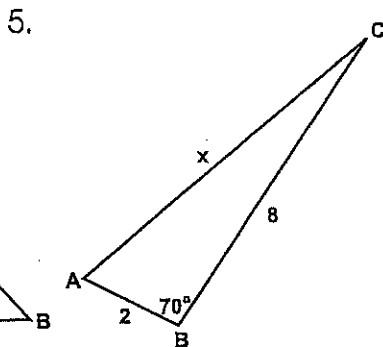
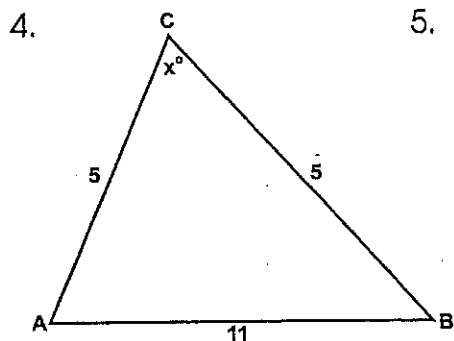
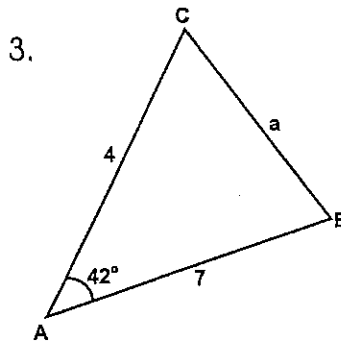
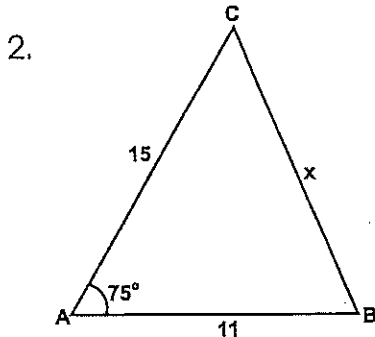
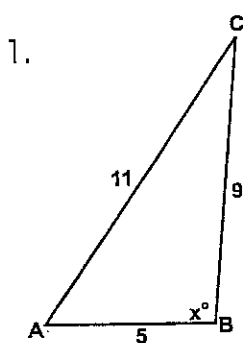
Solving Any Triangle

Law of cosines



In any triangle ABC,
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $c^2 = a^2 + b^2 - 2ab \cos C$

Use the law of cosines to state an equation to find the missing part, x.



Find the indicated part of $\triangle ABC$. Round angles to the nearest ^{degree} and lengths to the nearest tenths

6. $b = 12, c = 10, \angle A = 38^\circ, a = \underline{\hspace{2cm}}$
7. $a = 14, b = 15, c = 18, \angle A = \underline{\hspace{2cm}}$
8. $a = 12, c = 11, \angle B = 81^\circ, b = \underline{\hspace{2cm}}$
9. $a = 8, b = 9, c = 15, \angle C = \underline{\hspace{2cm}}$
10. $a = 5, b = 7, \angle C = 40^\circ, c = \underline{\hspace{2cm}}$
11. $c = 20, b = 30, \angle A = 140^\circ, a = \underline{\hspace{2cm}}$
12. $b = 2, a = 4, \angle C = 20^\circ, c = \underline{\hspace{2cm}}$
13. $a = 5, b = 9, c = 11, \angle C = \underline{\hspace{2cm}}$
14. $a = 1.5, b = 3, c = 2, \angle B = \underline{\hspace{2cm}}$
15. $a = .6, b = .8, c = 1.2, \angle A = \underline{\hspace{2cm}}$

Problem Solving: The Law of Sines and the Law of Cosines

Draw a picture and solve.

1. Juan and Romelia are standing at the seashore 10 miles apart. The coastline is a straight line between them. Both can see the same ship in the water. The angle between the coastline and the line between the ship and Juan is 35 degrees. The angle between the coastline and the line between the ship and Romelia is 45 degrees. How far is the ship from Juan?
2. Jack is on one side of a 200-foot-wide canyon and Jill is on the other. Jack and Jill can both see the trail guide at an angle of depression of 60 degrees. How far are they from the trail guide?
3. Tom, Dick, and Harry are camping in their tents. If the distance between Tom and Dick is 153 feet, the distance between Tom and Harry is 201 feet, and the distance between Dick and Harry is 175 feet, what is the angle between Dick, Harry, and Tom?
4. Three boats are at sea: Jenny one (J1), Jenny two (J2), and Jenny three (J3). The crew of J1 can see both J2 and J3. The angle between the line of sight to J2 and the line of sight to J3 is 45 degrees. If the distance between J1 and J2 is 2 miles and the distance between J1 and J3 is 4 miles, what is the distance between J2 and J3?
5. Airplane A is flying directly toward the airport which is 20 miles away. The pilot notices airplane B 45 degrees to her right. Airplane B is also flying directly toward the airport. The pilot of airplane B calculates that airplane A is 50 degrees to his left. Based on that information, how far is airplane B from the airport?