Trig Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$ $\quad \tan^2\theta = \sec^2 - 1$ $\quad \cot^2\theta = \csc^2\theta - 1$

$\cos^2\theta = 1 - \sin^2\theta$ $\quad \sec^2\theta - \tan^2\theta = 1$ $\quad \csc^2\theta - \cot^2\theta = 1$

Sin $2\theta =$

Cos $2\theta =$

Tan $2\theta =$

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Pre-Calculus II Notes Chapter 6

1. Give the domain of each of the following identities

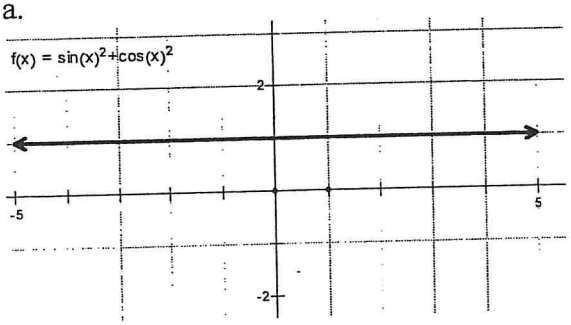
a. $\sin^2 x = 1 - \cos^2 x$ b. $\log t^3 = 3\log t$ c. $\dfrac{x^2-4}{x+2} = x-2$

2. Use your calculator to decide if the following identities are identities. If it is not an identity, show it is not with a counterexample. If it is an identity, identify its domain.

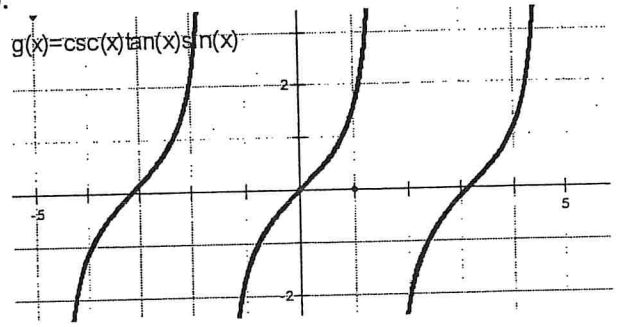
a. $\sec^2 x - \tan^2 x = 1$ b. $\sin\left(\dfrac{\pi}{2} - x\right) = \sin\left(x - \dfrac{\pi}{2}\right)$

3. A graph is given. What identity is suggested by these graphs?

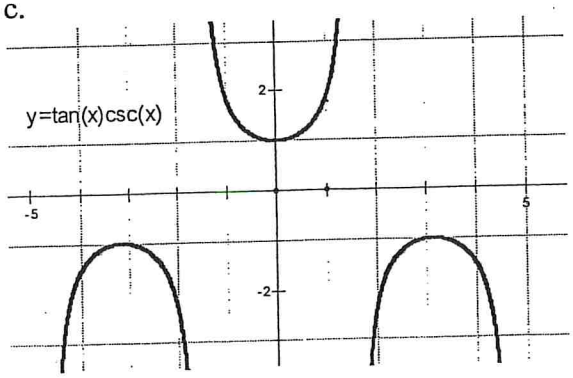
a.



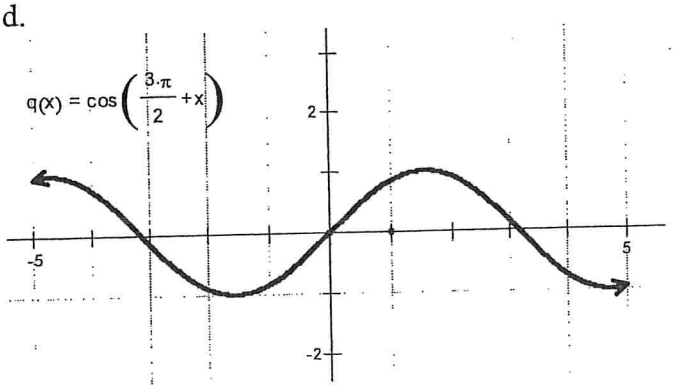
b.



c.



d.



Simplify each expression:

1. $\tan\theta\cos\theta$

2. $\tan(90^\circ - x)$

3. $\cos\left(\frac{\pi}{2} - x\right)$

4. $(1+\sin x)(1-\sin x)$

5. $\sin^2 x - 1$

6. $(\sec x + 1)(\sec x - 1)$

7. $\tan A \cot A$

8. $\cot y \sin y$

9. $\cot^2 x - \csc^2 x$

10. $\dfrac{1}{\cos(90^\circ - \theta)}$

11. $1 - \dfrac{\sin^2\theta}{\tan^2\theta}$

12. $\dfrac{1}{\cos^2\theta} - \dfrac{1}{\cot^2\theta}$

13. $\csc^2 x(1-\cos^2 x)$

14. $\cos\theta(\sec\theta - \cos\theta)$

15. $\dfrac{\sin x\cos x}{1-\cos^2 x}$

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16. $\dfrac{\tan x + \cot x}{\sec^2 x}$

17. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

18. $(\sec^2 \theta - 1)(\csc^2 \theta - 1)$

19. $\dfrac{\tan^2 \theta}{\sec \theta + 1} + 1$

20. $\cos^3 y + \cos y \sin^2 y$

21. $\dfrac{\sec y + \csc y}{1 + \tan y}$

22. $\dfrac{\cos \theta}{1 + \sin \theta} + \dfrac{1 + \sin \theta}{\cos \theta}$

23. $\dfrac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta - \cos^2 \theta}$

Simplify

1. $\sec^2\alpha - 1$

2. $\csc^2 t - \cot^2 t$

3. $\cos t \csc t$

4. $\sin\phi \sec\phi$

5. $\sin\theta \sec\theta \cot\theta$

6. $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x}$

7. $\dfrac{\csc x}{\sin x} - \dfrac{\cot x}{\tan x}$

8. $\dfrac{1}{\sin^2 x} - \dfrac{1}{\tan^2 x}$

9. $\dfrac{\sec t}{\cos t} - \sec t \cos t$

10. $1 - \dfrac{\cos^2\alpha}{\cot^2\alpha}$

11. $\dfrac{1+\tan^2\alpha}{\tan^2\alpha}$

12. $\dfrac{\sec^2 t - 1}{\sec^2 t}$

13. $\dfrac{\csc^2 x - \cot^2 x}{\sec x}$

14. $\cos^2 x^2 + \sin^2 x^2$

15. $\sec^2(\ln x) - \tan^2(\ln x)$

16. $\dfrac{\sin x}{\csc x} - \sin x \csc x$

17. $\dfrac{\sin\theta\cos\theta}{1-\sin^2\theta}$

18. $\tan^2\alpha - \cot^2\alpha + \csc^2\alpha$

19. $\dfrac{1-\sin^2\alpha}{1-\sin\alpha} - 1$

20. $\cos^2 t(\cot^2 t + 1)$

21. $\sec x - \sin x \tan x$

22. $\sin x(\csc x - \sin x)$

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Prove the Stated Identity

1. $\sin^2 x(1+\tan^2 x)=\tan^2 x$

2. $\tan\alpha(\tan\alpha+\cot\alpha)=\sec^2\alpha$

3. $\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$

4. $\cos^2 x-\sin^2 x=1-2\sin^2 x$

5. $\sec^2 x+\csc^2 x=\sec^2 x\csc^2 x$

6. $\tan^2 t-\sin^2 t=\tan^2 t\sin^2 t$

7. $\sec\alpha-\cos\alpha=\sin\alpha\tan\alpha$

8. $\csc\theta-\sin\theta=\cos\theta\cot\theta$

9. $\sec\alpha-\cos\alpha=\sin\alpha\tan\alpha$

10. $\sin^4 x+\cos^4 x+2\sin^2 s\cos^2 x=1$

11. $\dfrac{1}{1-\sin x}-\dfrac{1}{1+\sin x}=2\tan x\sec x$

12. $\dfrac{1-\tan\alpha}{1+\tan\alpha}=\dfrac{\cot\alpha-1}{\cot\alpha+1}$

13. $\sec t+\csc t=(\tan t+\cot t)(\cos t+\sin t)$

14. $\dfrac{\cos\theta}{\sec\theta+\tan\theta}=1-\sin\theta$

15. $\dfrac{1}{1+\sin s}+\dfrac{1}{1-\sin s}=2\sec^2 s$

Verify that each of the following is an identity

1. $(1+\sin\theta)(1-\sin\theta)=\dfrac{1}{\sec^2\theta}$

2. $\cos^2 x\cot^2 x=\cot^2 x-\cos^2 x$

3. $\tan^4 w+2\tan^2 w+1=\sec^4 w$

4. $\sin^2 x(\csc^2 x+\sec^2 x)=\sec^2 x$

5. $\dfrac{\sin x+\cos x}{1-\sin x}=\dfrac{1+\cot x}{\csc x-1}$

6. $\dfrac{1-\tan x}{1+\tan x}=\dfrac{\cot x-1}{\cot x+1}$

Prove Each Identity

1. $\cot^2\theta+\cos^2\theta+\sin^2\theta=\csc^2\theta$

2. $\dfrac{\cot\theta-\tan\theta}{\sin\theta\cos\theta}=\csc^2\theta-\sec^2\theta$

3. $\dfrac{\sin\theta}{\csc\theta}+\dfrac{\cos\theta}{\sec\theta}=\sin\theta\csc\theta$

4. $\dfrac{1-\sin^2\theta}{1+\cot^2\theta}=\sin^2\theta\cos^2\theta$