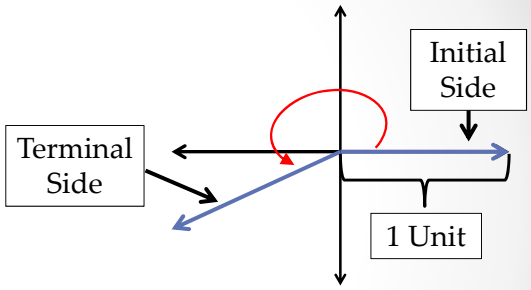
13-2: Angles and the Unit Circle

Algebra 2

Mr. Gallo

Angles in Standard Position

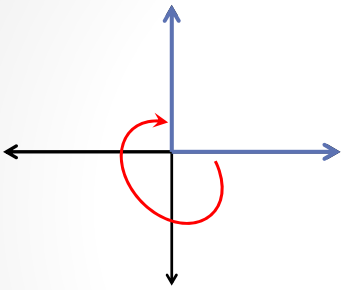
- Vertex at origin; one ray on positive *x*- axis.
- Parts of angles
  - Initial side
    - Ray on (+) *x*-axis
  - Terminal side
    - Other ray



- Measure of angle from initial side to terminal side
  - Counterclockwise (+ measure)
  - Clockwise (- measure)

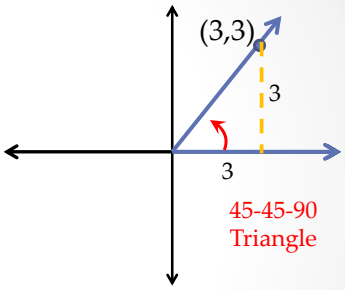
What is the measure of each angle?

1.



-270°

2.



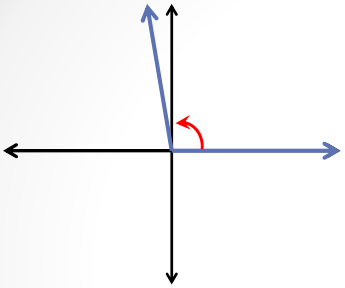
45°

Complete Got It? #1 p.837

225°

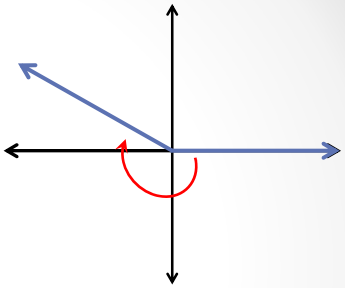
What is a sketch of each angle in standard position?

1.



100°

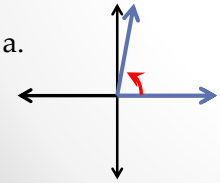
2.



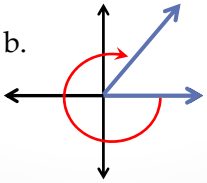
-215°

Complete Got It? #2 p.837

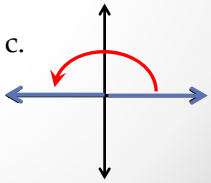
a.



b.

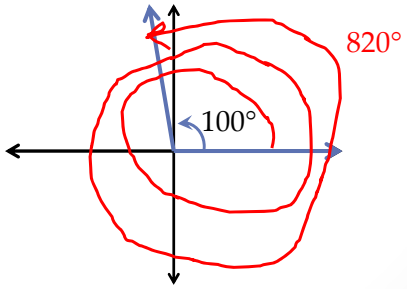


c.



Coterminal Angles

- Two angles in standard position with the same terminal side.
- Unlimited number can be identified by adding or subtracting 360°.



$360 + 360 + 100 = 820°$

Find the angles of smallest possible positive measure coterminal with each angle.

1. 908° 2. −75°

1. Add or subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°.

$$908 - 360 = 548$$
$$548 - 360 = 188$$

An angle of 188°is coterminal with an angle of 908°.

2. Add or subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°.

$$908 - 360 = 548$$
$$548 - 360 = 188$$

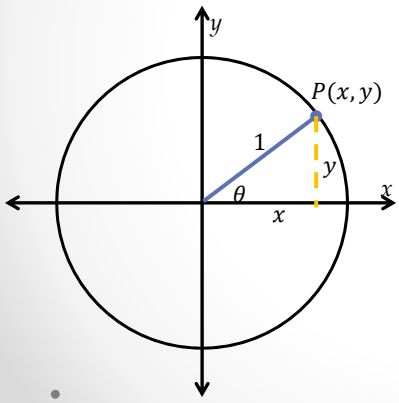
An angle of 285°is coterminal with an angle of −75°.

- Complete Got It? #3 p.838 c. 315°

Homework: p.840 #7-25

Angles and the Unit Circle

- Trig functions of angles are used to describe coordinates



To find the coordinates of P:

1. Draw height of triangle back to x-axis (creates reference angle, θ)
2. Hypotenuse has length of 1 unit, height = y and base = x
3. Use trig functions to find x and y

$$\cos\theta = \frac{x}{1} \qquad \sin\theta = \frac{y}{1}$$

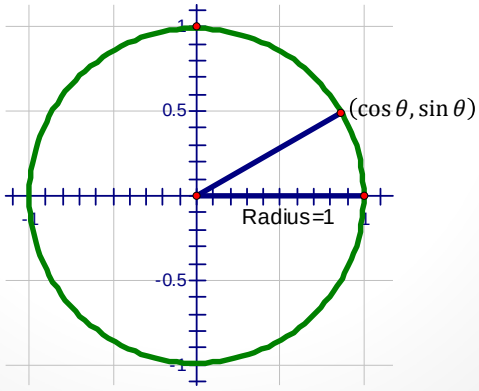
$$\cos\theta = x \qquad \sin\theta = y$$

4. Rewrite the coordinates of P as:

$$P(\cos\theta, \sin\theta)$$

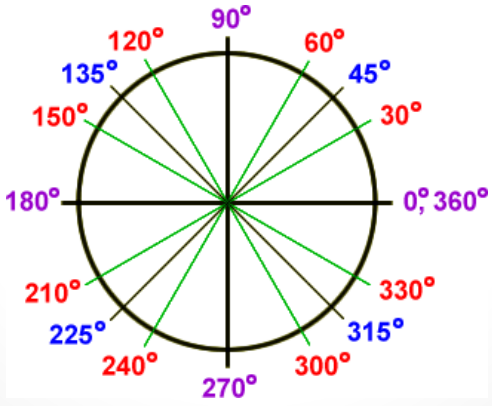
Unit Circle

- Has radius 1 unit and is centered at the origin
- All points on the circle are related to periodic functions $(\cos\theta, \sin\theta)$



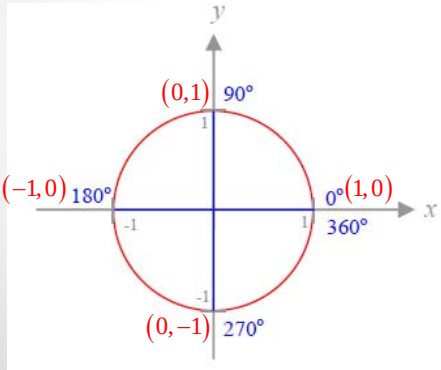
Unit Circle with Degrees

- Will deal with special angles
  - Multiples of $30°, 45°, 60°$ and $90°$



Finding Cosine and Sine of Quadrantal Angles

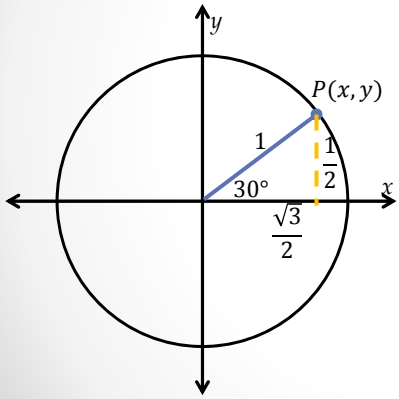
- Quadrantal Angles
  - Terminal side of angle falls on the *x*- or *y*-axis
  - 0°, 90°, 180° and 360°



Angles	Coordinates
0°	$(1,0)$
90°	$(0,1)$
180°	$(-1,0)$
270°	$(0,-1)$
360°	$(1,0)$

Finding Exact Values of Cosine and Sine

- Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of *P*:

1. Draw height of triangle back to *x*-axis (creates reference angle, 30°)
2. Hypotenuse has length of 1 unit, height = *y* and base = *x*
3. Use trig functions to find *x* and *y*

$$\cos 30 = \frac{\frac{\sqrt{3}}{2}}{1} \qquad \sin 30 = \frac{\frac{1}{2}}{1}$$

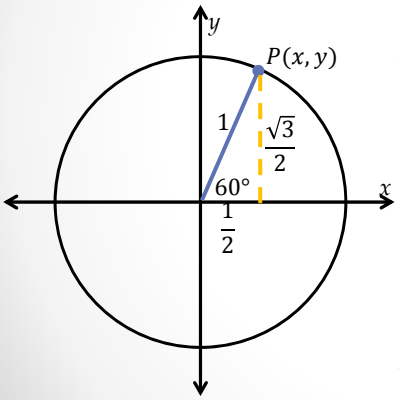
$$\cos 30 = \frac{\sqrt{3}}{2} \qquad \sin 30 = \frac{1}{2}$$

4. Rewrite the coordinates of *P* as:

$$P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Finding Exact Values of Cosine and Sine

- Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of P:

1. Draw height of triangle back to x-axis (creates reference angle, 60°)
2. Hypotenuse has length of 1 unit, height = y and base = x
3. Use trig functions to find x and y

$$\cos 60 = \frac{\frac{1}{2}}{1} \qquad \sin 60 = \frac{\frac{\sqrt{3}}{2}}{1}$$

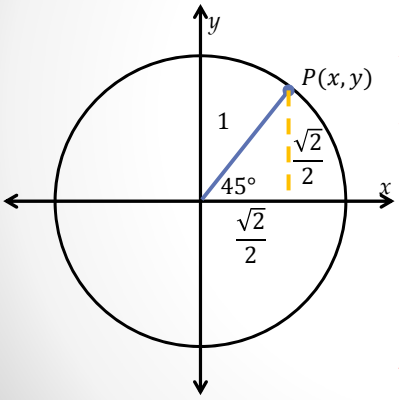
$$\cos 60 = \frac{1}{2} \qquad \sin 60 = \frac{\sqrt{3}}{2}$$

4. Rewrite the coordinates of P as:

$$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

Finding Exact Values of Cosine and Sine

- Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of P:

1. Draw height of triangle back to x-axis (creates reference angle, 45°)
2. Hypotenuse has length of 1 unit, height = y and base = x
3. Use trig functions to find x and y

$$\cos 45 = \frac{\frac{\sqrt{2}}{2}}{1} \qquad \sin 45 = \frac{\frac{\sqrt{2}}{2}}{1}$$

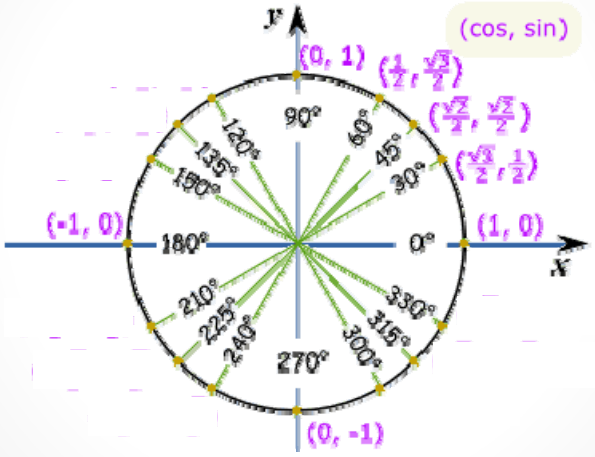
$$\cos 45 = \frac{\sqrt{2}}{2} \qquad \sin 45 = \frac{\sqrt{2}}{2}$$

4. Rewrite the coordinates of P as:

$$P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Complete Unit Circle

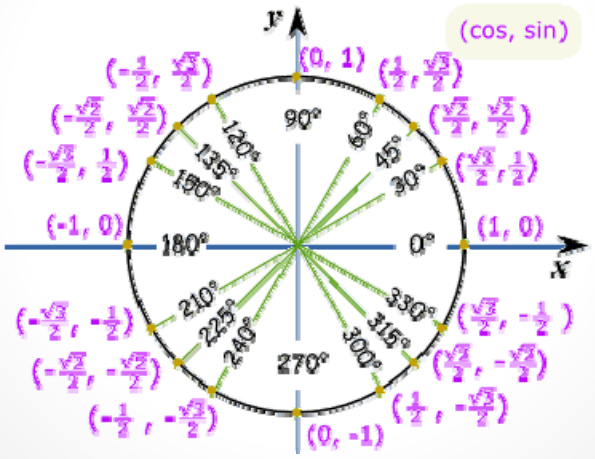
- Use properties of reflections over the *x*- and *y*-axis to complete the other quadrants.



Complete Got It? #5 $a.\ \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$ $b.\ -\frac{\sqrt{3}}{2}, \frac{1}{2}$

Complete Unit Circle

- Use properties of reflections over the *x*- and *y*-axis to complete the other quadrants.



Homework: p.840 #26-38, 51(a-c)