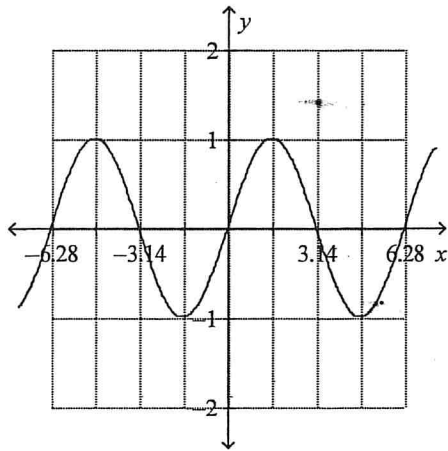


Lesson 4-6: Inverse Trigonometric Functions

(Recall: the sine of an angle is the same as the sine of the supplement.)

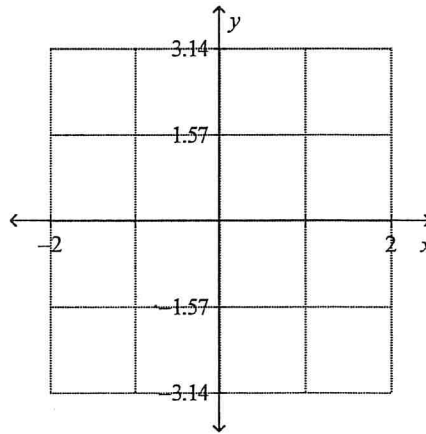
Graph $y = \sin(x)$. Notice it fails the HLT, so the inverse is not a function. Restrict it so that the section contains the entire range (-1 to 1), and passes HLT.



$f(x) = \sin x$

Domain:

Range:



$f(x) = \sin^{-1} x = \arcsin x$

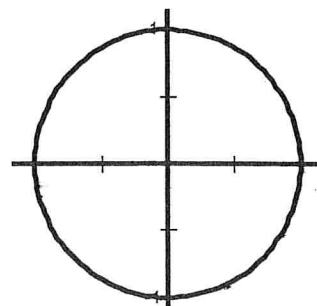
Domain:

Range:

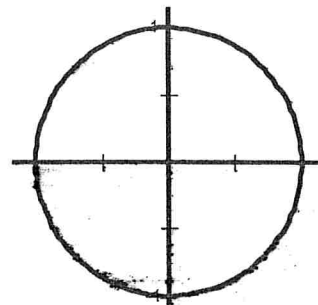
When restricting the domain, the following qualifications must be met:

1. Must include the values from 0 degrees to 90 degrees (to represent all acute angles that are possible in a right triangle.)
2. Must include the entire range of the sine graph from -1 to 1.
3. Make the function continuous (no breaks), if possible.

Example 1: Find exact value of $\sin^{-1} \frac{1}{2}$

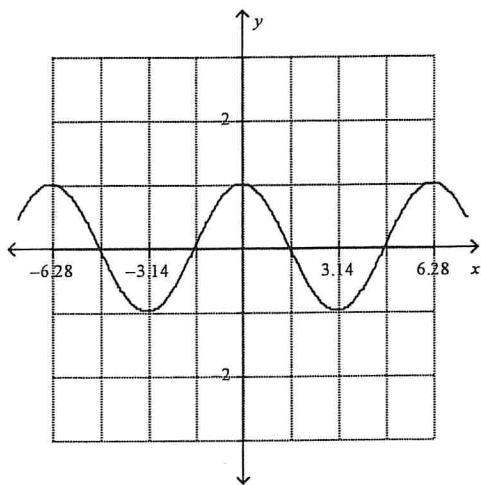


Example 2: Find exact value of $\sin^{-1} \left(\frac{-\sqrt{3}}{2} \right)$



Example 3: Find exact value of $\text{Arcsin } 1$.

Graph $y = \cos(x)$. Notice that since it fails the HLT, its inverse is not a function. Restrict it so that the section contains the entire range (-1 to 1) and passes the HLT.

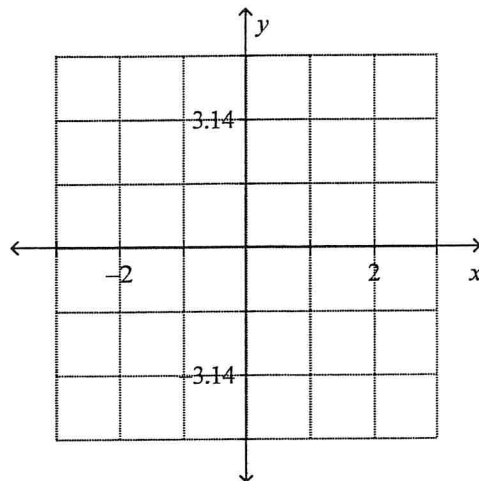


$$f(x) = \cos x$$

Domain:

Range:

Graph $y = \cos^{-1}(x)$ with domain $0 \leq x \leq \pi$



$$f(x) = \cos^{-1} x = \arccos x$$

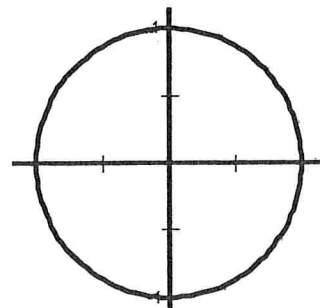
Domain:

Range:

When restricting the domain, the following qualifications must be met:

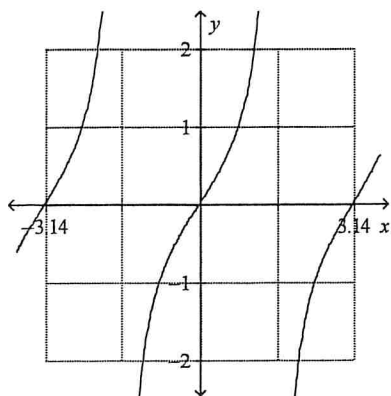
1. Must include the values from 0 degrees to 90 degrees (to represent all acute angles that are possible in a right triangle.)
2. Must include the entire range of the cosine graph from -1 to 1.
3. Make the function continuous (no breaks), if possible.

Example 1: Evaluate $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (exact answer)



Example 2: Evaluate $\cos^{-1}(-.1256)$, give an answer in degrees (approximate)

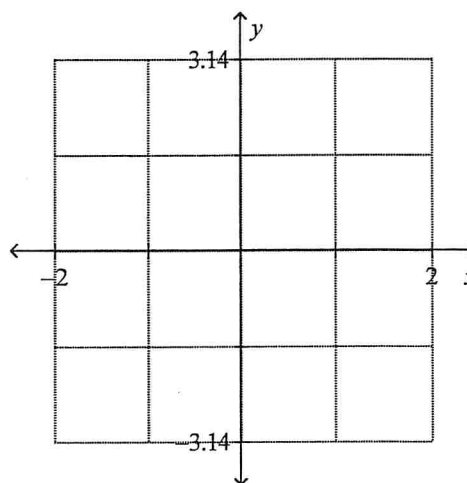
Graph $y = \tan(x)$. How do we restrict the tangent function so that the inverse is also a function and the entire range is contained (as the domain) in the new function?



$f(x) = \tan x$

Domain:

Range:



$f(x) = \tan^{-1} x = \arctan x$

Domain:

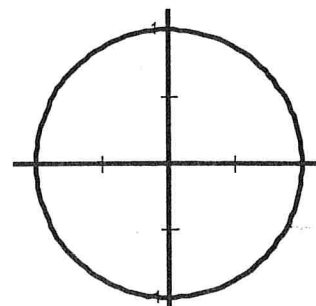
Range:

When restricting the domain, the following qualifications must be met:

1. Must include the values from 0 degrees to 90 degrees (to represent all acute angles that are possible in a right triangle.)
2. Must include the entire range of the tangent graph. (all reals)
3. Make the function continuous (no breaks), if possible.

(Find Exact values)

Example 1: $\text{Arctan } \frac{\sqrt{3}}{3}$



Example 2: $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

Precalculus Worksheet
Section 4.7 – Inverse Trig Functions

Name _____
Period _____

Evaluate the given expression without the aid of a calculator.

1. $\sin^{-1}\left(\frac{1}{2}\right)$

2. $\cos^{-1}\left(\frac{1}{2}\right)$

3. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

4. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

5. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

6. $\arctan(1)$

7. $\arcsin^{-1}\left(-\frac{1}{2}\right)$

8. $\arccos\left(-\frac{1}{2}\right)$

9. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

11. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

12. $\tan^{-1}(-1)$

13. $\sin^{-1}0$

14. $\cos^{-1}0$

15. $\tan^{-1}(-\sqrt{3})$

16. $\arcsin(1)$

17. $\arccos(1)$

18. $\tan^{-1}0$

19. $\arcsin(-1)$

20. $\arccos(-1)$

Find the exact value without a calculator.

21. $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ 22. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ 23. $\sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$

24. $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$ 25. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$ 26. $\arccos\left(\sin\left(\frac{\pi}{3}\right)\right)$

27. $\sin\left(\tan^{-1}(\sqrt{3})\right)$ 28. $\cos\left(\tan^{-1}(-1)\right)$ 29. $\tan^{-1}(\cos(\pi))$

Find an algebraic expression equivalent to the given expression.

30. $\tan\left(\arccos\left(\frac{x}{3}\right)\right)$ 31. $\sin(\arccos(x))$ 32. $\sin\left(\arctan\left(\frac{x}{2}\right)\right)$

Evaluate using your calculator to find the approximate value. Express your answer in degrees.

33. $\sin^{-1}(.8621)$ 34. $\tan^{-1}(.5893)$ 35. $\cos^{-1}(-.3218)$

36. $\arcsin(-.6821)$ 37. $\arctan(-1.6283)$ 38. $\arccos(.2814)$

Evaluate using your calculator to find the approximate value. Express your answer in radians

39. $\arcsin(.2618)$ 40. $\cos^{-1}(-.8090)$ 41. $\tan^{-1}(-1.7321)$

PRECALCULUS

Practice exercises

A. ANSWER EACH OF THE GIVEN QUESTIONS.

1. Given that $\cot \theta = \frac{-12}{5}$ and $\frac{\pi}{2} < \theta < \pi$ determine the exact values of the other five trigonometric values of θ .
2. Determine the exact value of $\cos(\arcsin \frac{9}{41})$.
3. Determine the exact value of $\cot(\text{Cos}^{-1} \frac{2}{7})$.
4. Determine the value of $\sin(\text{Tan}^{-1} \frac{r}{s})$
5. Determine the value of $\cos(\arcsin \frac{2x}{5})$

PRECALCULUS

Practice exercises

B. ANSWER EACH OF THE GIVEN QUESTIONS.

1. Given that $\sin \theta = \frac{-4\sqrt{2}}{9}$ and $\pi < \theta < \frac{3\pi}{2}$ determine the exact values of the other five trigonometric values of θ .

2. Determine the exact value of $\cos(\arctan \frac{\sqrt{7}}{3})$.

3. Determine the exact value of $\tan(\text{Cos}^{-1} \frac{3}{7})$.

4. Determine the value of $\sin(\text{Cot}^{-1} \frac{a}{b})$

5. Determine the value of $\cos(\arcsin \frac{3x}{4})$

Inverse Trigonometric Functions

3

1. Evaluate: $\arccos\left(-\frac{1}{2}\right)$

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $-\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$

(e) None of these

2. Evaluate: $\arcsin 0$

(a) $\frac{\pi}{2}$

(b) π

(c) 0

(d) $-\frac{\pi}{2}$

(e) None of these

3. Evaluate: $\arctan(-1)$

(a) $\frac{\pi}{4}$

(b) $-\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $\frac{7\pi}{4}$

(e) None of these

4. Evaluate: $\arcsin\left(-\frac{1}{2}\right)$

(a) $-\frac{\pi}{6}$

(d) $\frac{\pi}{6}$

(b) $\frac{11\pi}{6}$

(c) $\frac{5\pi}{6}$

(e) None of these

5. Evaluate: $\arccos\left(-\frac{1}{2}\right)$

(a) $-\frac{\pi}{6}$

(d) $\frac{\pi}{6}$

(b) $\frac{11\pi}{6}$

(c) $\frac{5\pi}{6}$

(e) None of these

6. Evaluate: $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

7. Evaluate: $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

8. Evaluate: $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

9. Evaluate: $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

10. Evaluate: $\arctan(-\sqrt{3})$

11. Evaluate: $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

12. Evaluate: $\arccos\left(\frac{\sqrt{2}}{2}\right)$

13. Evaluate: $\arctan\frac{\sqrt{3}}{3}$

14. Evaluate: $\arcsin(-1)$

15. Evaluate: $\arccos(-1)$

16. Evaluate: $\arctan(2.41)$

(a) 1.1775

(d) 0.9732

(b) -0.8978

(e) None of these

(c) 0.4149

4

17. Evaluate: $\arccos(-0.4777)$

(a) -1.0049

(d) 2.0688

(b) 1.0728

(e) None of these

(c) 2.0934

18. Evaluate: $\arcsin(-0.7182)$

(a) -0.5771

(d) 4.2318

(b) 1.3924

(e) None of these

(c) -0.8012

19. Evaluate: $\arccos(-0.923)$

(a) -1.1758

(d) 1.1758

(b) -0.3950

(e) None of these

(c) 2.7466

20. Evaluate: $\arctan 5.572$

(a) 1.3932

(d) -0.8616

(b) 1.7484

(e) None of these

(c) 0.1795

21. Evaluate: $\arccos(-0.8923)$

22. Evaluate: $\arctan(-3)$

23. Evaluate: $\sec[\arctan 3]$

(a) $\sqrt{10}$

(d) $\frac{\sqrt{10}}{3}$

(b) $\frac{\sqrt{2}}{4}$

(e) None of these

(c) $2\sqrt{2}$

24. Evaluate: $\cos\left[\arctan\left(-\frac{2}{3}\right)\right]$

(a) $-\frac{3\sqrt{3}}{13}$

(d) $\frac{2\sqrt{13}}{13}$

(b) $\frac{3\sqrt{13}}{13}$

(e) None of these

(c) $-\frac{2\sqrt{13}}{13}$

5

25. Evaluate: $\sin\left[\arccos\left(-\frac{2}{7}\right)\right]$

(a) $\frac{\sqrt{53}}{7}$

(b) $-\frac{\sqrt{53}}{7}$

(c) $\frac{3\sqrt{5}}{7}$

(d) $-\frac{3\sqrt{5}}{7}$

(e) None of these

26. Find the exact value: $\cos\left[\arctan\left(-\frac{3}{10}\right)\right]$

27. Find the exact value: $\sin(\arctan 3)$

28. Evaluate: $\sin\left(\arctan \frac{3}{8}\right)$

(a) $\frac{8}{3}$

(b) $\frac{\sqrt{73}}{8}$

(c) $\frac{3\sqrt{55}}{55}$

(d) $\frac{3\sqrt{73}}{73}$

(e) None of these

29. Evaluate: $\cos\left(\arctan \frac{2}{3}\right)$

(a) $\frac{2\sqrt{5}}{5}$

(b) $\frac{3\sqrt{13}}{13}$

(c) $\frac{2\sqrt{13}}{13}$

(d) $\frac{3\sqrt{5}}{3}$

(e) None of these

30. Evaluate: $\tan\left(\arccos \frac{3}{7}\right)$

(a) $\frac{3\sqrt{10}}{20}$

(b) $\frac{2\sqrt{10}}{3}$

(c) $\frac{\sqrt{58}}{7}$

(d) $\frac{\sqrt{58}}{3}$

(e) None of these

6

31. Evaluate: $\sec\left[\arctan\left(-\frac{2}{5}\right)\right]$

(a) $-\frac{5\sqrt{21}}{21}$

(b) $-\frac{\sqrt{21}}{2}$

(c) $\frac{\sqrt{21}}{5}$

(d) $\frac{\sqrt{29}}{2}$

(e) None of these

32. Evaluate: $\tan\left[\arcsin\left(-\frac{\sqrt{7}}{7}\right)\right]$

(a) $\sqrt{8}$

(b) $\sqrt{6}$

(c) $-\frac{\sqrt{6}}{6}$

(d) $-\frac{\sqrt{8}}{8}$

(e) None of these

33. Evaluate: $\sin\left[\arccos\left(-\frac{4}{9}\right)\right]$

(a) $\frac{\sqrt{65}}{9}$

(b) $-\frac{\sqrt{65}}{9}$

(c) $-\frac{9\sqrt{97}}{97}$

(d) $\frac{4\sqrt{97}}{97}$

(e) None of these

34. Evaluate: $\sin\left(\arctan\frac{x}{5}\right)$

(a) $\frac{x}{x+5}$

(b) $\frac{x}{\sqrt{x^2+25}}$

(c) $\frac{5}{\sqrt{x^2+25}}$

(d) $\frac{\sqrt{25-x^2}}{5}$

(e) None of these

35. Evaluate: $\csc\left(\arccos\frac{x}{5}\right)$

(a) $\frac{\sqrt{25-x^2}}{5}$

(b) $\frac{\sqrt{25+x^2}}{x}$

(c) $\frac{5}{\sqrt{25-x^2}}$

(d) $\frac{x}{x+5}$

(e) None of these

8

State the exact value of the following.

$$\csc \frac{3\pi}{2}$$

$$\csc \frac{5\pi}{4}$$

$$\sec \frac{5\pi}{3}$$

$$\cot \frac{11\pi}{6}$$

$$\cot \frac{7\pi}{4}$$

$$\sec\left(\frac{-3\pi}{4}\right)$$

$$\csc \frac{17\pi}{6}$$

$$\csc\left(-\frac{\pi}{6}\right)$$

$$\sec\left(\frac{-2\pi}{3}\right)$$

Find $\cos \theta$, $\tan \theta$, $\sec \theta$, $\csc \theta$, and $\cot \theta$ given that $\sin \theta = -\frac{84}{85}$ and $\pi < \theta < \frac{3\pi}{2}$.