1

PRECALCULUS II NAME: ____________________

Practice Exercises (2.9.4)

A. SOLVE EACH OF THE GIVEN EQUATIONS (correct to the nearest thousandth).

1. $5^x = 200$

2. $e^x = 95$

2. $6^{2x-1} + 24 = 99$

4. $5e^{1-x} - 15 = 39$

5. $\left(\frac{5}{8}\right)^x + 15 = 25$

6. $\frac{3}{5}e^{\sqrt{x}} - 24 = 40$

B. EVALUATE EACH OF THE GIVEN EXPRESSIONS (correct to the nearest thousandth).

7. $\log_{\sqrt{6}} 24$

8. $\log_{\pi} \sqrt[3]{2000}$

②

C. ANSWER EACH OF THE GIVEN QUESTIONS.

9. The population of Kenya in 2000 was approximately 47.6 million with an annual growth rate of 1.88%. If the population is assumed to change continuously, predict the year in which the population will reach 64 million.

10. A radioactive isotope has a half-life of 6.5 days. Determine the time over which a given amount of the substance would decay to 35% of its original volume.

11. Nicole deposited $3,600 into an account paying 5.25% interest to be compounded continuously. Find the time it takes for the account to triple in value.

12. If $5,000 was invested in an account with continuous compounding what interest rate would be required for the value of the account to increase to $7,500 in five years.

3

NAME ______________________________ DATE ____________ PERIOD ____________

3-4 Study Guide and Intervention

Exponential and Logarithmic Equations

Solve Exponential Equations **One-to-One Property of Exponential Functions:** For $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

This property will help you solve exponential equations. For example, you can express both sides of the equation as an exponent with the same base. Then use the property to set the exponents equal to each other and solve. If the bases are not the same, you can *exponentiate* each side of an equation and use logarithms to solve the equation.

Example 1

a. Solve $4^{x-1} = 16^x$.

$4^{x-1} = 16^x$	Original equation
$4^{x-1} = (4^2)^x$	$16 = 4^2$
$4^{x-1} = 4^{2x}$	Power of a Power
$x - 1 = 2x$	One-to-One Property
$-1 = x$	Subtract x from each side.

b. Solve $e^{2x} - 3e^x + 2 = 0$.

$e^{2x} - 3e^x + 2 = 0$	Original equation
$u^2 - 3u + 2 = 0$	Write in quadratic form.
$(u - 2)(u - 1) = 0$	Factor.
$u = 2$ or $u = 1$	Solve.
$e^x = 2$ or $e^x = 1$	Substitute for u.
$x = \ln 2$ or 0	Take the natural logarithm of each side.

Example 2 **Solve each equation. Round to the nearest hundredth if necessary.**

a. $3^x = 19$

$\log 3^x = \log 19$	Take the log of both sides.
$x \log 3 = \log 19$	Power Property
$x = \dfrac{\log 19}{\log 3}$	Divide each side by log 3.
$x \approx 2.68$	Use a calculator. Check this solution in the original equation.

b. $e^{8x+1} - 6 = 1$

$e^{8x+1} = 7$	Add 6 to both sides.
$\ln e^{8x+1} = \ln 7$	Take the ln of both sides.
$(8x + 1) \ln e = \ln 7$	Power Property
$8x + 1 = \ln 7$	$\ln e = 1$
$8x = \ln 7 - 1$	Subtract 1 from each side.
$x = \dfrac{\ln 7 - 1}{8} \approx 0.12$	Divide by 8 and use a calculator.

Exercises

Solve each equation. Round to the nearest hundredth.

1. $9^x = 3^{3x-4}$

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

3. $4^{3x-2} = \frac{1}{2}^{2x}$

4. $2e^{2x} + 12e^x - 54 = 0$

5. $9^{2x} = 12$

6. $2.4e^{x-6} = 9.3$

7. $3^{2x} = 6^{x-1}$

8. $e^{19x} = 23$

(4)

NAME ____________________ DATE ____________ PERIOD ____________

3-4 Practice

Exponential and Logarithmic Equations

Solve each equation.

1. $5^x = 125^{x-2}$

2. $\log_6 x + \log_6 9 = \log_6 54$

3. $\left(\frac{1}{9}\right)^{x+3} = 27^x$

4. $e^{2x} - e^x - 6 = 0$

5. $\log_x 64 = 3$

6. $\ln \frac{1}{e} = x$

7. $\ln (2x - 1) = \ln 16$

8. $3e^{4x} - 9e^{2x} - 15 = 0$

9. $\ln (x - 5) + \ln 4 = \ln x - \ln 2$

10. $4^{x+2} = 6^{-2x-3}$

11. $6e^{6x} - 17e^{3x} + 7 = 0$

12. $6 \ln (x + 2) - 3 = 21$

13. $4e^{2x} - 13e^x + 9 = 0$

14. $\log (2x + 1) + \log (x - 4) = \log (2x^2 - 4)$

15. $2^{-4x+1} = 3^{2x-3}$

16. $\log_5 (x + 4) + \log_5 x = \log_5 12$

17. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

18. $\ln 0.04x = -8$

Solve each equation. Round to the nearest hundredth.

19. $2^{9x} = 1210$

20. $4^{3x} = 1056$

21. $5^{x+3} - 4 = 19$

22. $3^{x-8} + 2 = 38$

23. $6^{2x-1} = 18$

24. $2^{3+2x} = 130$

25. BANKING Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is $A = Pe^{rt}$. If Ms. Cubbatz made the investment on January 1, 2005, and the account was worth \$12,000 on January 1, 2009, what was the original amount in the account?

26. FINANCIAL LITERACY If \$500 is deposited in a savings account providing an annual interest rate of 5.6% compounded quarterly, how long will it take for the account to be worth \$750?

Solving Exponential Equations Pre Calc (CP)

Take logs of both sides (or ln) to drop the exponent in front of the log, then solve.

EX 1: Solve $3^x = 525$

EX 4: Solve $3^{2x} = 50$

EX 5: $3e^{2x} = 48$

EX 6) $e^{3-5x} = 16$

EX 4: Suppose you invest $5000 in an account at 4% compounded continuously. How long will it take for the money in the account to reach $6000 if it is not touched?

EX 5: Suppose the half life of a substance is 400 years. If you start today with 2 grams, how long will it take until 1.78 grams remain?

6

Example 4: The half-life of a certain radioactive substance is 40 days. If 10 grams of the substance are present initially, how much of the substance will be present in 90 days?

$$InitialValue \cdot \left(\frac{1}{2}\right)^{\frac{time}{HalfLife}}$$

a. Use a table to organize the information you already know.

Number of Half-Life Periods	0	1	2	3
t = Number of Days After Decay Starts	0	40	80	120
f(t) = Amount Present (grams)	10	5	2.5	1.25

b. Calculate the Exponential Model for this data.

c. What is the initial value? What is the growth factor?

d. Answer the question being presented: How much of the substance will be present after 90 days?

1. A certain substance has a half-life of 24 years. If a sample of 80 grams is being observed, how much will remain in 50 years? When will only 5 grams remain?

a. In 50 years = __________

b. 5 grams remain = __________

P r

2. $1500 is put in a treasury note paying 5.5% interest compounded annually. How much is in the account after 15 years?

t

$$A(t) = P(1+r)^t$$

3. A farmer's tractor valued at $50,000 depreciates at an annual rate of 10% (yearly). When will its value be $15,000?

4. If John invests $2300 in a savings account with a 6% interest rate compounded annually, how long will it take until John's account has a balance of $4150?

5. How much time is required to double your money if it is invested at 6.25% compounded continuously?

6. Suppose that in any given year, the number of cases of a disease is reduced by 20%. If there are 10,000 cases today, how many years will it take

a) to reduce the number of cases to 1000?

b) to eliminate the disease; that is, to reduce the number of cases to less than 1?

8

Precalc 2 Name________________________________ ID: 1

Solving Equations using Logarithms

Solve each equation. Round your answers to the nearest ten-thousandth.

1) $6^x = 37$

2) $18^p = 54$

3) $4^n = 78$

4) $13^n = 27$

5) $9^x = 98$

6) $6^{m-4} - 5 = 32$

7) $3^{10x} + 10 = 97$

8) $9^{-8n} + 3 = 5$

9) $-4 \cdot 16^{-3n} = -25$

10) $-9 \cdot 12^{b+1} = -16$

Solve the logarithmic equation for x.

11. $\ln(2+x)=1$

12. $\log(x-4)=3$

13. $\log_3(2-x)=3$

14. $\log_2(x^2-x-2)=2$

15. $2\log x=\log 2+\log(3x-4)$

16. $\log_5 x+\log_5(x1)=\log_5 20$

17. $\log x+\log(x-3)=1$

18. $\ln(x-1)+\ln(x+2)=1$