Logarithmic Functions and Their Graphs

- You should know that a function of the form $y = \log_a x$, where $a > 0$, $a \neq 1$, and $x > 0$, is called a logarithm of x to base a.
- You should be able to convert from logarithmic form to exponential form and vice versa.

  $y = \log_a x \iff a^y = x$
- You should know the following properties of logarithms.
  (a) $\log_a 1 = 0$ since $a^0 = 1$.
  (b) $\log_a a = 1$ since $a^1 = a$.
  (c) $\log_a a^x = x$ since $a^x = a^x$.
  (d) If $\log_a x = \log_a y$, then $x = y$.
- You should know the definition of the natural logarithmic function.

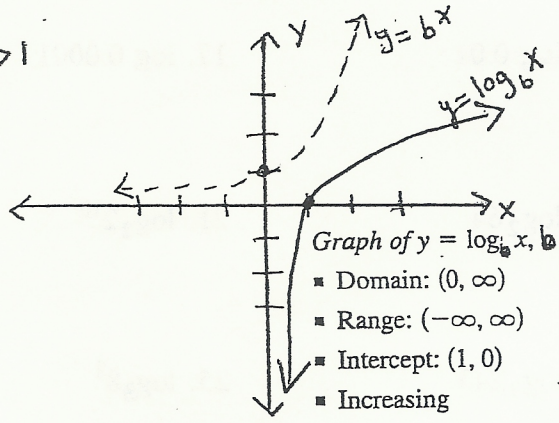
  $\log_e x = \ln x, x > 0$
- You should know the properties of the natural logarithmic function.
  (a) $\ln 1 = 0$ since $e^0 = 1$.
  (b) $\ln e = 1$ since $e^1 = e$.
  (c) $\ln e^x = x$ since $e^x = e^x$.
  (d) If $\ln x = \ln y$, then $x = y$.
- You should be able to graph logarithmic functions.

* Change of base *
$\log_a x = \frac{\log x}{\log a}$

Common log – base is 10

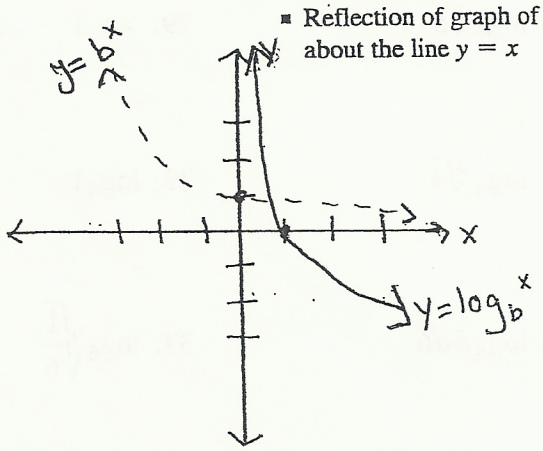
natural log – base is e
$e = (1+\frac{1}{x})^x \approx 2.718281\ldots$

$b > 1$



Graph of $y = \log_b x$, $b > 1$
- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- Intercept: $(1, 0)$
- Increasing
- y-axis is a vertical asymptote ($\log_b x \to -\infty$ as $x \to 0^+$)
- Continuous
- Reflection of graph of $y = b^x$ about the line $y = x$

$0 < b < 1$



Change each equation to logarithmic form.

1. $5^2 = 25$

2. $3^{-2} = \frac{1}{9}$

Change each equation to exponential form.

3. $\log_3 81 = 4$

4. $\log_4 8 = \frac{3}{2}$

Solve each equation.

5. $\log_2 x = 3$

6. $\log_x 8 = \frac{1}{2}$

Evaluate each expression.

7. $\log_{64} 8$

8. $\log_x 27 = -3$

Solve each equation.

9. $\log_3 2x = \log_3(3x - 5)$

10. $\log_9(x^2 + x) = \log_9 6$

(2)

PreCalculus II:

Logarithmic Form $\log_b y = x$ $\Leftrightarrow$ Exponential Form $b^x = y$ **NO CALCULATORS**

Find each logarithm without using a calculator

1. $\log_7 49$
2. $\log_2 16$
3. $\log_2 \frac{1}{8}$
4. $\log_5 \frac{1}{5}$
5. $\log_5 \sqrt{5}$

Solve each equation (Do not use a calculator)

6. $\log_5 x = 2$
7. $\log_6 x = 2$
8. $\log x = 2$
9. $\ln x = 2$
10. $\log_x 121 = 2$
11. $\log_x 64 = 3$
12. $\log_x \left(\frac{1}{2}\right) = -1$
13. $\log_x \sqrt{6} = \frac{1}{2}$

Find each logarithm. (Do not use a calculator)

14. $\log 100$
15. $\log 10,000$
16. $\log 0.01$
17. $\log 0.0001$
18. $\log_2 4$
19. $\log_2 32$
20. $\log_2 64$
21. $\log_2 2^{10}$
22. $\log_3 9$
23. $\log_3 27$
24. $\log_3 243$
25. $\log_8 8^8$
26. $\log_5 0.2$
27. $\log_5 \frac{1}{125}$
28. $\log_5 \sqrt[3]{5}$
29. $\log_5 1$
30. $\log_4 64$
31. $\log_4 \frac{-1}{64}$
32. $\log_4 \sqrt[4]{4}$
33. $\log_4 1$
34. $\log_6 36$
35. $\log_6 6$
36. $\log_6 6\sqrt{6}$
37. $\log_6 \sqrt[3]{\frac{1}{6}}$

PRECALC – LOG WORKSHEET – SECTION 6.3

A. Write in exponential form then solve for x.

1. $\log_5 125 = t$
2. $\log_5 \sqrt{5} = t$
3. $\log_5 .04 = t$
4. $\log_4 8 = t$
5. $\log_6 \frac{1}{6} = t$
6. $\log_6 36 = t$
7. $\log_6 \sqrt[5]{36} = t$
8. $\log_9 243 = t$
9. $\log t = 2$
10. $\log_2 64 = t$
11. $\log_4 2 = t$
12. $\log_2 \frac{1}{16} = t$
13. $\log_{\frac{1}{5}} 25 = t$
14. $\log_\pi 1 = t$
15. $\log_t 8 = 3$
16. $\log_{\frac{1}{2}} 4 = t$
17. $\log_2 t = 3$
18. $\log_5 t = -2$
19. $\log_t 27 = 3$
20. $\log_t (-1) = \frac{1}{2}$
21. $\log_t 625 = 4$
22. $\log_t (.1) = -1$
23. $\log_{\sqrt{3}} 27 = x$
24. $\log(x^2 + 9x) = 1$
25. $\log(4x - 4) = 2$

26. $\log_{64} x = \frac{1}{2}$

27. $\log_{\sqrt{2}} 16 = x$

28. $\log(4x+8) = 2$

29. $\log_4\left(\frac{1}{16}\right) = x$

30. $\log_{\sqrt{3}} 3 = x$

31. $\log(3x+1) = 3$

32. $\log_4\left(\frac{1}{64}\right) = x$

33. $\log_8 x = \frac{1}{3}$

Name_____ (5)

Assignment

Date_____ Period____

Identify the domain and range of each.

1) $y = \log_5 (2x - 4) + 5$

A) Domain: $x > 2$
Range: All reals

B) Domain: All reals
Range: $x < -2$

C) Domain: $x > -2$
Range: All reals

D) Domain: $x < 2$
Range: All reals

2) $y = \log_3 (3x + 17) + 4$

A) Domain: All reals
Range: $x > \frac{17}{3}$

B) Domain: $x > \frac{17}{3}$
Range: All reals

C) Domain: $x < \frac{17}{3}$
Range: All reals

D) Domain: $x > -\frac{17}{3}$
Range: All reals

3) $y = \log_3 (3x + 12) + 3$

A) Domain: $x > 4$
Range: All reals

B) Domain: $x < -1$
Range: All reals

C) Domain: $x > -4$
Range: All reals

D) Domain: $x < -4$
Range: All reals

4) $y = \log_4 (4x + 2) + 1$

A) Domain: $x > \frac{1}{2}$
Range: All reals

B) Domain: $x < \frac{1}{2}$
Range: All reals

C) Domain: All reals
Range: $x > \frac{1}{2}$

D) Domain: $x > -\frac{1}{2}$
Range: All reals

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5) $y = \log_4 (2x + 10)$

A) Domain: $x > 5$
Range: All reals

B) Domain: All reals
Range: $x < 0$

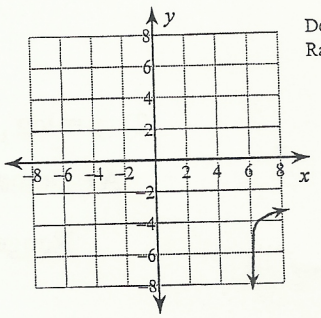
C) Domain: $x < 0$
Range: All reals

D) Domain: $x > -5$
Range: All reals

Identify the domain and range of each. Then sketch the graph.

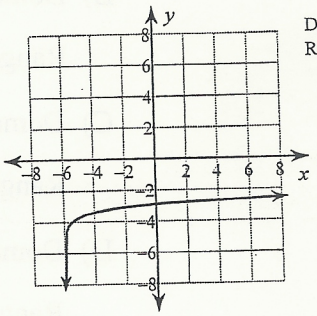
7) $y = \log (2x + 12) - 4$

A)



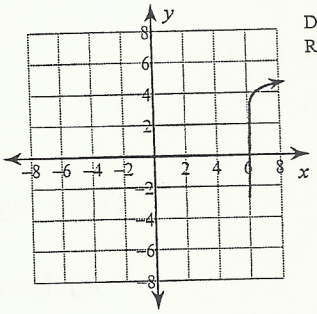
Domain: $x > 6$
Range: All reals

B)



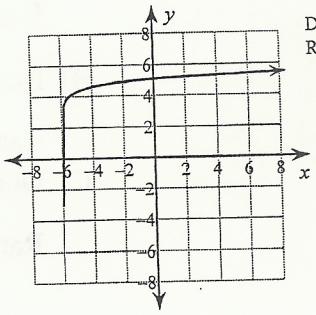
Domain: $x > -6$
Range: All reals

C)



Domain: $x > 6$
Range: All reals

D)

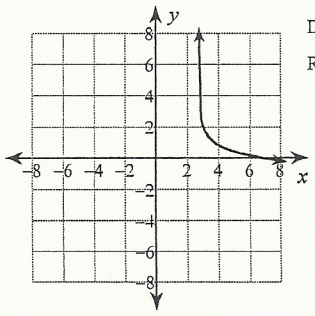


Domain: $x > -6$
Range: All reals

7

8) $y = \log_{\frac{1}{4}}(4x - 11) - 2$

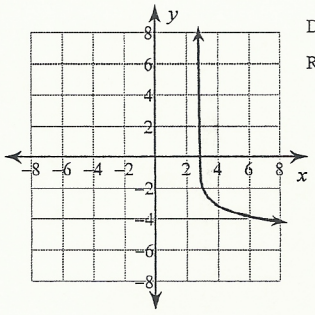
A)



Domain: $x > \frac{11}{4}$

Range: All reals

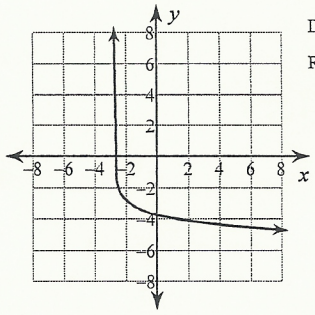
B)



Domain: $x > \frac{11}{4}$

Range: All reals

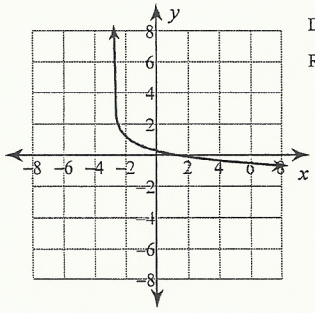
C)



Domain: $x > -\frac{11}{4}$

Range: All reals

D)

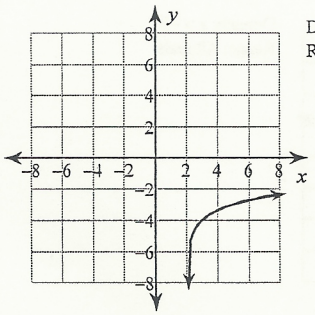


Domain: $x > -\frac{11}{4}$

Range: All reals

9) $y = \log_3(3x - 6) + 5$

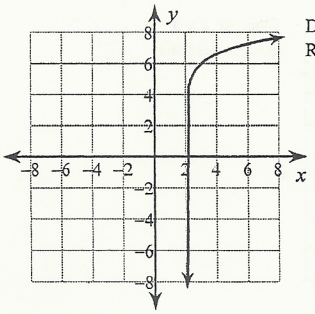
A)



Domain: $x > 2$

Range: All reals

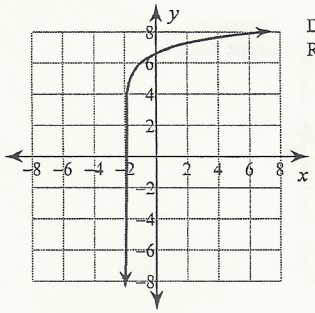
B)



Domain: $x > 2$

Range: All reals

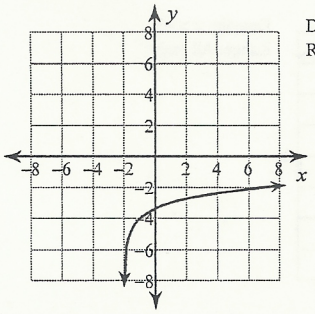
C)



Domain: $x > -2$

Range: All reals

D)

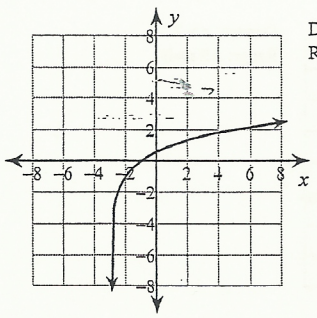


Domain: $x > -2$

Range: All reals

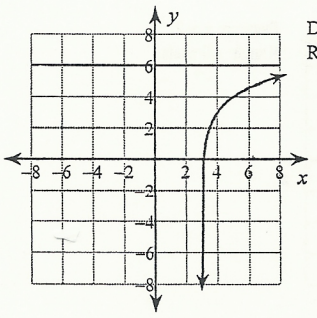
10) $y = \log_2 (2x + 6) + 2$

A)



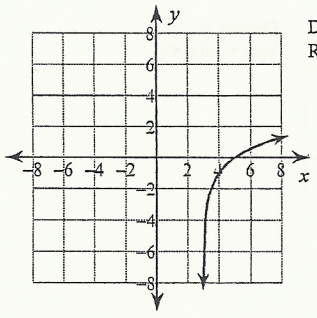
Domain: $x > -3$
Range: All reals

B)



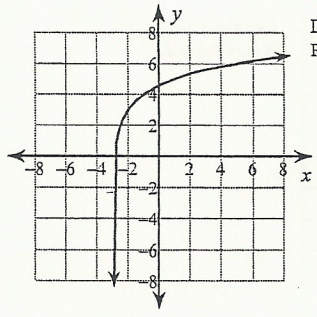
Domain: $x > 3$
Range: All reals

C)



Domain: $x > 3$
Range: All reals

D)



Domain: $x > -3$
Range: All reals

Solve each equation. Round your answers to the nearest ten-thousandth.

11) $\log x - \log 3 = \log 44$

16) $\log (x + 1) - \log x = \log 59$

13) $\log x - \log 4 = 1$

18) $\log x - \log (x - 2) = 1$

Simplify. Your answer should contain only positive exponents.

19) $-x^4y^3 \cdot 4x^3 \cdot -x^3y^3$

20) $4x^{-4}y^3 \cdot 2x$

21) $-2x^{-4}y^{-2} \cdot 2x^{-2}y^4$

22) $-uv \cdot 2u^3v^{-2}$

23) $-4u^{-4} \cdot v^{-1}$

Use a calculator to approximate each to the nearest thousandth.

24) $\log_7 5.7$

25) $\log_3 9$

26) $\log_4 26$

Condense each expression to a single logarithm.

27) $3\ln 3 + 6\ln 8$

28) $\frac{\log 12}{3} + \frac{\log 7}{3} + \frac{\log 5}{3}$

29) $12\log_4 8 - 4\log_4 5$

30) $\frac{\log_3 5}{2} + \frac{\log_3 7}{2} + \frac{\log_3 6}{2}$

31) $5\log_2 x + 3\log_2 y$

Expand each logarithm.

32) $\log_8 \left(\frac{x^4}{y}\right)^5$

33) $\log_8 \left(x^2y^2\right)$

(10)

A. EVALUATE EACH OF THE FOLLOWING EXPRESSIONS. (non-calculator)

1. $(-27)^{\frac{2}{3}}$
2. $9^{\frac{5}{2}}$
3. $4^{-\frac{3}{2}}$
4. $36^{-\frac{1}{2}}$
5. $5x^{0}$
6. $(5x)^{0}$
7. $\left(\frac{8}{27}\right)^{\frac{2}{3}}$
8. $(16)^{-\frac{5}{4}}$
9. $\log_{12} 12$
10. $\log_{6} 1$
11. $\log_{4}\left(\frac{1}{16}\right)$
12. $\log_{8} 16$
13. $\log_{7} \sqrt{7}$
14. $\log_{\sqrt{2}} 32$

B. DETERMINE THE VALUE OF x IN EACH OF THE GIVEN EQUATIONS.

15. $\log x = 3$
16. $\log x = -2$
17. $\log_{x} 125 = 3$
18. $\log_{x}\left(\frac{1}{16}\right) = -4$
19. $\log_{x} 9 = \frac{2}{3}$
20. $\log_{3}(4x+5) = 4$
21. $3^{2x-3} = 27$
22. $\left(\frac{1}{5}\right)^{x+2} = 25^{x}$
23. $4^{2x+1} = 8^{x-1}$
24. $\left(\frac{1}{9}\right)^{3-x} = 27^{2x+1}$

11

Logarithm Review

1. $3^x = \frac{1}{81}$ 2. $25^x = 625$ 3. $8^{3x+1} = 16^{x-1}$ 4. $3^{2x+1} = 27^{x+2}$

5. Express in exponential form: $\log_3 81 = 4$ _____

Express in logarithmic form: $2^5 = 32$ _____

6. Evaluate w/o a calculator

a) $\log_4 \frac{1}{64}$ b) $\log_3 \frac{1}{27}$ c) $\log_5 5\sqrt{5}$

7. Express in expanded form:

a) $\log_4 3w^5$ _____

b) $\log_5 \frac{5w}{z^2}$ _____

8. Express as a single log AND evaluate

a) $\log 250 + \log 4$ _____ _____

b) $\log_2 4 - \log_2 32$ _____ _____

c) $7\log_3 3 - 9\log_3 3$ _____ _____

d) $\log_{25} 7 - \log_{25} 35$ _____ _____

12

Logarithm Review

1. $3^x = \frac{1}{27}$ 2. $25^x = 125$ 3. $4^{3x+1} = 16^{x-1}$ 4. $9^{2x+1} = 27^{x+2}$

5. Express in exponential form: $\log_3 81 = 4$ _____

   Express in logarithmic form: $2^5 = 32$ _____

6. Evaluate w/o a calculator

   a) $\log_4 64$ b) $\log_3 \frac{1}{81}$ c) $\log_5 5\sqrt{5}$

7. Express in expanded form:

   a) $\log_4 3w$ _____

   b) $\log_5 \frac{5w^2}{z}$ _____

8. Express as a single log AND evaluate

   a) $\log 250 + \log 40$ _____ _____

   b) $\log_2 4 - \log_2 16$ _____ _____

   c) $7\log_3 3 - 8\log_3 3$ _____ _____

   d) $\log_{25} 7 - \log_{25} 35$ _____ _____

13

Solve the logarithmic equation for x.

11. $\ln(2 + x) = 1$

12. $\log(x - 4) = 3$

13. $\log_3 (2 - x) = 3$

14. $\log_2 (x^2 - x - 2) = 2$

15. $2\log x = \log 2 + \log (3x - 4)$

16. $\log_5 x + \log_5 (x + 1) = \log_5 20$

17 $\log x + \log (x - 3) = 1$

18. $\ln (x-1) + \ln (x + 2) = 1$

Properties of Powers and Properties of Logarithms

If $y = b^n$ then $\log_b y = n$.

Power Property	Logarithm Property
Zero Exponent $b^\circ = 1$	Logarithm of 1 $\log_b 1 = 0$
Product of Powers $b^m \cdot b^n = b^{m+n}$	Logarithm of a Product $\log_b(xy) = \log_b x + \log_b y$
Quotient of Powers $\frac{b^m}{b^n} = b^{m-n} \; (b \neq 0)$	Logarithm of a Quotient $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
Power of a Power $(b^m)^a = b^{am}$	Logarithm of a Power $\log_b(x^a) = a \log_b x$

If $b^x = b^y$ then $x = y$

If $\log_b x = \log_b y$ then $x = y$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$a(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

Compound interest Formula

$$a(t) = Pe^{rt}$$

Continuous growth Formula

$$\log_w N = \frac{\log N}{\log w} \quad \text{or} \quad \frac{\ln N}{\ln w}$$

Change of Base Rule