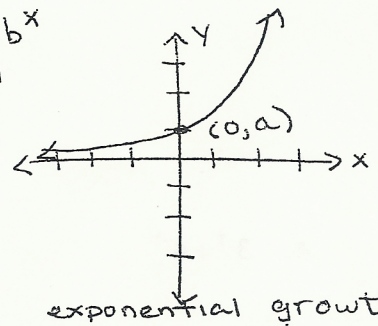
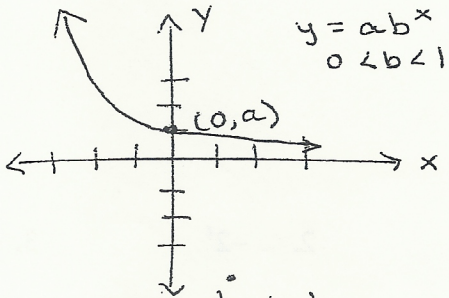
Exponential Functions and Their Graphs

- You should know that a function of the form $y = a^x$, where $a > 0$, $a \neq 1$, is called an exponential function with base a.
- You should be able to graph exponential functions.
- You should know formulas for compound interest.
  (a) For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
  (b) For continuous compoundings: $A = Pe^{rt}$.

$y = ab^x$
$b > 1$



exponential growth



exponential decay

- You should know the properties of exponents.
  (a) $a^1 = a$
  (b) $a^0 = 1, a \neq 0$
  (c) $a^m a^n = a^{m+n}$
  (d) $a^m / a^n = a^{m-n}, a \neq 0$
  (e) $a^{-n} = 1/a^n, a \neq 0$
  (f) $(a^m)^n = a^{mn}$
  (g) $(ab)^n = a^n b^n$
  (h) $(a/b)^n = a^n/b^n, b \neq 0$
  (i) $(a/b)^{-n} = (b/a)^n, a \neq 0, b \neq 0$
  (j) $|a^2| = |a|^2 = a^2$
- You should be able to write numbers in scientific notation, $c \times 10^n$, where $1 \leq c < 10$ and n is an integer.
- You should be able to use your calculator to evaluate expressions involving exponents.
- You should know the properties of radicals.
  (a) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{m/n}$
  (b) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
  (c) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
  (d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
  (e) $(\sqrt[n]{a})^n = a$
  (f) For n even, $\sqrt[n]{a^n} = |a|$.
  For n odd, $\sqrt[n]{a^n} = a$.
  (g) $a^{1/n} = \sqrt[n]{a}$
  (h) $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}, a \geq 0$
- You should be able to simplify radicals.
  (a) All possible factors have been removed from the radical sign.
  (b) All fractions have radical-free denominators.
  (c) The index for the radical has been reduced as far as possible
- You should be able to use your calculator to evaluate radicals.

(1)

Pre Calc Exponent Review WS

Simplify the following leaving answers as exact values. No Calculator. Assume all variables represent non-zero real numbers.

Part 1:

1. $a^9 \cdot a^{12}$ 2. $(-2x^2y)(-8x^3y^9)$ 3. $3x^0$ 4. $-2p^0+8z^0$

Part 2:

1. $(-2)^{-4}$ 2. -2^4 3. $(3x)^{-2}$ 4. $3^{-1}+4^{-1}$

Part 3: Write using only positive exponents.

1. $\frac{4^8}{4^2}$ 2. $\frac{7^3}{7^{-5}}$ 3. $\frac{s^2}{s^9}$ 4. $\frac{r^{-8}}{r^{-3}}$

Part 4:

1. $(3x^2y^5)^2$ 2. $\left(\frac{2t^4}{u^7}\right)^4$ 3. x^{-4} 4. $\left(\frac{2}{3}\right)^{-3}$

Part 5:

1. $z^{-4} \cdot z^{-11} \cdot z^{5}$
2. $\left(2^{-3}\right)^{2}$
3. $x^{-4} \cdot x^{6} \cdot x^{-2}$
4. $\dfrac{p^{-11}q^{3}}{p^{-5}q^{4}}$

Part 6

1. $100^{\frac{1}{2}}$
2. $-625^{\frac{3}{4}}$
3. $27^{\frac{-2}{3}}$
4. $-125^{\frac{-2}{3}}$

Part 7:

1. $9^{\frac{1}{6}} \cdot 9^{\frac{1}{3}}$
2. $\left(3^{\frac{3}{4}}\right)^{2}$
3. $\left(a^{2}b^{\frac{3}{5}}\right)^{10}$
4. $\sqrt[12]{64^{4}}$

NO CALCULATOR

Simplify each exponential expression. No negative exponents

1. $a^2 \cdot a^5$ _____

3. $(3x)^4$ _____

5. $\frac{2(v^2w^3)^2}{(3wx^5)^3}$ _____

7. $(z^2 \cdot z^{-3})^5$ _____

9. $(4a^4b^6)^{\frac{3}{2}}$ _____

11. $(r^4 \cdot r^2)^2$ _____

13. $(4jk^2)^3$ _____

15. $\frac{3(a^2b^{-2}c)^2}{4b^7c^{-3}}$ _____

Solve each equation. ★(Important)★

17. $6^{2x} = 36$ _____

19. $25^x = 125$ _____

21. $8^t = 2^5$ _____

23. $4^u = 128$ _____

25. $3^{\frac{z}{2}} = 27$ _____

27. $3^x \cdot 3^4 = 27$ _____

29. $2^{-5} \cdot 2^x = 16$ _____

31. $9^x \cdot 3^3 = 81$ _____

33. Suppose you invest $2500 in an account that pays 8% interest. What will the account balance be after five years if the interest is compounded:

a. annually? _____ b. monthly? _____

c. daily? _____ d. continuously? _____

exponents – one more time

1. $(x^3y^4)(x^2y^3)$

2. $\left(-\frac{2}{3}a^4\right)(-3a)(a^3y^7)$

3. $(-5x^4y^2)^3$

4. $\frac{y^9}{y^6}$

5. $\frac{r^6n^{-7}}{r^4n^2}$

6. $\frac{(4x^2y^3)^0}{8x^3}$

7. $\frac{6x^5y^7w^{-3}}{8x^2y^{-10}w^3}$

8. $\left(\frac{-2m^5}{3n^6}\right)^{-2}$

9. $\frac{-10m^{-1}y^0r}{14m^{-7}y^{-3}r^{-4}}$

10. $3^{2x} = 9^{2x-1}$ $\quad x =$

11. $9^{x-4} = \frac{1}{27}$ $\quad x =$

12. $4^{7a} = 16^{5a-6}$

13. $\frac{(3xy^3)^2}{9x^4y}$

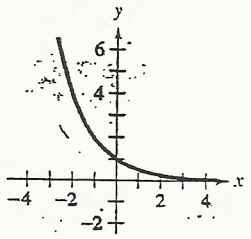
14. $\frac{5m^{-3}}{6^{-1}m^{-2}}$

15. $\frac{a^2(r^{-3}s)^{-2}}{a^5r^3s^3}$

★ $\left(\frac{1}{3}+\frac{1}{5}\right)^{-1}$

1. Match each graph with its equation

(a)



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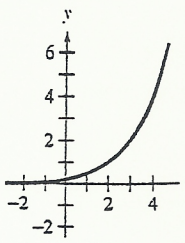

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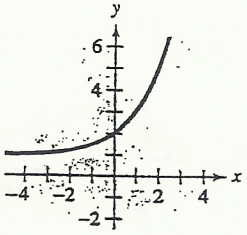

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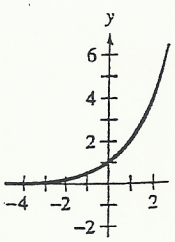


_____ $f(x) = 2^x$

_____ $f(x) = 2^{-x}$

_____ $f(x) = 2^x + 1$

_____ $f(x) = 2^{x-2}$

2. Which 2 functions are the same?

$f(x) = 3^{x-2}$

$g(x) = 3^x - 9$

$h(x) = \frac{1}{9}(3^x)$

3. Fill in the table.

n	1	2	4	12	365	Continuous
A						

$P = \$2500$, $r = 12\%$, $t = 10$ years

Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$

4. *Trust Fund* On the day of a child's birth, a deposit of $25,000 is made in a trust fund that pays 8.75% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

5. *Radioactive Decay* Let Q represent a mass of radium (^{226}Ra), whose half-life is 1620 years. The quantity of radium present after t years is given by

$Q = 25\left(\frac{1}{2}\right)^{t/1620}$.

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 1000 years.

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.

6. *Bacteria Growth* A certain type of bacteria increases according to the model

$P(t) = 100e^{0.2197t}$

where t is the time in hours. Find (a) $P(0)$, (b) $P(5)$ and (c) $P(10)$.

PreCalculus II Worksheet Name _____

(6)

Write each expression using radical signs and *no* negative exponents.

1. a) $x^{2/3}$ b) $x^{3/2}$ c) $5^{1/2} \cdot x^{-1/2}$

2. a) $3y^{2/5}$ b) $(3y)^{2/5}$ c) $a^{4/7} \cdot b^{-4/7}$ d) $a^{1/10} \cdot b^{-1/5}$

Write each expression using positive rational exponents.

3. a) $\sqrt{x^5}$ b) $\sqrt[3]{y^2}$ c) $\left(\sqrt[6]{2a}\right)^5$

4. a) $\sqrt[3]{8x^7}$ b) $\left(\sqrt[4]{16x}\right)^3$ c) $\sqrt[3]{27xy^2}$

Simplify

5. a) $\left(\frac{9}{25}\right)^{1/2}$ b) $\left(\frac{9}{25}\right)^{-1/2}$ c) $\left(\frac{9}{25}\right)^{5/2}$

6. a) $\left(\frac{27}{8}\right)^{1/3}$ b) $\left(\frac{27}{8}\right)^{2/3}$ c) $\left(\frac{27}{8}\right)^{-2/3}$

7. **Consumer Economics** The cost of a certain brand of camera has been increasing at 8% per year. If a camera now costs \$150, find the cost: $P(1+r)^{-t}$ -t goes back in time

   a) 2 years and 6 months from now b) 4 years and 3 months ago

8. **Business** The value of a computer depreciates at the rate of 25% per year. If a computer is now worth \$2400, find its approximate value:

   a) 3 years and 6 months from now b) 20 months ago

PRECALCULUS NAME: _____ (7)

Practice exercises

A. SIMPLIFY EACH OF THE GIVEN EXPRESSIONS.

1. $(5x^4)(12x^5)$

2. $(3x^5)^3(-2x^3)^4$

3. $\frac{x^{-3}}{y^{-4}}$

4. $\frac{12x^3y^{-2}}{8x^{-2}y^5}$

B. EVALUATE EACH OF THE GIVEN EXPRESSIONS. (noncalculator)

5. $12x^0$

6. $(\frac{3}{4})^{-2}$

7. $8^{\frac{2}{3}}$

8. $25^{\frac{-3}{2}}$

9. $(\frac{1}{8})^{\frac{-4}{3}}$

10. $(\frac{9}{4})^{\frac{5}{2}}$

11. $(\frac{81}{225})^{\frac{-1}{2}}$

12. $(\frac{16}{81})^{\frac{-3}{4}}$

13. $(\frac{27}{125})^{\frac{2}{3}}$

14. $(\frac{9}{25})^{\frac{-3}{2}}$

C. SOLVE EACH OF THE GIVEN EQUATIONS. (noncalculator)

15. $2^{x-1} = \frac{1}{8}$

16. $25^{x-2} = 125$

17. $8^{1-x} = 16^{x+1}$

18. $9^{x-1} = (\frac{1}{27})^{2-x}$

D. ANSWER THE GIVEN QUESTION.

19. State the domain and range of the function $f(x) = x^{\frac{3}{2}}$.

20. Find the volume (correct to the nearest hundredth) of a cube which has a surface area of 375 square centimeters.

EXPONENTIAL GROWTH AND DECAY WORKSHEET

PRECALC I – MEISTER NAME_____

1. There were 20,000 people living in Long Hill Township in 1990. The rate of growth is 2.7%.

   a. What is the initial value? _____ What is the growth rate? _____

   b. State the equation _____

   c. What will be the population in 2003? _____

   d. What was the population in 1980? _____

2. Justin bought a truck costing $21,000 in 1997. It depreciates at 15% each year.

   a. What is the initial value? _____ What is the growth rate? _____

   b. State the equation _____

   c. What will be the value of the truck in 2004? _____

   d. What was the value of the truck in 1996, when it was new? _____

3. A particular bacteria population doubles every 6 hours. There were 4,300 bacteria in the beginning.

   a. State the equation _____

   b. How many bacteria will there be after 12 hours? _____

   c. How many bacteria were there 4 hours before the experiment began? _____

4. Rossilium has a half-life of 15 minutes. You start with 78 grams.

   a. State the equation. _____

   b. How much will there be after 2 hours? _____

   c. How much was there 30 minutes before the experiment? _____

5. The half-life of radon is 385 days.

   a. What fraction of a sample with A_0 atoms remain at the end of 7.7 days? _____

   b. After 23.1 days? _____

6. A particular bacteria population doubles every 5 hours. There are 1,200 bacteria to begin with.

   a. State the equation _____

   b. Find the number of bacteria after 4 hours. _____

   c. When will the population reach 100,000? _____

7. In 1995, there were 32,000 people living in Cityville. The population triples every 25 years.

   a. State the equation _____

   b. What will the population be in 2005? _____

   c. When will the population reach 55,000? _____

Precalculus Name_____ ID: 1

3-1 Exponential Functions

Date_____ Period____

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

1) $x^{\frac{3}{2}}y^4 \cdot x^2y^{-2} \cdot \left(x^{-\frac{3}{2}}y^{\frac{4}{3}}\right)^{\frac{1}{3}}$

2) $\left(b^{\frac{1}{4}} \cdot a^{-2}b^{\frac{1}{3}}\right)^{-\frac{4}{3}}$

3) $x^{\frac{2}{3}}y^0 \cdot \left(xy^{\frac{1}{3}}\right)^2$

4) $\left(\left(x^2y^{\frac{3}{2}}\right)^0 \cdot x^{-2}y^{\frac{7}{4}}\right)^{\frac{3}{2}}$

5) $\left(a^{-\frac{1}{4}}b^2 \cdot b\right)^0$

6) $y^{-2} \cdot \left(x^{-\frac{3}{2}}y^{\frac{1}{2}}\right)^0$

7) $\dfrac{m^2n^{\frac{1}{3}}}{\left(m^{\frac{1}{4}}\right)^{\frac{2}{3}} \cdot m^{\frac{7}{4}}n^3}$

8) $\dfrac{\left(u^{-\frac{3}{2}}\right)^2 \cdot u^{-\frac{1}{3}}v^{-\frac{5}{4}}}{u^0v^{-\frac{5}{4}} \cdot u^{\frac{3}{2}}}$

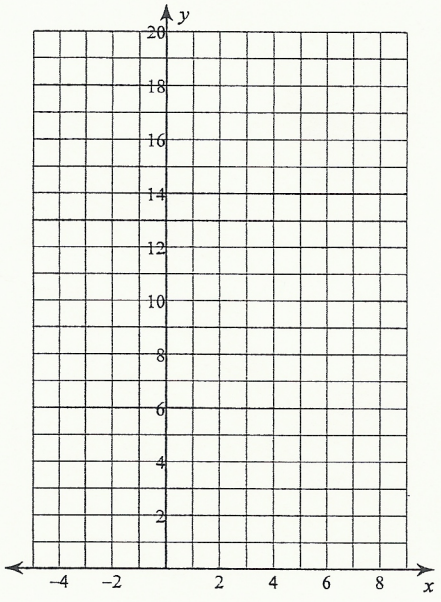
12

9) $\dfrac{\left(x^{-\frac{5}{3}}\right)^{-2}}{y^{3}x^{-2}}$

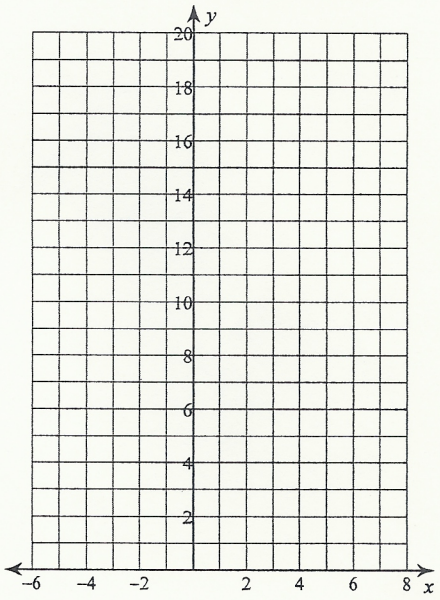
10) $\dfrac{x^{\frac{5}{3}}y^{-2}}{x^{-\frac{3}{4}}y^{-\frac{1}{2}}\cdot\left(x^{-\frac{5}{4}}\right)^{-\frac{3}{2}}}$

Sketch the graph of each function.

11) $f(x)=\dfrac{1}{3}\cdot 6^{x-2}+1$



12) $f(x)=4\cdot\left(\dfrac{1}{2}\right)^{x-1}+2$



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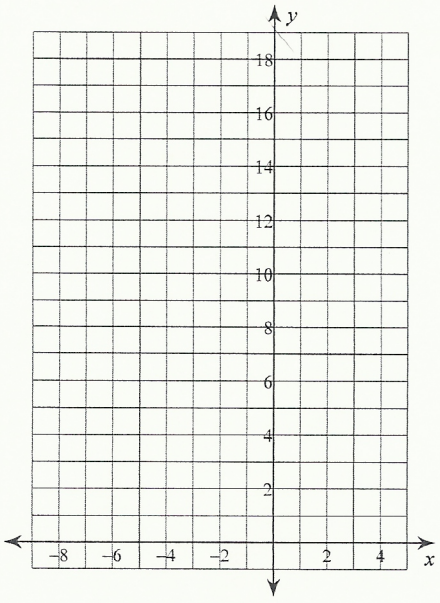
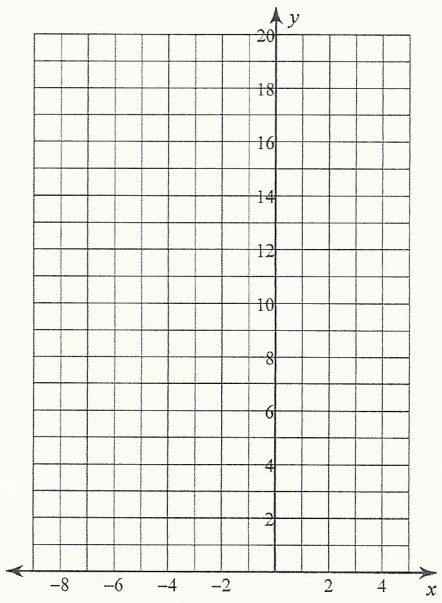
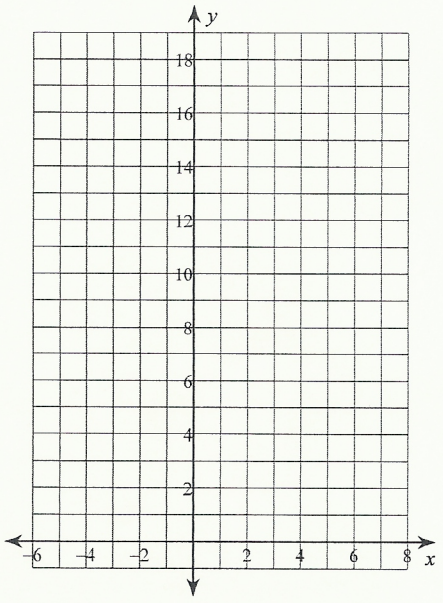
13) $f(x) = 2 \cdot 2^{x+2} - 1$



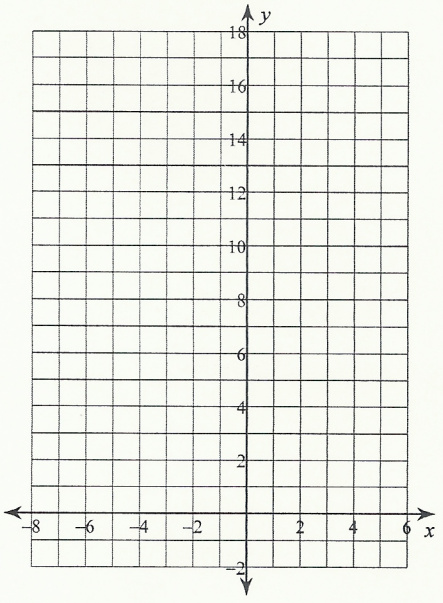
14) $f(x) = 4 \cdot 2^{x+2} + 2$



15) $f(x) = \frac{1}{4} \cdot 2^{x-1} - 1$



16) $f(x) = 4 \cdot \left(\frac{1}{2}\right)^{x+1} - 2$



3-1 Exponential Growth and Decay Word Problems

1. Bacteria can multiply at an alarming rate when each bacterial split into two new cells, thus doubling. If we start with only one bacterium which can double every hour, how many bacteria will we have by the end of one day?

2. Find a bank account balance after 12 years if the account starts with \$100 and has an annual interest rate of 4% compounded quarterly.

3. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

4. The population of Winnemucca, Nevada can be modeled by $P = 6191(1.04)^t$ where $t$is the number of years since 1990. What was the population in 1990? By what percentage did the population increase by each year?

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5. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

6. During normal breathing about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

7. An adult takes 400 mg of ibuprofen. Each hour the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left in the person's system after 6 hours?

8. You deposit $1600 in a bank account which has an interest rate of 4% compounded monthly. How much is in the account after 3 years?

9. You by a new computer for $2100. The computer depreciates at a rate of 50% annually. When will the computer have a value of $600? *(Hint: Use the intersection function on your calculator)*

10. You drink a beverage with 120mg of caffeine. Each hour the amount of caffeine in your system decreases by about 12%. How long until you have 10mg of caffeine left in your system?

11. The foundation of your house has about 1200 termites living in it. The termite population grows at a rate of about 2.4% per day. How long until the population of termites living in your foundation doubles?

12. If the population in 1990 was 782,248 and the population in 2000 was 894,943, when will the population reach 1.1 million?

skip

17

13. Suppose a culture of 100 bacteria doubles every 5 hours. Predict when the population will reach 50,000.

14. The ½ -life of a certain radioactive substance is 5 days. There are 12 g present initially. When will there be less than 1 g remaining?

PROPERTIES OF EXPONENTIAL FUNCTIONS: $f(x) = a \cdot b^x$

1. Domain is the set of all real numbers $\mathbb{R}$

2. Range is the set of POSITIVE real numbers (not 0)

3. Because the range is the set of positive reals, every positive real can be expressed as a multiple of the power of b.

**4. Graph crosses the point (0, a)

5. The graph does NOT cross or touch the x-axis

6. The function is strictly increasing (if $b > 1$) or strictly decreasing (if $0 < b < 1$)
   i.e. If $b = 1.3$, $b = 4$, $b = 15$, etc., the function will start near the x-axis and increase to infinity.
   If $b = .5$, $\frac{3}{4}$, etc., the function will start near infinity and decrease down toward the x-axis.

7. Growth: As x gets larger, f(x) increases without bound.
   This is when $b > 1$.
   Decay: As x gets smaller, f(x) increases without bound.
   This is when $0 < b < 1$.

8. There is a horizontal asymptote at $y = 0$.
   ($y = 0$ is the x-axis)