

1-3 Evaluating Limits, Precalculus

Estimate each one-sided or two-sided limit, if it exists.

1. $\lim_{x \rightarrow 0^+} (4 - \sqrt{x})$

2. $\lim_{x \rightarrow 3^+} \frac{3-x}{|x-3|}$

3. $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

4. $\lim_{x \rightarrow -1^-} \frac{x+7}{x^2 + 8x + 7}$

5. $\lim_{x \rightarrow -1^+} \frac{x+7}{x^2 + 8x + 7}$

6. $\lim_{x \rightarrow -2} \frac{1}{(x+2)^2}$

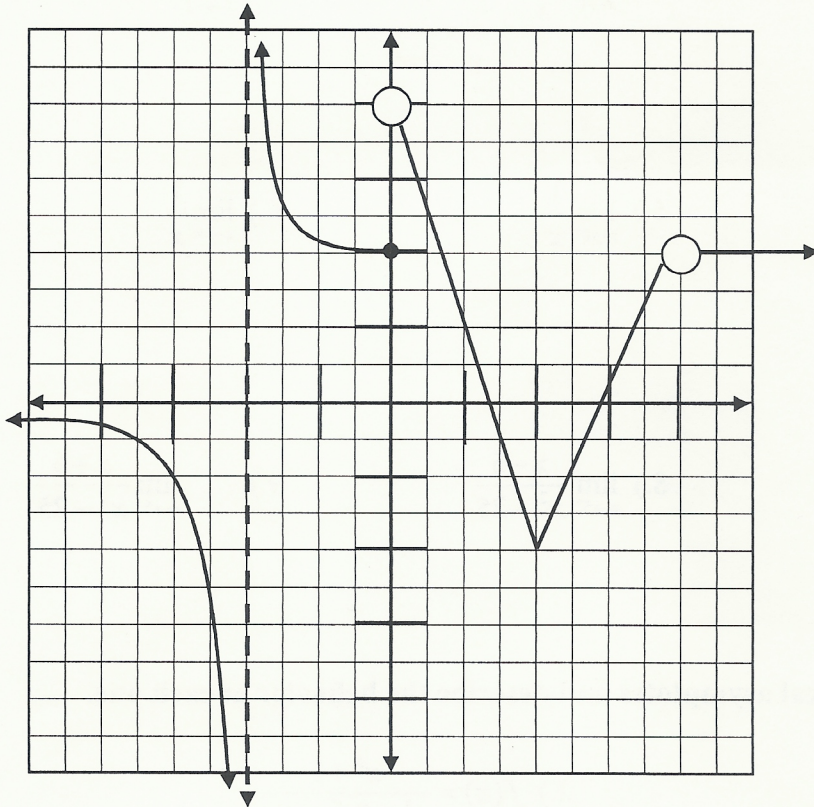
7. $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$

8. $\lim_{x \rightarrow 3} \frac{x^3 + 27}{x^2 - 9}$

PRECALCULUS
WS 1.3

NAME _____

I. Consider the function $f(x)$ graphed below (each line equals $1/2$). Find the following limits.



1.) $\lim_{x \rightarrow -2^+} f(x) =$ _____

2.) $\lim_{x \rightarrow -\infty} f(x) =$ _____

3.) $\lim_{x \rightarrow 0^+} f(x) =$ _____

4.) $\lim_{x \rightarrow \infty} f(x) =$ _____

5.) $\lim_{x \rightarrow 4} f(x) =$ _____

6.) $\lim_{x \rightarrow 0^-} f(x) =$ _____

7.) $\lim_{x \rightarrow 2} f(x) =$ _____

8.) $\lim_{x \rightarrow -2^-} f(x) =$ _____

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II. Use your graphing calculator and evaluate the following limits.

1.) $\lim_{x \rightarrow 2} 3x - 1$

2.) $\lim_{x \rightarrow 1} \frac{x+1}{x}$

3.) $\lim_{x \rightarrow -2^-} 2x - 4$

4.) $\lim_{x \rightarrow 0^-} \frac{1}{x}$

5.) $\lim_{x \rightarrow 0^+} \frac{1}{x}$

6.) $\lim_{x \rightarrow \infty} \frac{1}{x}$

7.) $\lim_{x \rightarrow -\infty} \frac{5x^2 + 1}{x^2 - 3x}$

8.) $\lim_{x \rightarrow 5^+} \frac{x-5}{x^2-25}$

9.) $\lim_{x \rightarrow 5^-} \frac{x+5}{x^2-25}$

III. Find the vertical asymptotes and describe the behavior at each one.

1.) $f(x) = \frac{x+5}{x^2-25}$

2.) $f(x) = \frac{x^3+1}{x^3+5x^2+4x}$

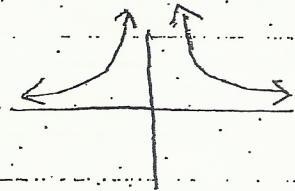
IV. Find the horizontal asymptotes algebraically.

1.) $f(x) = \frac{4x^3-1}{2x^2-3x^3}$

2.) $f(x) = \frac{4x^3-1}{2x^4-3x^3}$

3.) $f(x) = \frac{4x^3-1}{2x^2-3x}$

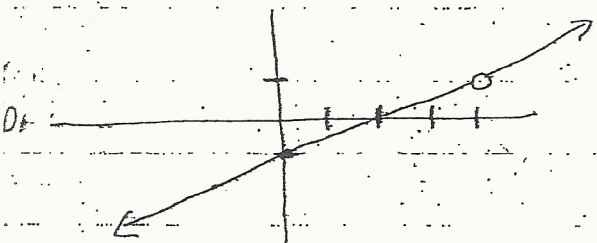
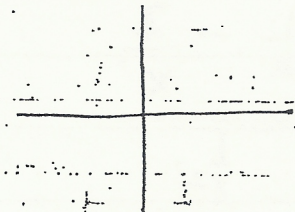
g(x) = $\frac{x^2}{x+1} - \frac{x}{x+1}$ Simplify + give restrictions

f(x) = $\frac{1}{x^2}$  Identify the equation of the discontinuity
Is it essential or removable?

f(x) = $\frac{3x+30}{x+10}$ simplify / identify restrictions and graph
ess or remou.

f(x) = $\frac{x(x-1)}{(x-1)^2}$ T/F There is a removable disc. @ x=1

f(x) = $\frac{x+2}{3x-5}$ Domain = Disc = Graph



Give an equation for the function on the left.

1-3 WSB

$$1. \frac{9}{x-y} + \frac{21}{y-x} + \frac{7}{4x-4y}$$

8. simplify + state restrictions

$$1. \frac{7}{3-x} + \frac{11}{x-3}$$

$$1. N = \sqrt{3}$$

$$10. \frac{2+\sqrt{3}}{4-\sqrt{3}}$$

$$1. \frac{\frac{1}{x} - \frac{2}{x}}{\frac{1}{x^2}}$$

ADV PRECALC

Name _____

1.3 WS 2

Determine whether each function is continuous at the given x -value(s). If it is discontinuous, identify the type of discontinuity.

1.) $f(x) = \frac{-2x}{x^2 - 9}$; at $x = 0$ and $x = -3$

2.) $f(x) = \frac{x+4}{x^2 + 2x - 8}$; at $x = 2$ and $x = -4$

3.) $f(x) = \frac{x+2}{x^2 - x - 6}$; at $x = 3$ and $x = -2$

4.) $f(x) = \frac{|x+2|}{x+2}$; at $x = 2$ and $x = -2$

Determine the end behavior of each function as x approaches both positive and negative infinity.

1.) $g(x) = x^4 - 3x^3 + 2x^2 - x + 1$

2.) $h(x) = 1 - 2x^2 - 4x^3$

3.) $m(x) = \frac{3}{x+2}$

4.) $f(x) = \frac{2+4x}{2x-5}$

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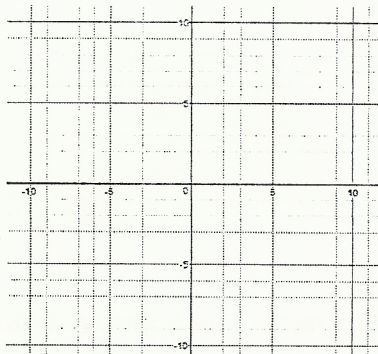
5.) $g(x) = \cos x$

6.) $h(x) = \frac{x^3 - 4x^2 - 29x - 24}{x^2 - 2x - 15}$

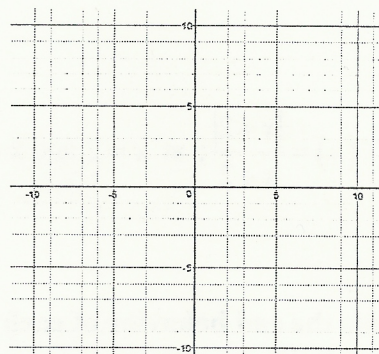
7.) $f(x) = \frac{2x^2 + 3x}{x^2 + 4} + 3$

Sketch a graph of each of the following functions, indicating holes and removable discontinuities, and dashed lines for any asymptotes.

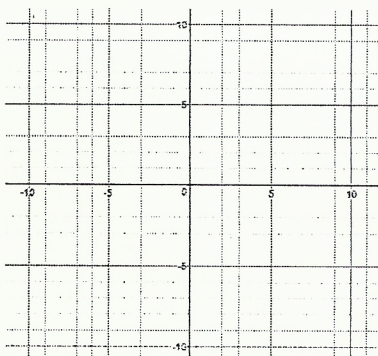
1.) $f(x) = \frac{x+1}{x^2-1}$



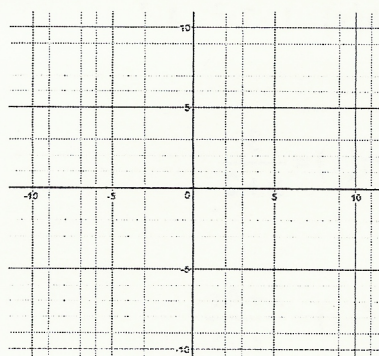
2.) $f(x) = \frac{x^2 + 2x}{x^3 - 4x}$



2.) $h(x) = \frac{x^2 + 5x - 6}{x + 6}$



4.) $f(x) = \frac{x-2}{x^2-4}$



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1-3 End Behavior WS, Precalculus

Describe the end behavior in limit notation.

1. $g(x) = \frac{2x^2 + 1}{x^2 - 1}$

2. $f(x) = \frac{x^3 + 2x^2 + 3x + 1}{4x^3 + 3x + 2}$

3. $g(x) = \frac{2x + 6}{x^2 + 6x + 9}$

4. $h(x) = \frac{3x^2 + 5x - 9}{2x + 6}$

5. $f(x) = \frac{x^2 + x - 10}{x + 2}$

6. $f(x) = \frac{x + 3}{x^2 - 2x - 15}$

7. $g(x) = \frac{3x - 5}{x + 1}$

8. $h(x) = \frac{x^2 - 16}{x - 4}$

9. $j(x) = \frac{6x^4}{x^2 - 3}$

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10. $g(x) = \frac{x}{2x-4}$

11. $f(x) = -2x^3 + 3x + 4$

12. $g(x) = 3x^6 + 2x^2 + 1$

13. $h(x) = -5x^4 + 3x^2 + 2x - 1$

14. $j(x) = 3x^5 + 4x^4 + 3x^2 - 5$

A. Determine the end behavior of each of the given functions.

1. $f(x) = 2x^2 + 8x - 5$

2. $g(x) = \left(\frac{3}{4}\right)^x - 2$

3. $h(x) = x^3 - x - 6$

4. $p(x) = \frac{20}{1 + e^{-x}}$

5. $q(x) = \frac{2x}{x^2 - 4}$

6. $r(x) = \frac{4}{x} + 4$

B. Find each of the indicated limits.

7. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 - 5}{x^2} \right)$

8. $\lim_{x \rightarrow -\infty} (\cos 2x)$

9. $\lim_{x \rightarrow \infty} \frac{5x - 1}{2x + 1}$

10. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

C. Answer each of the given questions.

11. Given that f is an even function such that $\lim_{x \rightarrow -\infty} f(x) = \infty$, determine $\lim_{x \rightarrow \infty} f(x)$.

1-3: Lesson Master, Precalculus

1. Consider the function g defined by $g(x) = \frac{-2x}{4x^2 - 9}$.

a. Identify the vertical asymptotes of the graph of g . _____

b. Use limit notation to describe the behavior of g near the asymptotes.

c. Use limit notation to describe the end behavior of g .

d. Sketch a graph of $g(x)$.

2. Given $f(x) = \frac{x+4}{x^2 + 2x - 8}$, sketch a graph of $f(x)$ and find:

a. $\lim_{x \rightarrow -4^-} f(x)$ _____

b. $\lim_{x \rightarrow -4^+} f(x)$ _____

c. $\lim_{x \rightarrow 2} f(x)$ _____

d. $\lim_{x \rightarrow 2^+} f(x)$ _____

3. Given $f(x) = \frac{x+2}{x^2 - x - 6}$, f has a(n) _____ discontinuity at $x = -2$ and

a(n) _____ discontinuity at $x = 3$.

In 4 and 5, classify any discontinuities as infinite or removable, and at each removable discontinuity, refine the function to make it continuous.

4. $f(a) = \frac{2a^2 + a - 3}{a + 1}$

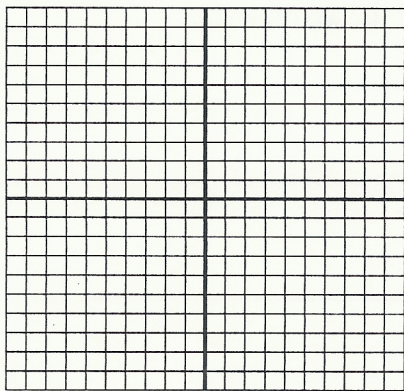
5. $f(b) = \frac{2b^2 + b - 3}{b - 1}$

6. Find a rule function that has a removable discontinuity at $x = -2$ and no infinite discontinuities.

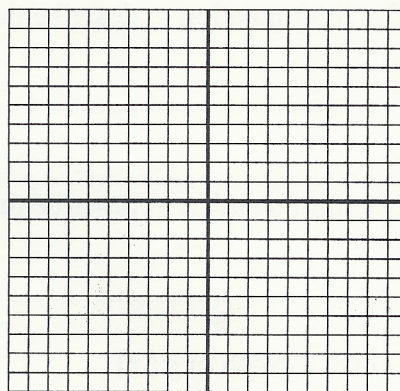
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In 7-10, sketch a graph of the function, indicating holes at the removable discontinuities and dashed lines for any asymptotes.

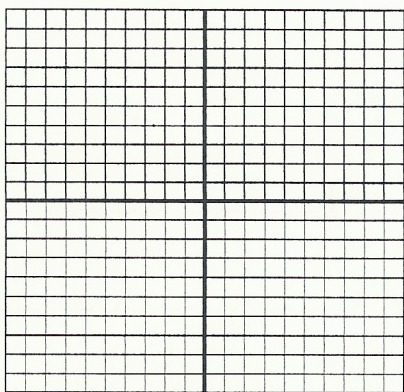
7. $f(x) = \frac{x+1}{x^2-1}$



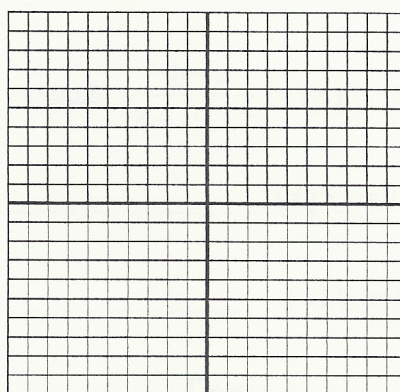
8. $f(x) = \frac{x^2+2x}{x^3-4x}$



9. $h(x) = \frac{x^2+5x-6}{x+6}$



10. $f(x) = \frac{x-2}{x^2-4}$



In 11 and 12: a. find equations for any vertical asymptotes, b. use limit notation to describe the behavior of the function near any asymptote and c. use limit notation to describe the behavior of the function near any removable discontinuity.

11. The function in Question 8.

a. _____

b. _____

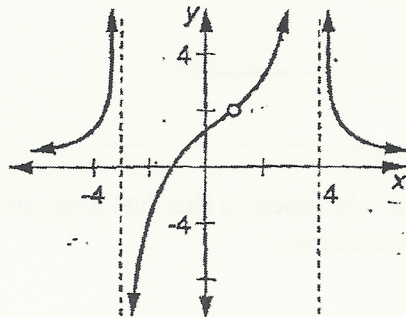
c. _____

12. The function graphed at the right

a. _____

b. _____

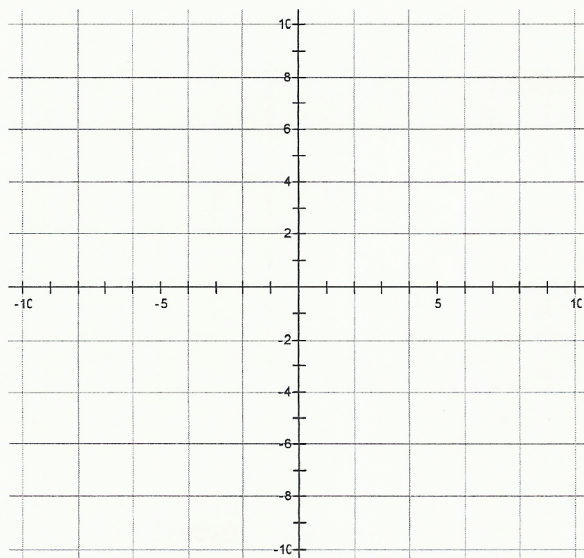
c. _____



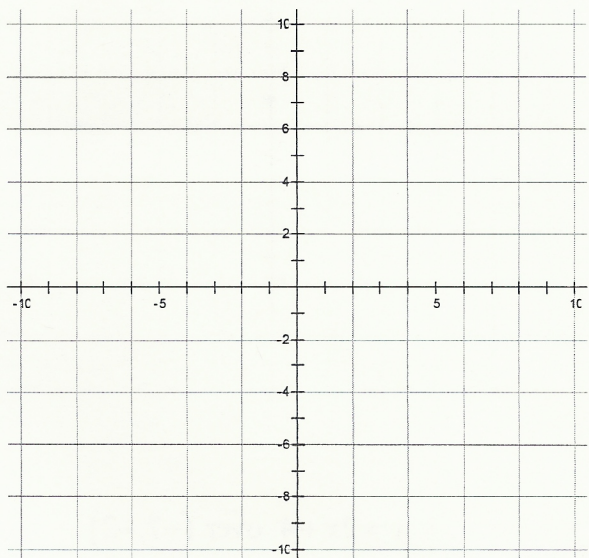
Piecewise Functions

Graph the equations over the given intervals

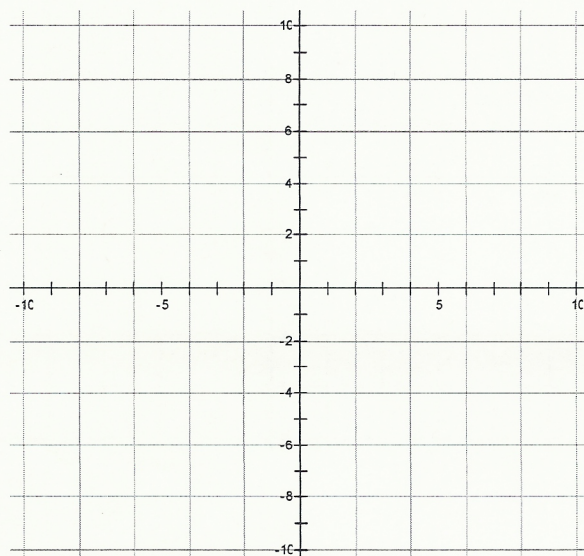
1. $y = 2x + 3$ over $[-2, 3)$



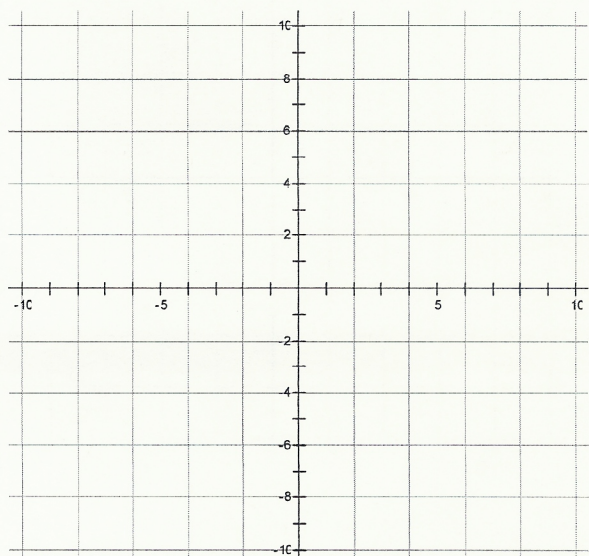
2. $y = -\frac{1}{3}x - 4$ over $(-2, 1)$



3. $y = x^2$ over $[-2, 2]$

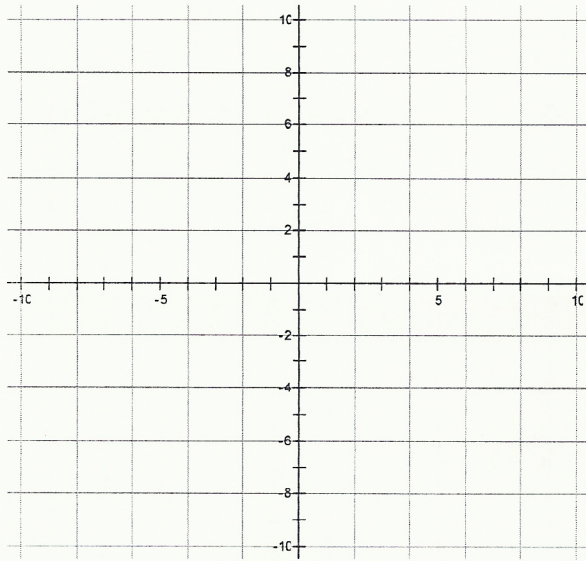


4. $y = x^2$ over $(-1, 3)$

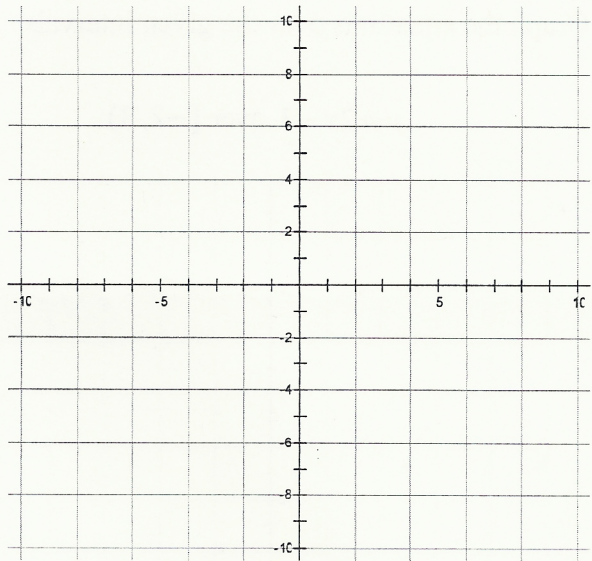


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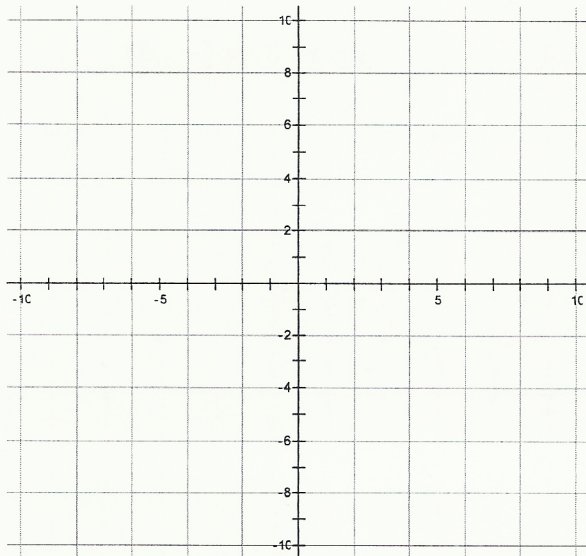
5. $y = \frac{1}{x}$ over $[1,6]$



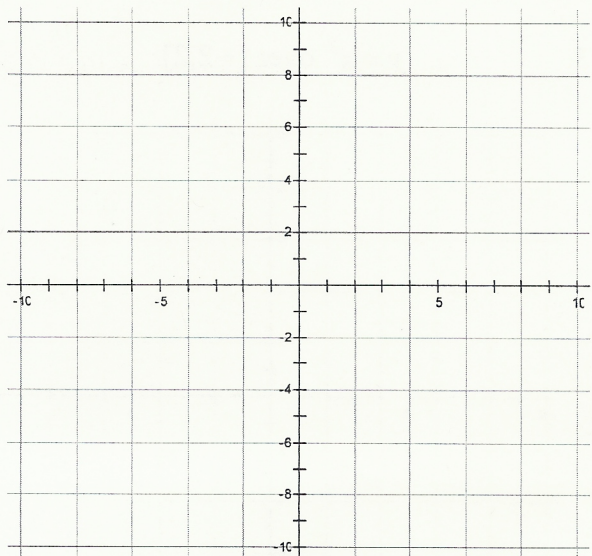
6. $y = \frac{1}{x^2}$ over $(-1,1)$



7. $y = 2x + 4$ over $(-7, -2)$
 $y = x^2$ over $[-2, 1]$



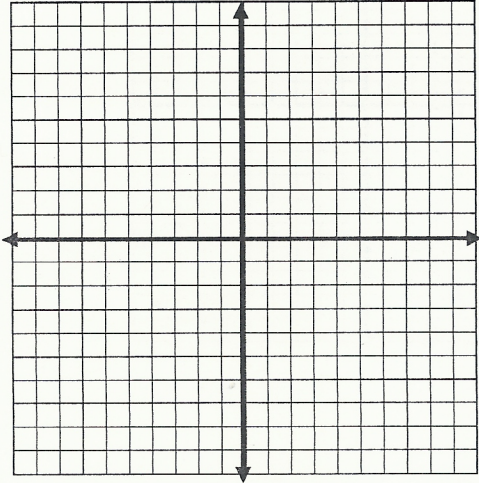
8. $y = \sqrt{x}$ over $(1, 4]$
 $y = x^3$ over $[-2, 1]$



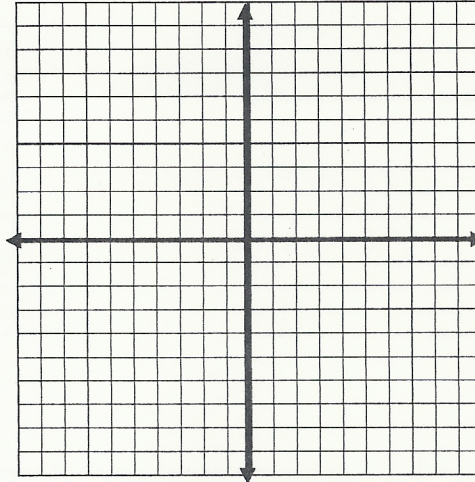
Piecewise Function Homework Sheet

I. Graph each of the piecewise functions:

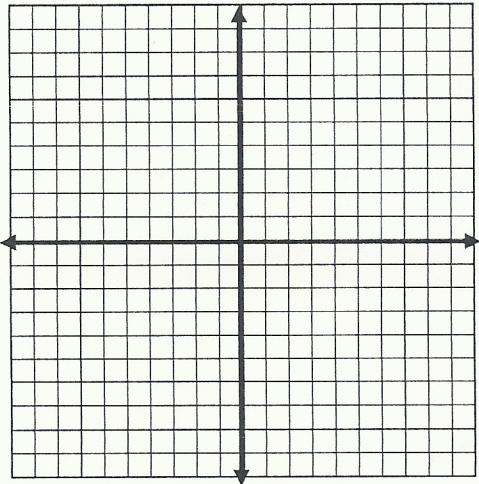
1. $f(x) = \begin{cases} 0 & x < 2 \\ 1 & x \geq 2 \end{cases}$



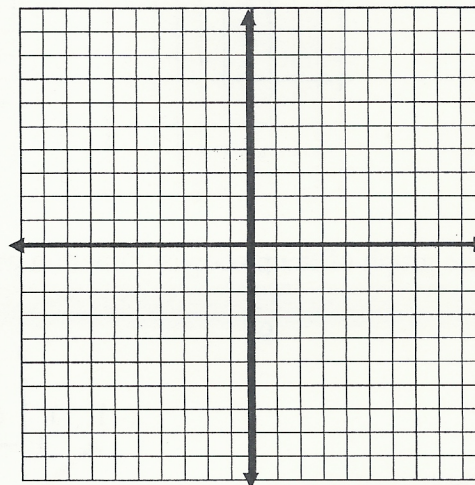
2. $g(x) = \begin{cases} 3 & x < 2 \\ x-1 & x \geq 2 \end{cases}$



3. $f(x) = \begin{cases} -1 & x < -1 \\ 1 & -1 \leq x \leq 1 \\ -1 & x > 1 \end{cases}$



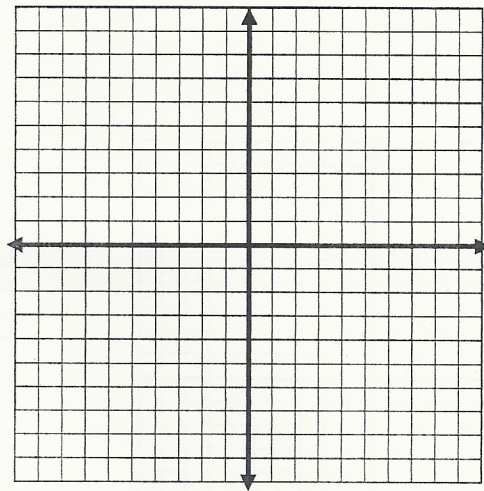
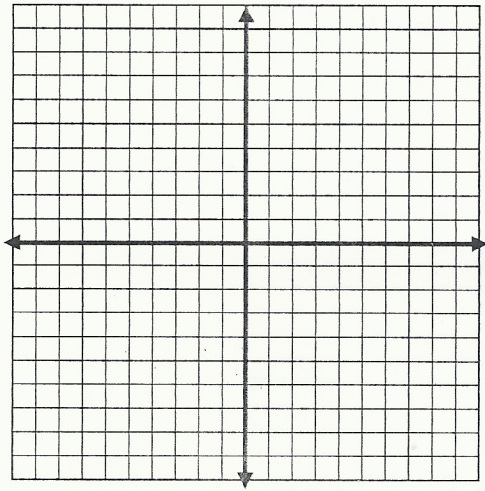
4. $p(x) = \begin{cases} 2 & x \leq -1 \\ x^2 & x > -1 \end{cases}$



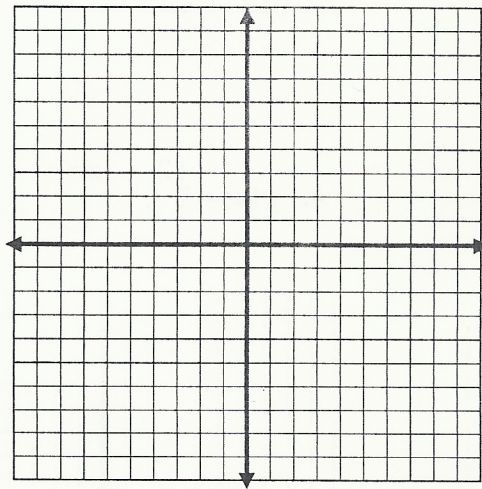
14

$$5. f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \leq x \leq 2 \\ 6-x & x > 2 \end{cases}$$

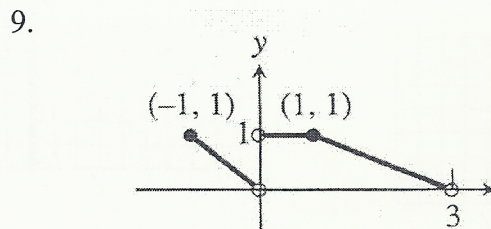
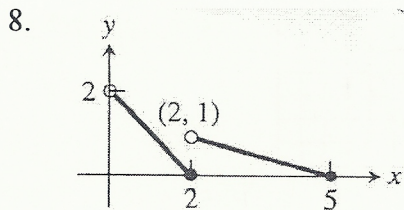
$$6. m(x) = \begin{cases} -x & x \leq 0 \\ 9-x^2 & 0 < x \leq 3 \\ x-3 & x > 3 \end{cases}$$



$$7. y(x) = \begin{cases} x+2 & x \leq -1 \\ x^2 & x > -1 \end{cases}$$



Give an equation for the graph of the piecewise function graphed below:

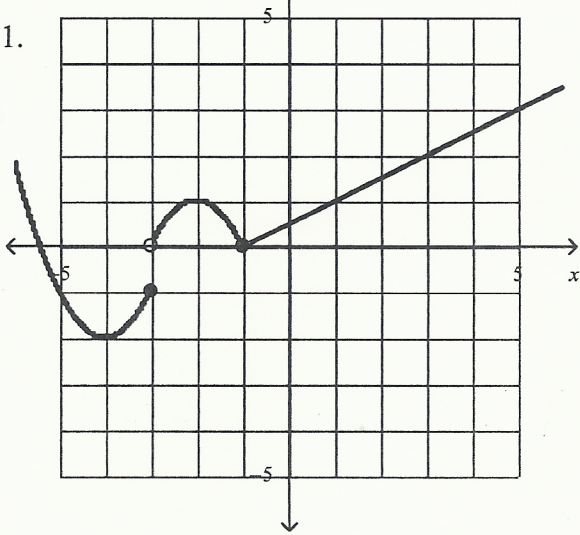


Piecewise Functions: Equations from Graphs

Precalculus

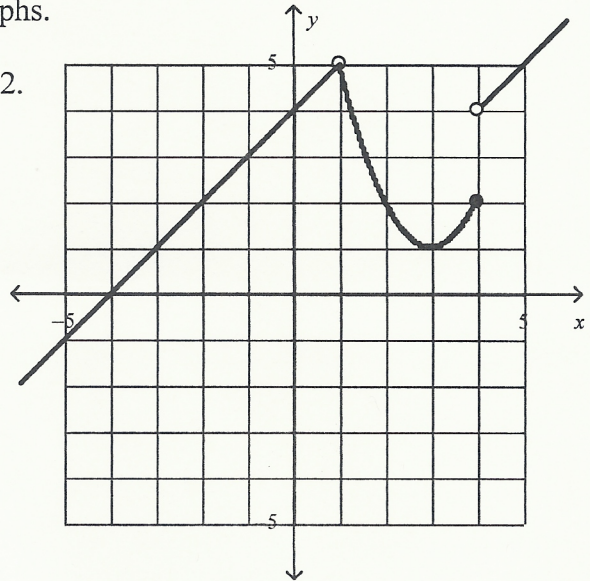
Write the equations for the following piecewise graphs.

1.



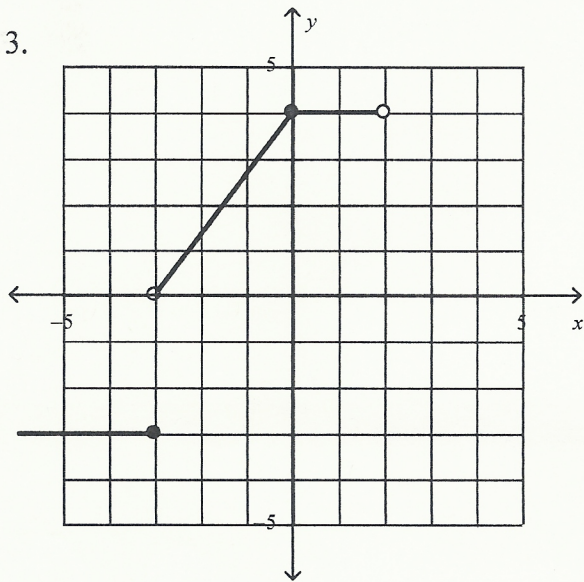
$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$

2.



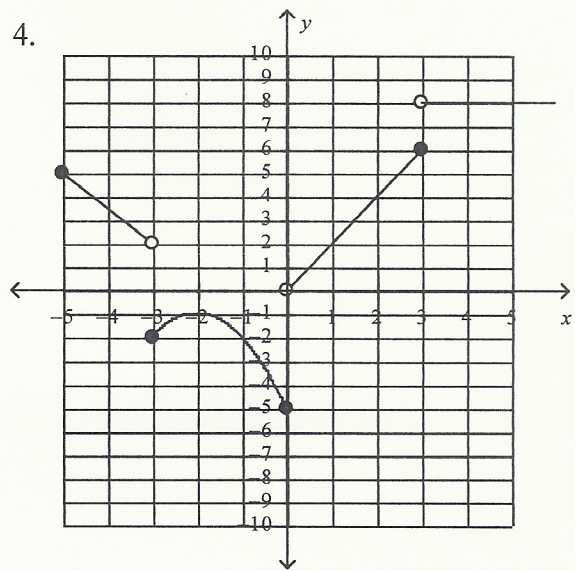
$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$

3.



$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$

4.



$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$