

5-2: POLYNOMIALS, LINEAR FACTORS AND ZEROS

Algebra 2

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WRITING A POLYNOMIAL IN FACTORED FORM

What is the factored form of $x^3 + x^2 - 12x$?

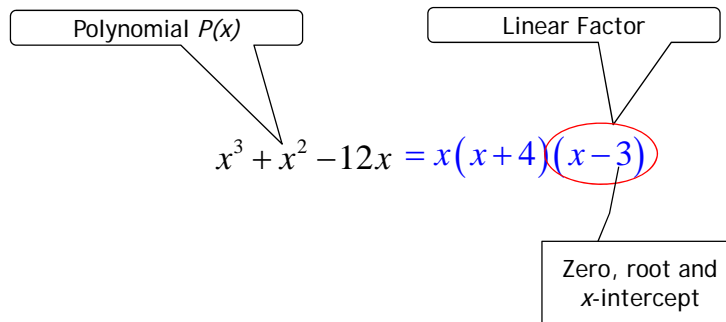
$$x^3 + x^2 - 12x = x(x^2 + x - 12) \quad \text{Factor out the GCF, } x$$

$$= x(x+4)(x-3) \quad \text{Factor } x^2 + x - 12$$

The factored form of $x^3 + x^2 - 12x$ allows you to find the zeros, roots (solutions) and x-intercepts of the polynomial.

The following are equivalent statements about a real number b and a polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

- $x - b$ is a linear factor of the polynomial $P(x)$.
- b is a zero of the polynomial function $y = P(x)$.
- b is a root (solution) of the polynomial equation $P(x) = 0$.
- b is an x-intercept of the graph $y = P(x)$.



FINDING ZEROS OF A POLYNOMIAL FUNCTION

What are the zeros of $y = (x - 3)(x + 4)(x - 1)$? Graph the function.

Use the Zero Product Property

$$0 = (x - 3)(x + 4)(x - 1)$$

The zeros are: 3, -4 and 1

Find points for x -values between the zeros: -2, 0 and 2.

$$(-2 - 3)(-2 + 4)(-2 - 1) = 30$$

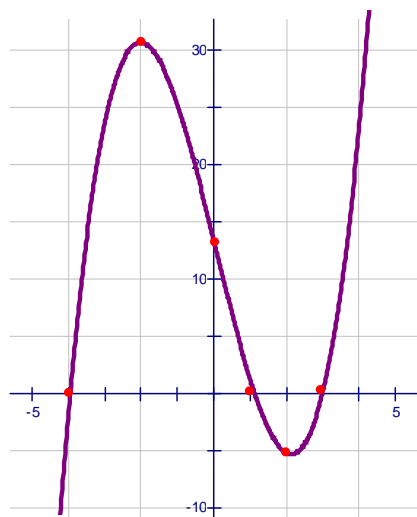
$$(0 - 3)(0 + 4)(0 - 1) = 12$$

$$(2 - 3)(2 + 4)(2 - 1) = -6$$

What is the end behavior.

Falls to the left, rises to the right

Sketch the graph.



WRITING A POLYNOMIAL FUNCTION FROM ITS ZEROS

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

- Allows factors of a polynomial to be written from the zeros
- Multiply the factors to write the polynomial function.

What is a polynomial function in standard form with zeros 1, -1 and 4?

$$f(x) = (x-1)(x+1)(x-4)$$

Write a linear factor for each zero.

$$f(x) = (x-1)(x^2 - 3x - 4)$$

Multiply $(x+1)$ and $(x-4)$.

$$f(x) = x^3 - 4x^2 - x + 4$$

Multiply $(x-1)$ and (x^2-3x-4) .

Complete Got It? #3 a & b p. 290

a. $f(x) = x^2 - 9$ b. $P(x) = x^3 - 3x^2 - 9x + 27$

MULTIPLICITY OF ZERO

- a is a zero of multiplicity n
 - Means that $x - a$ appears n times as a factor

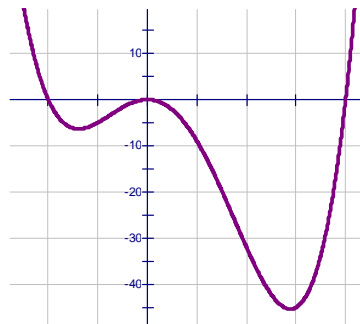
$$f(x) = x^4 - 2x^3 - 8x^2$$

$$f(x) = x^2(x-4)(x+2)$$

The zeroes are -2, 0, 4

Multiplicity of each:

- 2 & 4- Multiplicity of 1
- 0- Multiplicity of 2

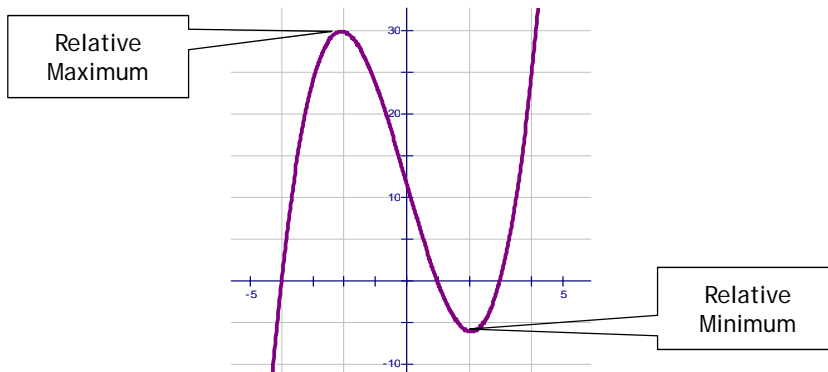


- a is a zero of *even* multiplicity
 - Graph touches x -axis at a and turns around
- a is a zero of *odd* multiplicity
 - Graph crosses x -axis at a

HOMEWORK: p. 293 #7-33 odd, 57-59

GRAPHS OF POLYNOMIAL FUNCTIONS

- ◉ Relative Maximum:
 - The value at an up-to-down turning point.
- ◉ Relative Minimum:
 - The value at a down-to-up turning point.



A designer wants to make a rectangular prism box with maximum volume, while keeping the sum of its length, width and height 12 in. The length must be 3 times the height. What should each dimension be?

$$x = \text{height of box} \quad 3x = \text{length of box}$$

$$12 - (x + 3x) = \text{width of box}$$

$$V = l \times w \times h$$

$$V = x(3x)(12 - (x + 3x))$$

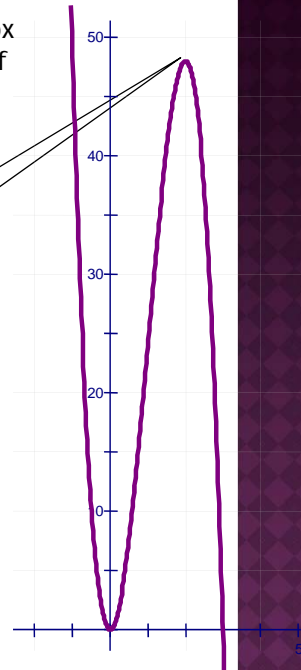
$$V = x(3x)(12 - 4x)$$

$$V = 3x^2(12 - 4x)$$

$$V = 36x^2 - 12x^3$$

2 in. = height
6 in. = length
4 in = width

Relative
Maximum
(2,48)



HOMEWORK: p. 293 #35-39, 44-47, 61-69 odd