3-2: Solving Systems Algebraically

Mr. Gallo
Algebra 2

Solving Systems with The Substitution Method

Given the system
$$\begin{cases} x + y = 6 \\ y = x + 2 \end{cases}$$

1) Substitute the expression for y from the second equation into the first

$$x + x + 2 = 6$$

2) Simplify and solve for x:

$$2x + 2 = 6$$
$$2x = 4$$
$$x = 2$$

3) Substitute the solution for x into the second equation and solve for y

$$y = x + 2$$
$$y = 2 + 2$$
$$y = 4$$

3) Check the solution to the system in both equations:

$$4 = 2 + 2 \qquad 4 + 2 = 6$$

Example: Solve the following system of equations:

$$\begin{cases} 5x - 3y = -1 \\ x + y = 3 \end{cases}$$

$$x + y = 3$$
$$y = -x + 3$$

1) Substitute

5x-3y = -1

5x-3(-x+3) = -1

2) Simplify and solve for x:

5x+3x-9 = -1

8x-9 = -1

8x = 8

x = 1

3) Substitute the solution for x into the first equation and solve for y

x = 1

y = -1+3

y = 2

(1, 2) is the solution

4) Check the solution to the system in both equations:

5(1)-3(2) = -1

1+2 = 3

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(-2.5, 2.5) is the solution

Solving Systems of Linear Equations

You are in charge of ordering labels for a small business. A company that makes labels charges a yearly fee plus a cost per label. You paid $375 last year for 300 labels. This year you ordered 1,000 labels and paid $725. What are the yearly fee and cost per label, assuming the prices did not change?

Let y = the yearly cost

Let l = the cost per label

$$\begin{cases} y + 300l = 375 \\ y + 1000l = 725 \end{cases}$$

1) Solve one equation for y

y + 300l = 375

y = - 300l + 375

2) Substitute

- 300l + 375 + 1000l = 725

3) Simplify and solve for l:

$$700l + 375 = 725$$
$$700l = 350$$
$$l = .5$$

4) Substitute and solve for y:

y = -300(.5)+375

y = -150+375

y = 225

The yearly fee is $225 and the cost per label is $.50

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$.95 per download; $5.50 one-time registration fee

Homework: p. 146 #10-21, 72-76 even

p. 146 #10-21, 72-76 even

10. (0.5, 2.5)

11. (–2, 4)

12. (20, 4)

13. (0.75, 2.5)

14. (10, −1)

15. (8, −1)

16. (–6, −9)

17. (–2, −5)

18. (–6, −6)

19. seven \$1-bills; eight \$5-bills

20a. Let m = number of multiple choice and r = number of extended response, then

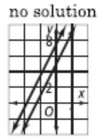
$$\begin{cases} m + r = 20 \\ 2m + 6r = 60 \end{cases}$$

20b. 15 multiple choice; 5 extended response

21. 3 vans and 2 sedans

72. no solution

74. no solution



76. function

Solving Systems Using the Elimination Method

This method is most useful when the equations are written in _Standard Form__.

Solve the System:

$$\begin{cases} x + y = 9 \\ 2x - y = 2 \end{cases}$$

$$+ \quad \overline{3x \quad = 11}$$

$$x = \frac{11}{3}$$

Notice the coefficients of y in each equation is 1 and -1. When the two equations are added together their sum is _0__

Adding the **two** equations then results in an equation with only _one variable__.

When the coefficients of a variable are opposites, _add_ the two equations and solve for the remaining variable.

Solve the System:

$$\begin{cases} x + y = 9 \\ 2x - y = 2 \end{cases}$$

$$+ \quad \overline{3x \quad = 11}$$

$$x = \frac{11}{3}$$

$$x + y = 9$$

$$\frac{11}{3} + y = 9$$

$$y = \frac{16}{3}$$

$$2\left(\frac{11}{3}\right) - y = 2$$

$$\frac{22}{3} - \frac{16}{3} \stackrel{?}{=} 2$$

$$\frac{6}{3} = 2$$

Substitute the answer in either _original_ equation and solve for the remaining variable.

The solution is: $\left(\frac{11}{3}, \frac{16}{3}\right)$

Complete Got It? #3 p. 144 (4,0) is the solution

Solving Equivalent Systems:

Solve the System:

$$\begin{cases}(3x+2y=10)2\\(2x+7y=18)\text{-}3\end{cases}$$

$$6x+4y=20$$

$$+\ -6x-21y=-54$$

$$-17y=-34$$

$$y=2$$

$$2x+7(2)=18$$

$$2x+14=18$$

$$2x=4$$

$$x=2$$

The solution is: $(2,2)$

1) Pick one variable and find the opposites of its coefficients.

2) Multiply each equation by the factor which will produce the LCM for the chosen variable.

3) Add to eliminate one of the variables.

4) Solve the new equation for the remaining variable.

5) Substitute the value into one of the original equations and solve for the other variable.

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(-2, 3) is the solution

1) $\begin{cases}x=3-2y\\3x+6y=6\end{cases}$

$$3(3-2y)+6y=6$$

$$9-6y+6y=6$$

$$9=6$$

No Solution

Inconsistent System

2) $\begin{cases}\frac{x}{3}+2y=12\\y=6-\frac{x}{6}\end{cases}$

$$\frac{x}{3}+2\left(6-\frac{x}{6}\right)=12$$

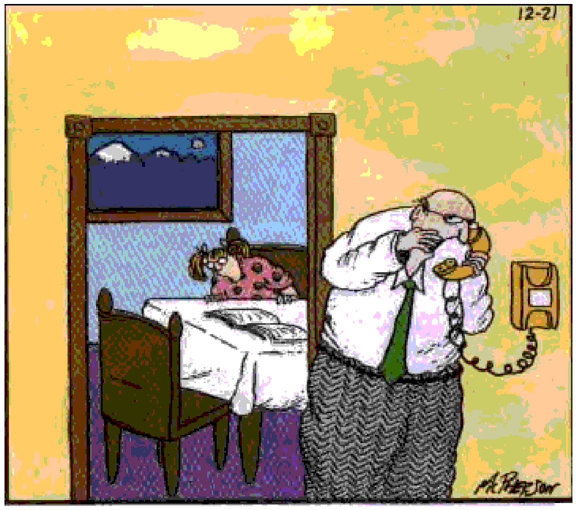
$$\frac{x}{3}+12-\frac{x}{3}=12$$

$$12=12$$

Infinitely Many Solutions

Consistent System

Homework: p.146 #22-26 even, 31-41 odd, 44, 45, 49, 59-61, 73-77 odd



"Uh, yeah, Homework Help Line? I need to have you explain the Quadratic Equation in roughly the amount of time it takes to get a cup of coffee."