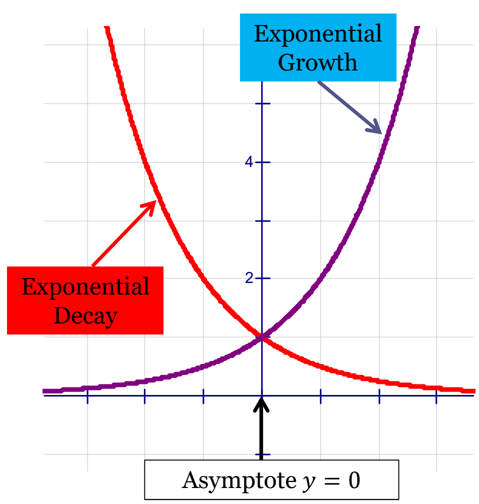
3-1: Exponential Functions

Precalculus
Mr. Gallo

Exponential Function

- Types:
  - Exponential Growth
    as ***x*** increases, ***y*** increases
  - Exponential Decay
    as ***x*** increases, ***y*** decreases approaching zero
- Asymptote
  - Line the graph approaches but never reaches.



Exponential Function

For the function $f(x) = ab^x$,

- if $a > 0$ and $b > 1$, the function represents:

$\boxed{\textit{exponential growth}}$ $\lim\limits_{x\to-\infty} f(x) = 0$ $\lim\limits_{x\to\infty} f(x) = \infty$

- if $a > 0$ and $0 < b < 1$, the function represents:

$\boxed{\textit{exponential decay}}$ $\lim\limits_{x\to-\infty} f(x) = \infty$ $\lim\limits_{x\to\infty} f(x) = 0$

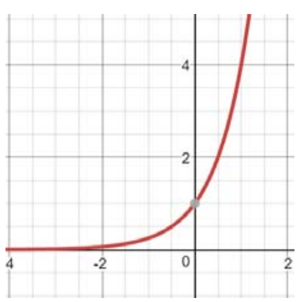
In either case, the y-intercept is $(0, a)$, the domain is$(-\infty, \infty)$, the range is $(0, \infty)$ and the asymptote is $y = 0$

Identify $y = 0.7^x$ as an example of exponential growth or decay. What is the $\boldsymbol{y}$- intercept? $a = 1$ $b = 0.7$

$\boxed{\text{Since } a > 0 \text{ and } 0 < b < 1\text{, this is decay.}}$ $\boxed{y\text{-intercept is } (0,1)}$

Sketch and analyze the graph of these functions.

$f(x) = 4^x$

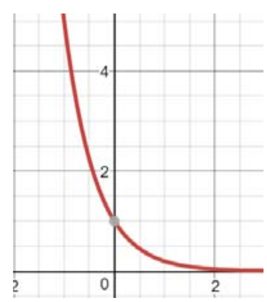


D: $(-\infty, \infty)$
R: $(0, \infty)$
y-int: $(0,1)$
x-int: None
EB: $\lim\limits_{x\to\infty} f(x) = \infty$

$\lim\limits_{x\to-\infty} f(x) = 0$

Extrema: None
Asym.: $y = 0$
Cont: $(-\infty, \infty)$

$g(x) = \left(\frac{1}{5}\right)^x$



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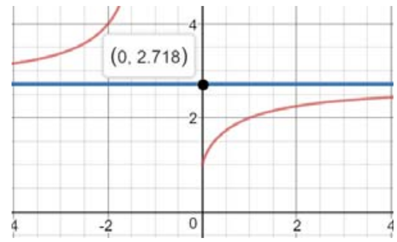
$\lim\limits_{x\to-\infty} f(x) = \infty$

Extrema: None
Asym.: $y = 0$
Cont: $(-\infty, \infty)$

Natural Base Exponential Functions

- Have ***e*** for a base
  - Most commonly used base
  - Simplifies calculations
- ***e***
  - Irrational number
  - $e \approx 2.71828$
  - Is an asymptote for graph of $y = \left(1 + \frac{1}{x}\right)^x$
- Functions have same properties as other exponential functions.
  - Has the form $f(x) = e^x$



Families of Exponential Functions

Families of Exponential Functions				
Parent Function				$y = b^x$
Dilation		Vertical	Horizontal	$y = ab^{cx}$
Stretch		$\lvert a \rvert > 1$	$0 < \lvert c \rvert < 1$	
Shrink		$0 < \lvert a \rvert < 1$	$\lvert c \rvert > 1$	
Reflection		$a < 0$; x-axis	$c < 0$; y-axis	
Translations (Horizontal by h; Vertical by k)				$y = b^{(x-h)} + k$
All transformations combined				$y = ab^{(c(x-h))} + k$

Describe the transformations from the parent function $f(x) = \left(\frac{1}{2}\right)^x$

$$f(x) = -2\left(\frac{1}{2}\right)^{x+1}$$

-Reflected over the *x*-axis

-Vertical stretch by a factor of 2

-Translated 1 unit left

Describe the transformations from the parent function $g(x) = e^x$

$$f(x) = e^{3x} - 1$$

-Translated 1 unit down

-Horizontal shrink by a factor of $\frac{1}{3}$

Homework: p. 166 #1-19 odd

Exponential Growth and Decay

In the equation $y = ab^x$,

- ***a*** is the <u>initial amount</u>.

- Exponential Growth: $b > 1$
  - ***b*** is the <u>growth factor</u>.
    - Increase written as a decimal is $\boldsymbol{r}$, <u>rate of increase</u> or <u>growth rate</u>. $b = 1 + r$ for exponential growth

- Exponential Decay: $0 < b < 1$
  - ***b*** is the <u>decay factor</u>.
  - Decay written as a decimal is $\boldsymbol{r}$, <u>rate of decay</u>.

$b = 1 + r$ for exponential decay because $\boldsymbol{r}$ is expressed as a <u>negative</u> quantity.

Compound Interest Formula

If a principal P is invested at an annual interest rate r (in decimal form) compounded n times a year, then the balance A in the account after t years is given by:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

To allow for quarterly, monthly or even daily compoundings, let n be the number of times the interest is compounded each year

- Rate per compounding $\frac{r}{n}$ is a fraction of the annual rate r
- Number of compoundings after t years is nt

Mrs. Salisman invested $2000 into an educational account for her daughter when she was an infant. The account has a 5% interest rate. If Mrs. Salisman does not make any other deposits or withdrawals, what will the account balance be after 18 years if the interest is compounded:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

a. Quarterly?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 2000\left(1+\frac{.05}{4}\right)^{4*18} = \$4891.84$$

b. Monthly?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 2000\left(1+\frac{.05}{12}\right)^{12*18} = \$4910.02$$

c. Daily?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 2000\left(1+\frac{.05}{365}\right)^{365*18} = \$4918.90$$

Continuously Compounded Interest

If a principal P is invested at an annual interest rate r (in decimal form) compounded continuously, then the balance A in the account after t years is given by:

$$A(t) = Pe^{rt}$$

Used when there is no waiting period between interest payments.

Mrs. Salisman found an account that will pay the 5% interest compounded continuously on her $2000 educational investment. What will be her account balance after 18 years?

P=2000 r=.05 t=18

$$A(t) = Pe^{rt}$$

$$A(t) = 2000e^{.05(18)}$$

$$A(t) \approx 4919.21$$

The account will have $4919.21

Exponential Growth or Decay

If an initial quantity N_0 grows or decays at an exponential rate r or k (as a decimal), then the final amount N after a time t is given by the following formulas:

$$N = N_0(1+r)^t$$

- If r is a **growth rate**, then $r > 0$
- If r is a **decay rate**, then $r < 0$

$$N = N_0 e^{kt}$$

- If k is a **continuous growth rate**, then $k > 0$
- If k is a **continuous decay rate**, then $k < 0$

Can be used to model investments, populations of people, animals, bacteria and amounts of radioactive material.

Models apply to any situation where growth is proportional to the initial size of the quantity being considered.

A state's population is declining at a rate of 2.6% annually. The state currently has a population of approximately 11 million people. If the population continues to decline at this rate, predict the population of the state in 15 and 30 years.

a. 2.6% annually $N = N_0(1+r)^t$

$$N = N_0(1+r)^t = 11{,}000{,}000(1-.026)^{15} = 7{,}409{,}298$$

$$N = N_0(1+r)^t = 11{,}000{,}000(1-.026)^{30} = 4{,}990{,}699$$

b. 2.6% continuously $N = N_0 e^{kt}$

$$N = N_0 e^{kt} = 11{,}000{,}000e^{-.026*15} = 7{,}447{,}626$$

$$N = N_0 e^{kt} = 11{,}000{,}000e^{-.026*30} = 5{,}042{,}466$$

Homework: p. 166 #21-26 & WS #6 #1-8

The table shows the population growth of deer in a forest from 2000 to 2010.

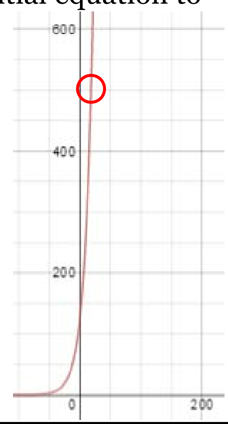
Deer Population	
Year	Deer
2000	125
2010	264

a. If the number of deer is increasing at an exponential rate, identify the rate of increase and write an exponential equation to model this situation.

$$264 = 125(1+r)^{10} \qquad N = 125(1.078)^t$$

$$\sqrt[10]{\frac{264}{125}} - 1 = r$$

$$.078 = r$$



b. Use your model to predict how many years it will take for the number of deer to reach 500.

$\approx$ 18.5 years

Homework: p. 166 #27, 29, 35-38, 40