

LIMITS 4: TRIGONOMETRIC FUNCTIONS AND PATCHING

I. Limit Theorems

A. $\lim_{\theta \rightarrow 0} \cos \theta = 1$

B. $\lim_{\theta \rightarrow 0} \sin \theta = 0$

C. $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

D. $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} = 1$

E. $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$

II. Patching

In order to make our trigonometric limits look like A-E of I, we may need to “PATCH” the trig expression. After, we apply our limit properties and verify on our calculator.

Examples -

$$\begin{aligned} 1. \lim_{x \rightarrow 0} \frac{\sin 3x}{x} &= \lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{} \cdot x} = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{3} \cdot x} \\ &= 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \\ &= 3(1) \\ &= 1 \cdot 3 = \boxed{3} \end{aligned}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x}{5x} =$$

$$\begin{aligned} &\frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{} \cdot x} \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{3} \cdot x} \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \\ &= \frac{3}{5} (1) = \boxed{\frac{3}{5}} \end{aligned}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x} =$$

$$\begin{aligned} &\lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{}} \cdot \frac{\boxed{}}{\sin 2x} \\ &= \frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin 3x}{\boxed{3x}} \cdot \frac{\boxed{2x}}{\sin 2x} \\ &= \frac{3}{2} (1)(1) = \boxed{\frac{3}{2}} \end{aligned}$$

$$4. \lim_{x \rightarrow 0} \frac{\sin^3 x}{x} = \lim_{x \rightarrow 0} \sin^2 x \cdot \frac{\sin x}{x} = 0 \cdot 1 = \boxed{0}$$

$$5. \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x \cos 3x} =$$

$$\lim_{x \rightarrow 0} \frac{1}{\cos 3x} \cdot \frac{\sin 3x}{\sin 2x} \cdot \frac{\boxed{}}{} =$$

$$\frac{3}{2} \lim_{x \rightarrow 0} \frac{1}{\cos 3x} \cdot \frac{\sin 3x}{\boxed{3x}} \cdot \frac{\boxed{2x}}{\sin 2x} = \boxed{\frac{3}{2}}$$

$$6. \lim_{x \rightarrow 0} \frac{\sec 3x \tan 2x}{7x} = \lim_{x \rightarrow 0} \sec 3x \cdot \tan 2x \cdot \frac{1}{7x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\cos 3x} \cdot \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{7x}$$

$$= \frac{1}{7} \lim_{x \rightarrow 0} \frac{1}{\cos 3x \cos 2x} \cdot \frac{\sin 2x}{\boxed{} \cdot x}$$

$$= \frac{2}{7} \lim_{x \rightarrow 0} \frac{1}{\cos 3x \cos 2x} \cdot \frac{\sin 2x}{\boxed{2x}} = \boxed{\frac{2}{7}}$$