LIMITS 3: INFINITE LIMITS AND ASYMPTOTES

I. Theorems:

A. $\lim\limits_{x \to \pm\infty} \frac{1}{x^n} = 0$

B. $\lim\limits_{x \to \pm\infty} k = k$

C. If $\lim\limits_{x \to \pm\infty} f(x) = L_1$ and $\lim\limits_{x \to \pm\infty} g(x) = L_2 \Rightarrow$ the sum, difference, constant, and power properties all apply.

D. $\lim\limits_{x \to \pm\infty} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to \pm\infty} f(x)}$ if the root exists

II. Vertical and Horizontal asymptotes

A. *Definition: the line x=a is a vertical asymptote of the graph of the function f(x) iff*

$$\lim_{x\to a^+} f(x)=\pm\infty \quad \text{or} \quad \lim_{x\to a^-} f(x)=\pm\infty$$

B. *Definition: the line y=b is a horizontal asymptote of the graph of f(x) iff*

$$\lim_{x\to\pm\infty} f(x)=b$$

C. *Find the vertical and horizontal asymptotes for the functions below and describe the behavior at each asymptote.*

1. $f(x)=\dfrac{2x^2-1}{x^2+3}$

- V.A. – None
- H.A. y = 2 Why?

$$\lim_{x\to\pm\infty} f(x)=\frac{x^2\left(\frac{2x^2}{x^2}-\frac{1}{x^2}\right)}{x^2\left(\frac{x^2}{x^2}+\frac{3}{x^2}\right)}=$$

$$\lim_{x\to\pm\infty}\frac{2-\frac{1}{x^2}}{1+\frac{3}{x^2}}=\frac{2-0}{1+0}=\boxed{2}$$

$$2.\ f(x)=\frac{x-3}{x+3}$$

V.A. → *x = -3*

H.A. → *y = 1*

$$\lim_{x\to -3^+} = \frac{-3^+ - 3}{\left(-3^+\right)+3} = \frac{-6}{0^+} = -\infty$$

$$\lim_{x\to -3^-} = \frac{-3^- - 3}{\left(-3^-\right)+3} = \frac{-6}{0^-} = \infty$$

$$\lim_{x\to\pm\infty} = \frac{x-3}{x+3} = \frac{x\left(\frac{x}{x}-\frac{3}{x}\right)}{x\left(\frac{x}{x}+\frac{3}{x}\right)} =$$

$$\lim_{x\to\pm\infty} = \frac{1-\frac{3}{x}}{1+\frac{3}{x}} = \frac{1-0}{1+0} = 1$$

D. *Example 3: Evaluate the following limit*

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+40}}{x-35} = \lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{\frac{x^2}{x^2}+\frac{40}{x^2}}}{x\left(\frac{x}{x}-\frac{35}{x}\right)}$$

$$= \lim_{x\to-\infty}\frac{|x|}{x}\cdot\lim_{x\to-\infty}\frac{\sqrt{1+\frac{40}{x^2}}}{1-\frac{35}{x}}$$

$$= -\lim_{x\to-\infty}\frac{\sqrt{1+0}}{1-0}$$

$$= \boxed{-1}$$