4-6: Inverse Trigonometric Functions

CP Precalculus

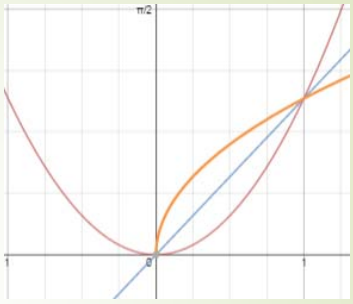
Mr. Gallo

The graphs of sine and arcsine

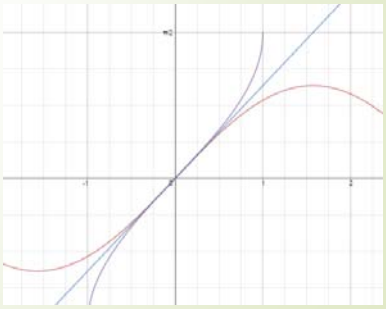
- The graph of $y = \sin\theta$
  - Fails horizontal line test
  - Not 1 to 1
  - $y = \sin^{-1} x$ not a function
- Restrict domain of $y = \sin x$ to make $y = \sin^{-1} x$ a function
  - Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range:$[-1,1]$

Graph of $y = \sin^{-1} x$

- Remember inverses are reflections of the graph over the line $y = x$.



- $y = \sin^{-1} x$ is the reflection of $y = \sin x$ over the line $y = x$



- Domain: $[-1,1]$
- Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Using the Unit Circle to Evaluate Inverse Functions

- Possible angle measures of $y = \sin^{-1} x$ located on right half of unit circle because of restricted domain.

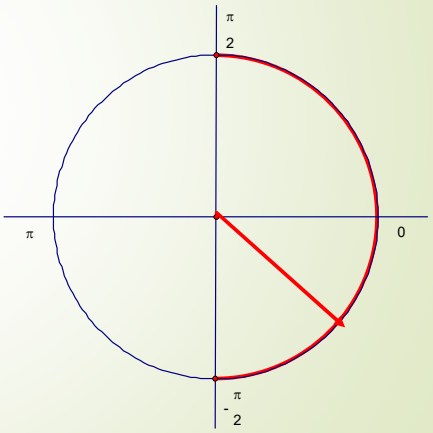
$$\sin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

What are the possible values for θ?

$$\frac{5\pi}{4} \text{ and } \frac{7\pi}{4}$$

Neither is in the domain so we use a negative angle measure instead. Therefore,

$$\frac{7\pi}{4} = -\frac{\pi}{4}$$



The graphs of cosine and arccosine

- The graph of $y = \cos\theta$
  - Fails horizontal line test
  - Not 1 to 1
  - $y = \cos^{-1} x$ not a function
- Restrict domain of $y = \cos x$ to make $y = \cos^{-1} x$ a function
  - Domain: $[0, \pi]$ Range:$[-1,1]$

Graph of $y = \cos^{-1} x$

- Remember inverses are reflections of the graph over the line $y = x$.
- $y = \cos^{-1} x$ is the reflection of $y = \cos x$ over the line $y = x$
- Domain: $[-1,1]$
- Range: $[0, \pi]$

Using the Unit Circle to Evaluate Inverse Functions

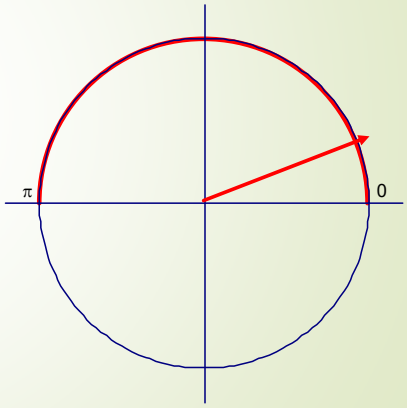
- Possible angle measures of $y = \cos^{-1} x$ located on top half of unit circle because of restricted domain.

$$\cos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

What are the possible values for θ?

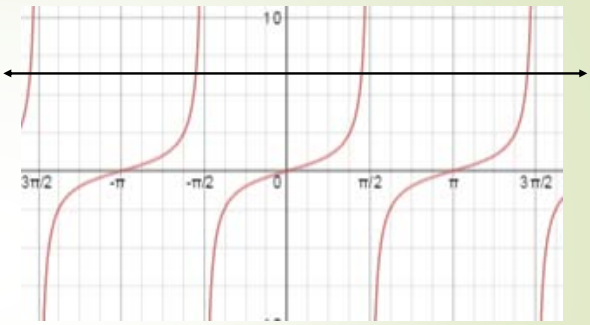
$$\frac{\pi}{6} \text{ and } \frac{11\pi}{6}$$

Only one is in the domain.



The graphs of tangent and arctangent

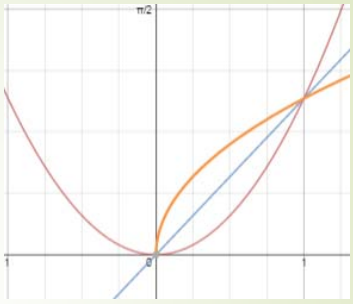
- The graph of $y = \tan\theta$
  - Fails horizontal line test
  - Not 1 to 1
  - $y = \tan^{-1} x$ not a function



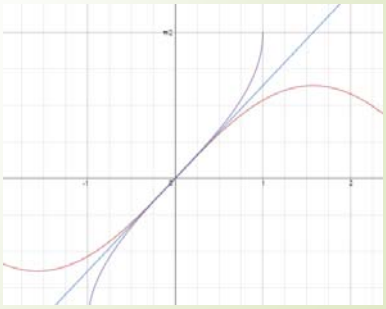
- Restrict domain of $y = \tan x$ to make $y = \tan^{-1} x$ a function
  - Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range:$(-\infty, \infty)$

Graph of $y = \sin^{-1} x$

- Remember inverses are reflections of the graph over the line $y = x$.



- $y = \sin^{-1} x$ is the reflection of $y = \sin x$ over the line $y = x$



- Domain: $[-1,1]$
- Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Using the Unit Circle to Evaluate Inverse Functions

- Possible angle measures of $y = \sin^{-1} x$ located on right half of unit circle because of restricted domain.

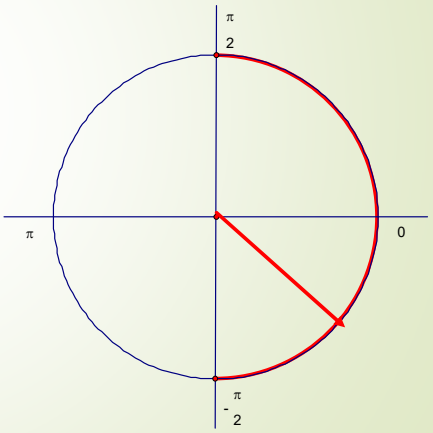
$$\tan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{4}$$

What are the possible values for θ?

$$\frac{5\pi}{4} \text{ and } \frac{7\pi}{4}$$

Neither is in the domain so we use a negative angle measure instead. Therefore,

$$\frac{7\pi}{4} = -\frac{\pi}{4}$$



Homework: p.288 #1-14