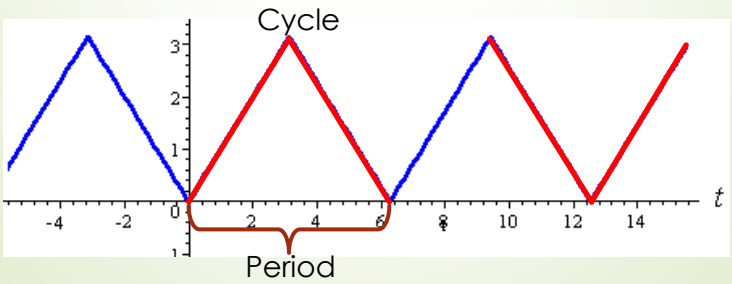
4-4: Graphing Sine and Cosine Functions

CP Precalculus

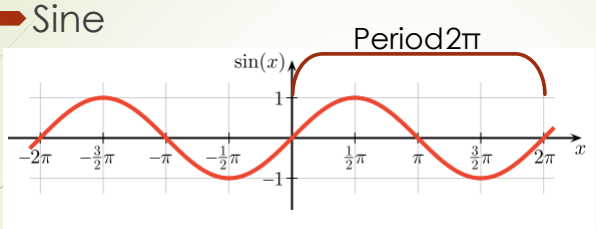
Mr. Gallo

- Periodic Function
  - Function which repeats a pattern of *y*-values at regular intervals.
- Cycle
  - One complete pattern
  - Can begin at any point on the graph.
- Period
  - Horizontal length of one cycle

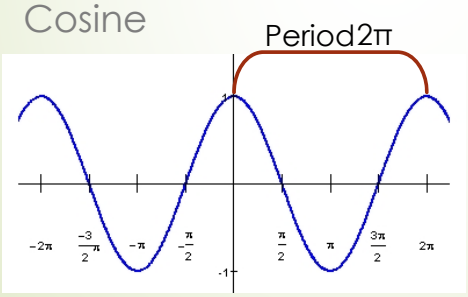


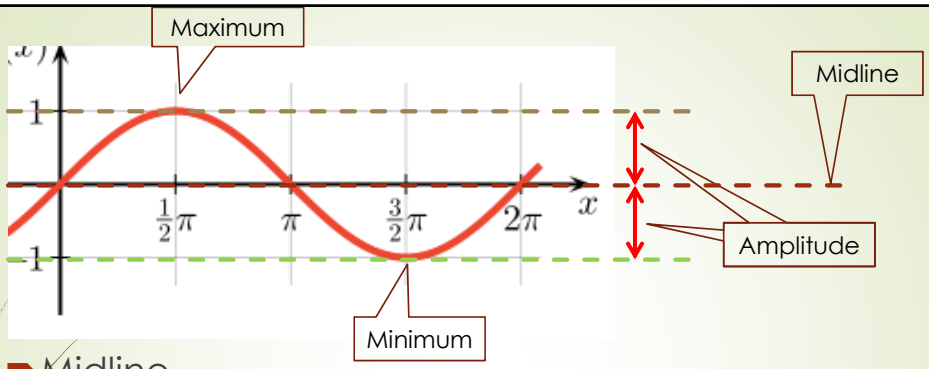
Examples of Periodic Functions

- Sine



- Cosine





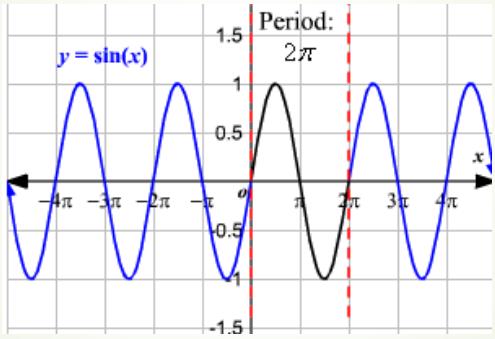
- Midline
  - Horizontal line halfway between the min and max *y*-values.

$$\text{midline} = \frac{1}{2}(\max + \min)$$

- Amplitude
  - Measures amount of variation in function values
  - Half the distance between the min and max values

$$\text{amplitude} = \frac{1}{2}(\max - \min)$$

The Graph of $y = \sin(x)$

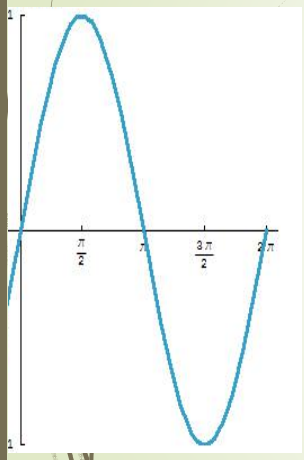


Graphing y = a sin(bx)

|a| = amplitude of function.

b = number of complete cycles from 0 to 2π

$\frac{2\pi}{b}$ is the period of the function.



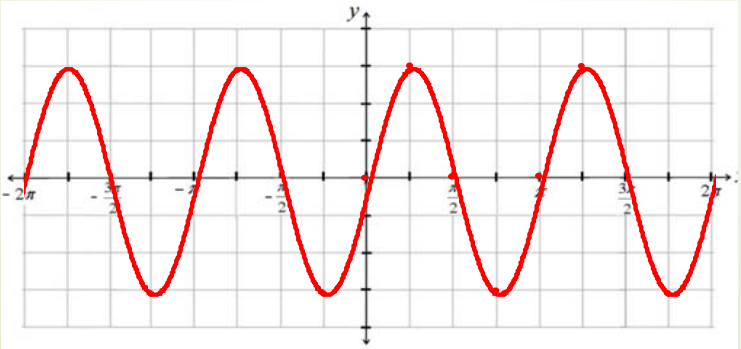
1. Calculate the amplitude, # of cycles, and period in order to graph a sine function.
2. For one period, identify the x-values for: *zero*, *max*, *zero*, *min*, *zero* **by dividing the period by 4**.
3. Above the *max* x-value, plot a point using the amplitude as your height.
4. Below the *min* x-value, plot a point using the amplitude as your height.
5. Connect the points with a smooth curve.
6. Repeat the cycle across the rest of the graph.

Graph $y = 3\sin(2\theta)$

Amplitude: 3 # of cycles from 0 to 2π: 2

Period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Zero: 0 Max: $\frac{\pi}{4}$ Zero: $\frac{\pi}{2}$ Min: $\frac{3\pi}{4}$ Zero: π

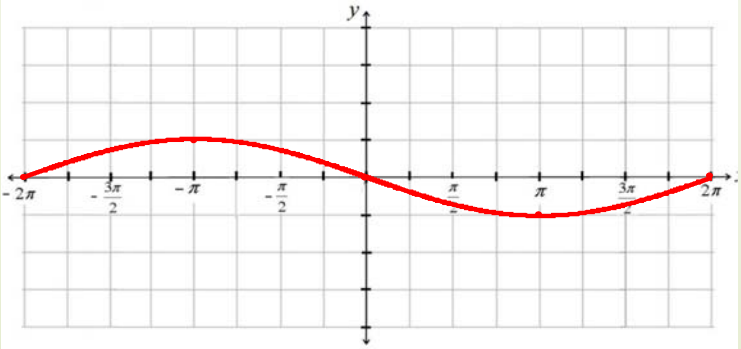


Graph $y = -\sin\left(\frac{\theta}{2}\right)$

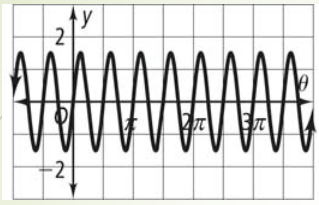
Amplitude: 1 # of cycles from 0 to 2π: $\frac{1}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1/2} = 4\pi$

Zero: -2π Max: $-\pi$ Zero: 0 Min: π Zero: 2π



(a) Determine the period and amplitude of each graph.
(b) Determine the equation of the sine curve in each graph.

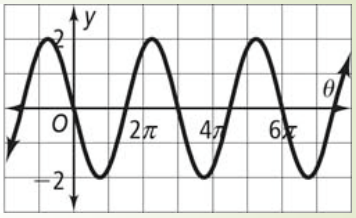


a). Period $\frac{2\pi}{4} = \frac{\pi}{2}$

Amplitude $\frac{1.5-(-1.5)}{2} = \frac{3}{2}$

b). $y = a\sin(b\theta)$

$y = \frac{3}{2}\sin(4\theta)$



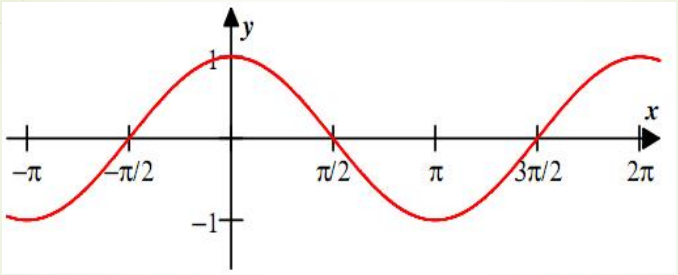
a). Period 3π

Amplitude $\frac{2-(-2)}{2} = 2$

b). $y = a\sin(b\theta)$

$y = -2\sin\left(\frac{2}{3}\theta\right)$

The Graph of y = cos(x)



Graphing $y = a\cos(bx)$

$|a|$= amplitude of function.
b= number of complete cycles from 0 to 2π
$\frac{2\pi}{b}$ is the period of the function

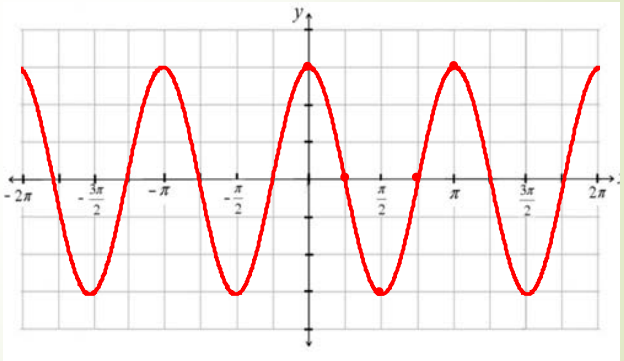
- Follow the same steps for graphing cosine, except the five values will be *max, zero, min, zero, max.*
- *Note: negative "a" means that the graph starts BELOW the x-axis first.*

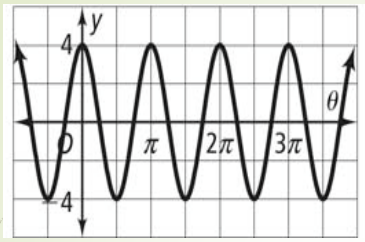
Graph $y = 3\cos(2\theta)$

Amplitude: 3 # of cycles from 0 to 2π: 2

Period: $\frac{2\pi}{2} = \pi$

- Max: 0
- Zero: $\pi/4$
- Min: $\pi/2$
- Zero: $3\pi/4$
- Max: π



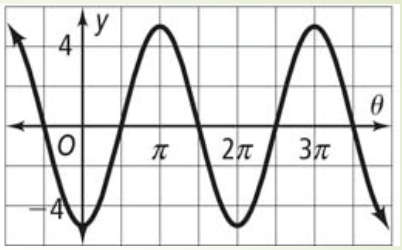


a). Period: $\frac{2\pi}{2} = \pi$

Amplitude: 4

b) Equation:

$y = 4\cos(2\theta)$



a). Period: $\frac{2\pi}{1} = 2\pi$

Amplitude: 5

b) Equation:

$y = -5\cos(\theta)$

Properties of the Sine Function

Sine Function

Domain: **Range:**

y-intercept:

x-intercepts:

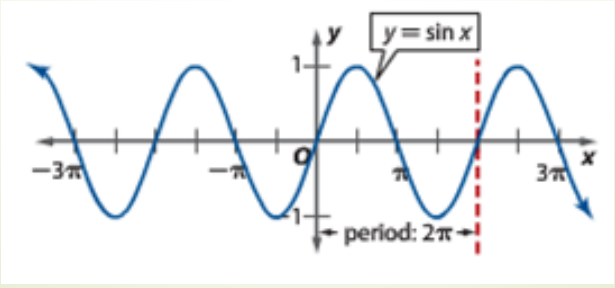
Continuity:

Symmetry:

Extrema:

End Behavior:

Oscillation:



Properties of the Cosine Function

Cosine Function

Domain: **Range:**

***y*-intercept:**

***x*-intercepts:**

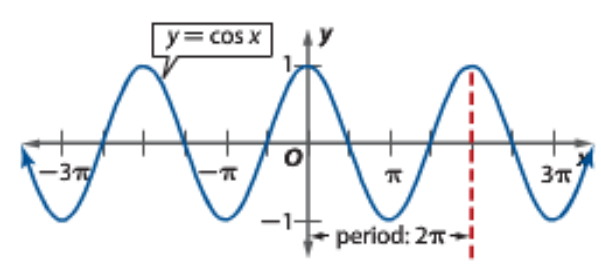
Continuity:

Symmetry:

Extrema:

End Behavior:

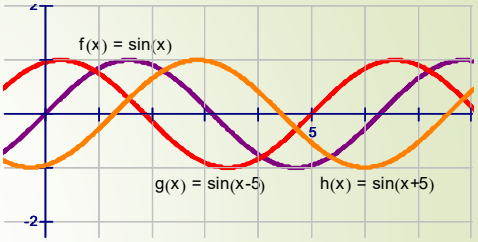
Oscillation:



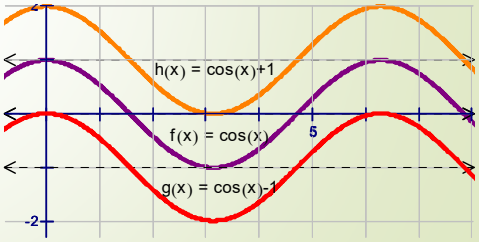
Homework: 4-4 Homework WS1 #7-15

Translations

- Horizontal Translations
  - Called **phase shifts**

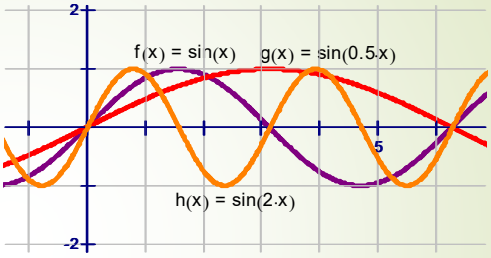


- Vertical Translations
  - Shifts the **midline** up and down

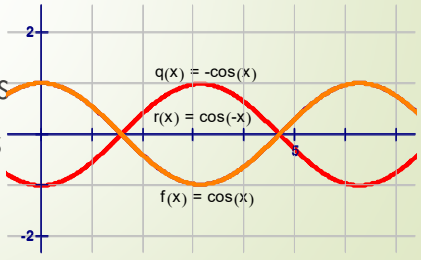


Stretches, Compressions and Reflections

- Stretches
  - $0 < b < 1$
- Compressions
  - $1 < b$



- Reflections
  - *-a* reflects over the *x*-axis
  - *-b* reflects over the *y*-axis



Families of Sine and Cosine Functions

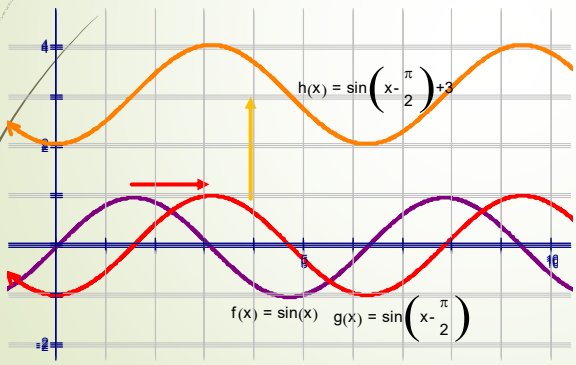
Parent Function	Transformed Function
$y = \sin x$	$y = a\sin b(x-h)+k$
$y = \cos x$	$y = a\cos b(x-h)+k$

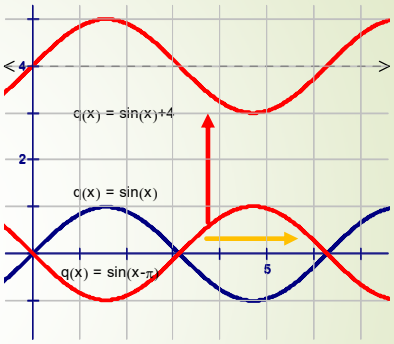
- $|a|$ = amplitude (vertical stretch or shrink)
- $\frac{2\pi}{|b|}$ = period (when x is in radians and $b > 0$)
- $\frac{|b|}{2\pi}$ = Frequency (reciprocal of period)
- h = phase shift, or horizontal shift
- k = vertical shift ($y = k$ is the midline)

Use the graph of the parent function $y = \sin(x)$. What is the graph of each translation in the interval $0 \le x \le 2\pi$?

a. $y = \sin x + 4$

b. $y = \sin(x - \pi)$



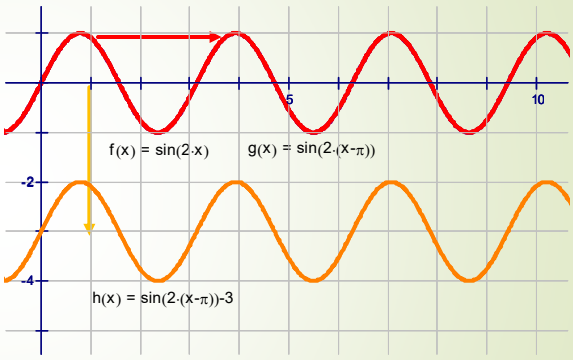


c. $y = \sin\left(x - \frac{\pi}{2}\right) + 3$

Use the graph of the parent function $y = \sin(2x)$. What is the graph of each translation in the interval $0 \le x \le 2\pi$?

d. $y = \sin 2(x - \pi) - 3$

1. Begin with a graph of $y = \sin(2x)$.
2. Translate the graph of $y = \sin(2x)$ to the right π units.
3. Then translate the graph down 3 units.



The table gives the average temperature in Los Angeles, California *x* months after the start of the calendar year ($0 \le x \le 12$). What cosine function models the temperature as a function of *x*?

Month	Temp (°F)
1	57
2	58
3	59
4	62
5	64
6	67
7	72
8	73
9	72
10	67
11	64
12	59

Amplitude: $\frac{1}{2}(\max - \min) = \frac{1}{2}(73 - 57) = 8$

Period= $\frac{2\pi}{b}$

$12 = \frac{2\pi}{b}$

$\frac{\pi}{6} = b$

Phase Shift:
Max value for $y = \cos(x)$ is $(0,1)$
Max value for this data set is $(8,73)$
$h = 8 - 0 = 8$, which is the phase shift

Vertical Translation:
Midline for $y = \cos(x)$ is 0.
Midline for this data set is $\frac{73+57}{2} = 65$.
$k = 65 - 0 = 65$, which is the vertical translation of the midline.

$$y = a\cos b(x - h) + k$$
$$y = 8\cos\frac{\pi}{6}(x - 8) + 65$$

Homework: WS15 #16-29; Graph odd #'s