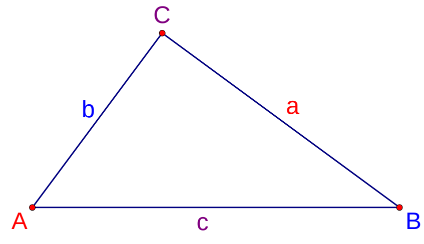
4-7: THE LAW OF SINES AND THE LAW OF COSINES

Precalculus
Mr. Gallo

THE LAW OF SINES (AAS, ASA OR SSA TRIANGLES)

For any ΔABC, let the lengths of the sides opposite angles *A*, *B* and C be *a*, *b* and c, respectively. Then the Law of Sines relates the sine of each angle to the length of the opposite side.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Note: The Law of Sines is used with triangles that ***aren't*** right triangles.

ASA Triangle: In ΔABC, $m\angle C = 85°$, $m\angle B = 25°$ and $a = 7$. To the nearest tenth, find the length of the other two sides.

Use Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \rightarrow \frac{\sin A}{a} = \frac{\sin 25^\circ}{b} = \frac{\sin 85^\circ}{c}$$

Find $m\angle A$ and rewrite

$$m\angle A = 180 - 25 - 85 = 70^\circ \qquad \frac{\sin 70^\circ}{7} = \frac{\sin 25^\circ}{b} = \frac{\sin 85^\circ}{c}$$

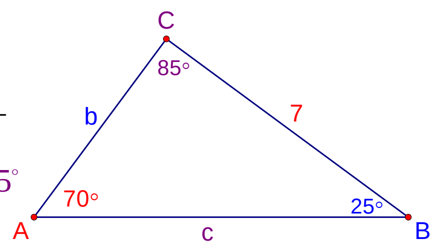
Use proportions to solve for missing side lengths.

$$\frac{\sin 70^\circ}{7} = \frac{\sin 25^\circ}{b} \qquad \frac{\sin 70^\circ}{7} = \frac{\sin 85^\circ}{c}$$

$$b\sin 70^\circ = 7\sin 25^\circ \qquad c\sin 70^\circ = 7\sin 85^\circ$$

$$b = \frac{7\sin 25^\circ}{\sin 70^\circ} \qquad c = \frac{7\sin 85^\circ}{\sin 70^\circ}$$

$$b \approx 3.1 \qquad c \approx 7.4$$

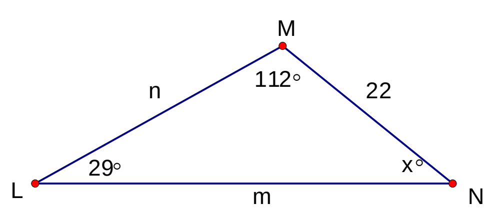


AAS Triangle: In ΔLMN, $MN = 22$, $m\angle M = 112°$ and $m\angle L = 29°$. Solve the triangle.

$$m\angle N = 180 - (112 + 29) = 39°$$

$$\frac{\sin 29}{22} = \frac{\sin 112}{m} \qquad \frac{\sin 29}{22} = \frac{\sin 39}{n}$$

$$m = \frac{22\sin 112}{\sin 29} \approx 42.1 \qquad n = \frac{22\sin 39}{\sin 29} \approx 28.6$$



Two wildlife spotters are 2 mi. apart on an east-west line. The spotter in the eastern spot sees a bear 62° north of west, and the other spotter sees the bear 48° north of east. How far is the bear from each spotter?

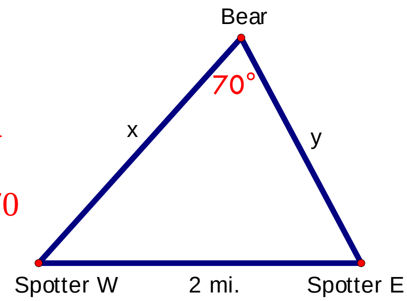
$$m\angle WBE = 180 - 48 - 62 = 70^{\circ}$$

$$\frac{\sin 48}{y} = \frac{\sin 70}{2} \qquad \frac{\sin 62}{x} = \frac{\sin 70}{2}$$

$$2\sin 48 = y\sin 70 \qquad 2\sin 62 = x\sin 70$$

$$\frac{2\sin 48}{\sin 70} = y \qquad \frac{2\sin 62}{\sin 70} = x$$

$$1.58 \approx y \qquad 1.88 \approx x$$



The bear is about 1.6 mi. from the eastern spotter and about 1.9 mi. from the western spotter.

Homework: p. 298 # 1-9; Law of Sines WS

SSA TRIANGLES:

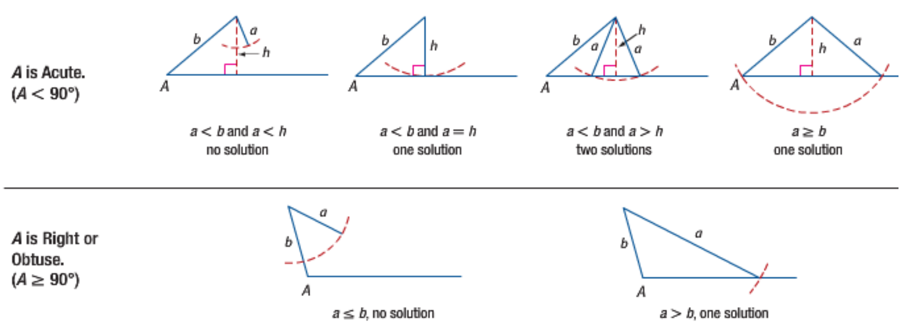
Does not define a unique triangle

- Called the **ambiguous case**

Three possible answers:

- No solution, one solution or two solutions

Consider a triangle in which a, b, and A are given. For the acute case, $\sin A = \frac{h}{b}$, so $h = b \sin A$.

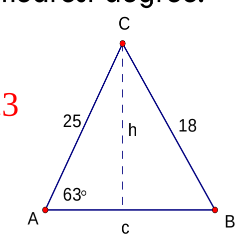


Find all solutions for the given triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

a. $A = 63°$, $a = 18$ and $b = 25$

$A < 90°$ $a < b$ Find h: $\sin 63 = \frac{h}{25} \approx 22.3$

$a < h$, so no solution



b. $A = 105°$, $a = 73$ and $b = 55$

$A \geq 90°$ $a > b$, so one solution

$$\frac{\sin 105}{73} = \frac{\sin B}{55}$$

$$B = \sin^{-1}\left(\frac{55 \sin 105}{73}\right)$$

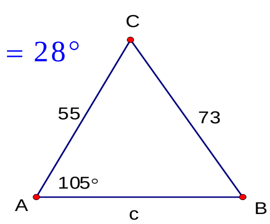
$$B \approx 47°$$

$$C = 180 - (105 + 47) = 28°$$

$$\frac{\sin 105}{73} = \frac{\sin 28}{c}$$

$$c = \frac{73 \sin 28}{\sin 105}$$

$$c \approx 35.5$$

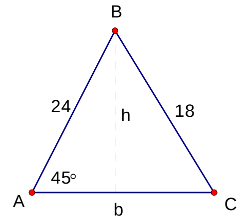


Find all solutions for the given triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

$A = 45°,\ a = 18 \text{ and } c = 24$

$A < 90°\quad a < c\quad \text{Find } h:\ \sin 45 = \frac{h}{18} \approx 17$

$a > h$, so two solutions



1. $C < 90°$

$$\frac{\sin 45}{18} = \frac{\sin C}{24}$$

$$C = \sin^{-1}\left(\frac{24 \sin 45}{18}\right)$$

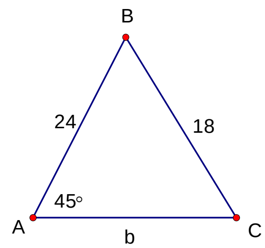
$$C \approx 71°$$

$$B = 180 - (45 + 71) = 64°$$

$$\frac{\sin 45}{18} = \frac{\sin 64}{b}$$

$$b = \frac{18 \sin 64}{\sin 45}$$

$$b \approx 22.9$$



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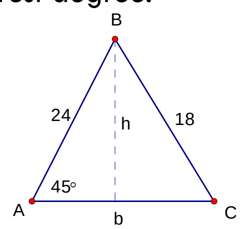

2. $C > 90°$

From $\angle C$ in case 1

$$b = \frac{18 \sin 26}{\sin 45}$$

$$\frac{\sin 45}{18} = \frac{\sin 26}{b}$$

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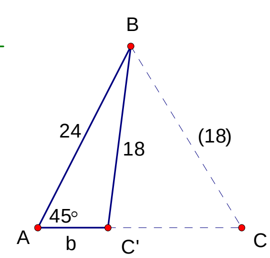
$$b = \frac{18 \sin 26}{\sin 45}$$

2. $C > 90°$



Homework: p. 298 # 11-17 odd, 18-24

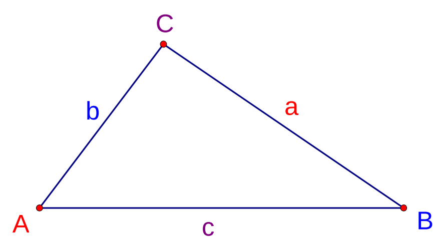
THE LAW OF COSINES (SAS OR SSS)

For any ΔABC, the Law of Cosines relates the cosine of each angle to the side lengths of the triangle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



THE SAS CONDITION:

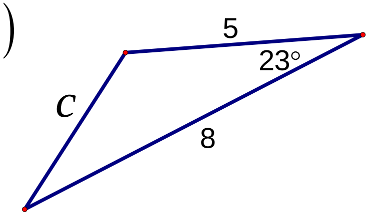
Example 1: Find the missing side in the triangle below:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (5)^2 + (8)^2 - 2(5)(8)\cos(23^\circ)$$

$$c^2 \approx 15.36$$

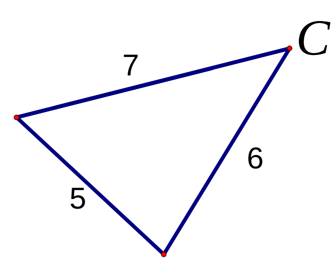
$$\boxed{c \approx 3.91}$$



THE SSS CONDITION

Example 2: Find the smallest angle in the triangle below:

Note: the shortest angle will be across from the 5, so we label that C



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$(5)^2 = (7)^2 + (6)^2 - 2(7)(6)\cos C$$

$$25 = 49 + 36 - 84\cos C$$

$$-60 = -84\cos C$$

$$\frac{-60}{-84} = \cos C$$

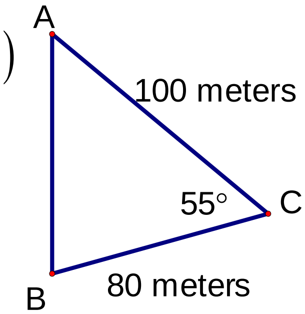
$$C = \cos^{-1}\left(\frac{5}{7}\right) \approx \boxed{44.41^\circ}$$

The distances from a sailboat, at point C, to two points A and B on the shore are known to be 100 meters and 80 meters respectively. If the angle ACB measures 55 degrees, find the distance between points A and B on the shore.

$$c^2 = (100)^2 + (80)^2 - 2(100)(80)\cos(55^\circ)$$

$$c^2 \approx 7222.78$$

$$\boxed{c \approx 84.99m}$$



FINDING THE AREA OF AN OBLIQUE TRIANGLE

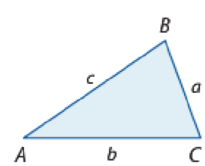
When all three sides of a triangle are known:

- Heron's Formula

If the measures of the sides of $\triangle ABC$ are a, b, and c, then the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$.



Given SAS:

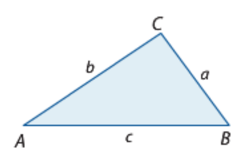
Words The area of a triangle is one half the product of the lengths of two sides and the sine of their included angle.

Symbols

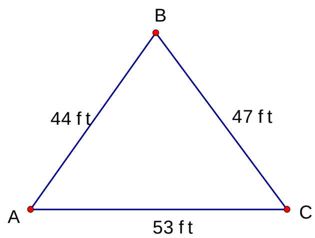
$\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2}ac\sin B$

$\text{Area} = \frac{1}{2}ab\sin C$



Find the area of the following triangles:



$$s = \frac{a+b+c}{2} = \frac{44+47+53}{2} = 72$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

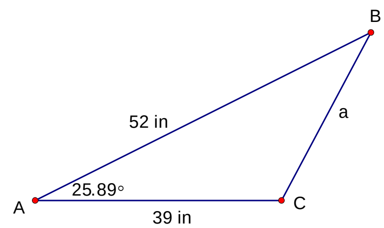
$$A = \sqrt{72(72-44)(72-47)(72-53)}$$

$$A = 978.6\,ft^2$$

$$A = \frac{1}{2}bc\sin A$$

$$A = \frac{1}{2}(52)(39)\sin(25.89)$$

$$A = 442.8\,in^2$$



Homework: p. 298 #28-42 even, 46-50 even