4-2: DEGREES AND RADIANS

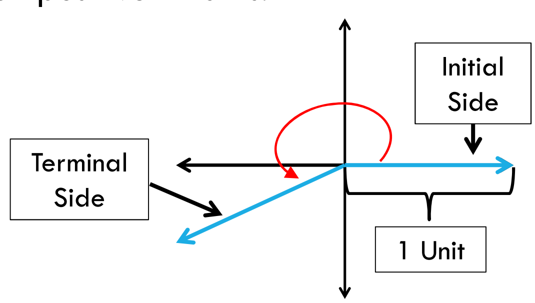
Precalculus
Mr. Gallo

ANGLES IN STANDARD POSITION

Vertex at origin; one ray on positive *x*- axis.

Parts of angles

- Initial side
  - Ray on (+) *x*-axis
- Terminal side
  - Other ray

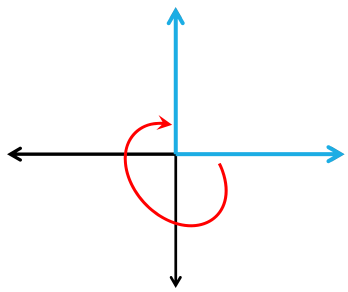


Measure of angle from initial side to terminal side

- Counterclockwise (+ measure)
- Clockwise (- measure)

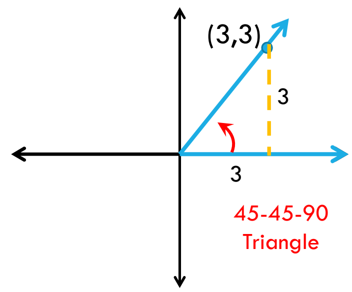
What is the measure of each angle?

1.



-270°

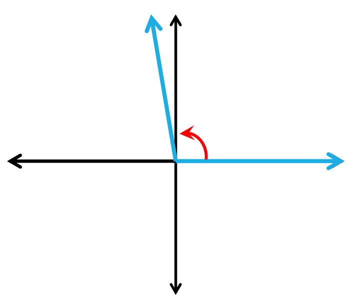
2.



45°

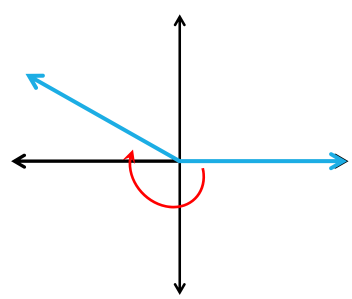
What is a sketch of each angle in standard position?

1.



100°

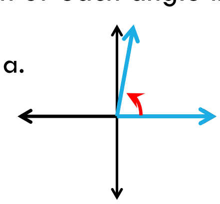
2.



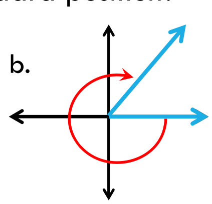
-215°

What is a sketch of each angle in standard position? $a. 85°$ $b. -320°$ $c. 180°$

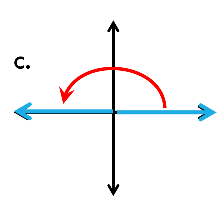
a.



b.



c.



CONVERTING BETWEEN DMS (DEGREE-MINUTE-SECOND) FORM AND DEGREES

Decimal degrees can be converted to Degrees-Minutes-Seconds (DMS) form

- $1° = 60$ minutes $1\ Degree = 60'$
- 1 minute =60 seconds $1\ Minute = 60''$

Write each decimal degree measure in DMS form and each DMS measure in decimal form to the nearest thousandth.

$$
\begin{aligned}
329.125° &= 329° + .125°\left(\frac{60'}{1°}\right) \\
&= 329° + 7.5' \\
&= 329° + 7' + .5\left(\frac{60''}{1'}\right) \\
&= 329° + 7' + 30''
\end{aligned}
$$

$$329.125° = 329°\,7'\,30''$$

$$
\begin{aligned}
35°12'7'' &= 35° + 12'\left(\frac{1°}{60'}\right) + 7''\left(\frac{1°}{3600''}\right) \\
&= 35° + .2 + .002 \\
&= 35.202°
\end{aligned}
$$

$$35°12'\,7'' = 35.202°$$

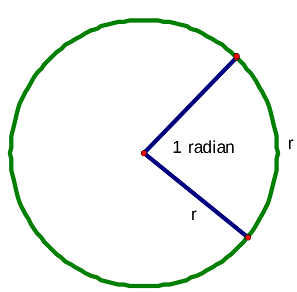
RADIAN

Another way to measure a central angle

The intercepted arc has a length equal to the radius of the circle

Measures the amount of rotation from the initial side to the terminal side of an angle

The radian measure of a circle is 2π radians and a semicircle is π radians.



CONVERTING BETWEEN RADIANS AND DEGREES

To convert degrees to radians, multiply degrees by:

$$\frac{\pi \text{ radians}}{180^\circ}$$

To convert radians to degrees, multiply radians by:

$$\frac{180^\circ}{\pi \text{ radians}}$$

1. What is the degree measure of an angle of $-\frac{7\pi}{30}$ radians?

$$-\frac{7\pi}{30} \bullet \frac{180}{\pi} = -42^\circ$$

2. What is the radian measure of an angle of 81°?

$$81 \bullet \frac{\pi}{180} = \frac{9\pi}{20} \text{ radians}$$

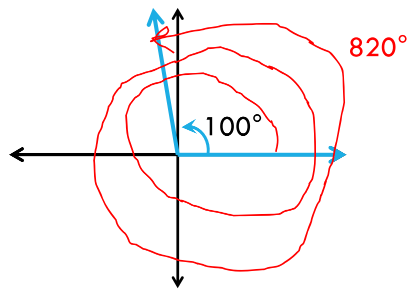
Complete Guided Practice p.233

$a. \frac{7\pi}{6}$ $b. -\frac{\pi}{3}$ $c. 240^\circ$ $d. -30^\circ$

COTERMINAL ANGLES

Two angles in standard position with the same terminal side.

Unlimited number can be identified by adding or subtracting $360°$.



$$360 + 360 + 100 = 820°$$

Find the angles of smallest possible positive measure coterminal with each angle.

1. $908°$ 2. $-75°$

1. Add or subtract $360°$ as many times as needed to obtain an angle with measure greater than $0°$ but less than $360°$.

$$908 - 360 = 548$$
$$548 - 360 = 188$$

An angle of $188°$ is coterminal with an angle of $908°$.

2. Add or subtract $360°$ as many times as needed to obtain an angle with measure greater than $0°$ but less than $360°$.

$$-75 + 360 = 285$$

An angle of $285°$ is coterminal with an angle of $-75°$.

Complete Guided Practice p.234

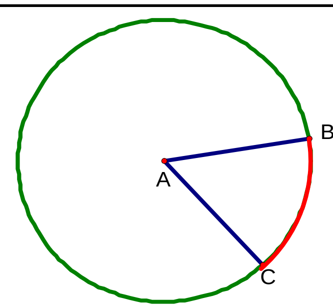
$a. -30° + 360n°;\ 330°;\ -390°$ $\quad b. \frac{3\pi}{4} + 2n\pi;\ \frac{11\pi}{4};\ -\frac{5\pi}{4}$

Homework: p.238 #1-25 odd, 26

ARC LENGTH

If θ is a central angle (measured in radians) in a circle of radius r, the the length of the intercepted arc s, is given by:

$$s = r\theta$$



1. In a circle with radius 30 cm, what is the arc length formed by an angle measuring $\frac{\pi}{3}$ radians?

$$s = r\theta = \frac{\pi}{3}(30) = 10\pi \approx 31.4\ cm$$

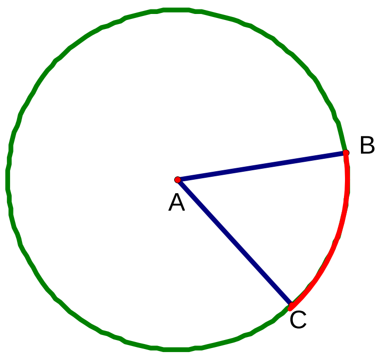
2. Find the exact radius of an intercepted arc of a circle measuring $\frac{\pi}{2}$ radians with an arc length of 16π meters.

$$16\pi = \frac{\pi}{2} r \Rightarrow 32m = r$$

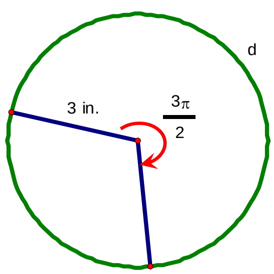
ARC LENGTH

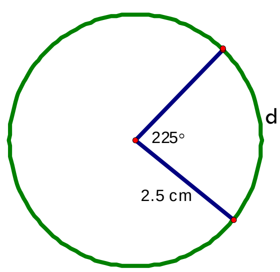
For a circle of radius *r* and a central angle of measure θ (in radians), the length *s* of the intercepted arc is:

$$s = r\theta$$



WHAT IS LENGTH *D* TO THE NEAREST TENTH?





$$s = r\theta$$

$$s = 3\left(\frac{3\pi}{2}\right)$$

$$s = \frac{9\pi}{2}$$

$$s \approx 14.1 \text{ in.}$$

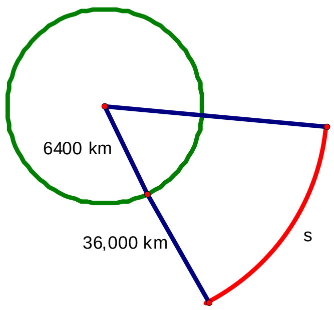
$$s = r\theta$$

$$s = 2.5\left(225 \cdot \frac{\pi}{180}\right) = 2.5\left(\frac{5\pi}{4}\right)$$

$$s = \frac{12.5\pi}{4}$$

$$s \approx 9.8 \text{ cm}$$

A satellite in geosynchronous orbit travels one Earth circumference in a full day. From a point on the ground. The satellite appears stationary overhead. The orbital height for a geosynchronous satellite is about 36,000 km. The radius of Earth is 6400 km. About how far does the satellite travel in 8 hours? Assume the length of an Earth day is exactly 24 hours.



$$\theta = 8\left(\frac{2\pi}{24}\right)$$

$$\theta = \frac{2\pi}{3}$$

$$s = r\theta$$

$$s = 42,400\left(\frac{2\pi}{3}\right)$$

$$s = 88,802.35$$

$$s \approx 89,000 \text{ km}$$

LINEAR AND ANGULAR SPEED

Suppose an object moves at a constant speed along a circular path of radius r.

If s is the arc length traveled by the object during time t, then the object's ***linear speed*** v is given by: $v = \frac{s}{t}$

If θ is the angle of rotation (in radians) through which the object moves during time t, then the ***angular speed*** ω of the object is given by:

$$\omega = \frac{\theta}{t}$$

A typical vinyl record has a diameter of 30 cm. When played on a turn table, the record spins at $33\frac{1}{3}$ revolutions per minute.

a. Find the angular speed, in radians per minute, of a record as it plays. Round to the nearest tenth.

$$\theta = \frac{100}{3} \bullet 2\pi = \frac{200\pi}{3} \qquad \omega = \frac{\theta}{t} = \frac{\frac{200\pi}{3}}{1 \text{ min}} = \frac{200\pi}{3} \approx 209.4 \text{ radians/minutes}$$

b. Find the linear speed at the outer edge of the record as it spins, in centimeters per second.

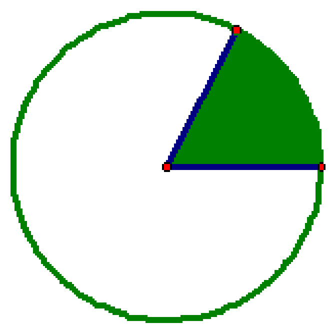
$$s = r\theta = \frac{200\pi}{3}(15) = 1000\pi \qquad v = \frac{s}{t} = \frac{1000\pi}{60 \text{ sec}} = \frac{50\pi}{3} \approx 52.36 \; cm/\text{sec}$$

AREA OF A SECTOR

Sector:

- region bounded by 2 radii and an arc of a circle.
- Area A, of a sector of a circle with radius r, and central angle θ, measured in radians is:

$$A = \frac{1}{2}r^2\theta$$



1. Find the area of a sector of a circle with a central angle measuring $\frac{\pi}{4}$ radians and a radius of 12".

$$A = \frac{1}{2}(12)^2\left(\frac{\pi}{4}\right) = 18\pi \approx 56.55 \; in^2$$

Homework: p.238 #27-33 odd, 43-49 odd, 51-54, 58-60