3-2: Logarithmic Functions

CP Precalculus
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Logarithm

- Exponent to which ***b*** must be raised to get ***x***.
- Logarithm is inverse of exponential function.
  - Used to rewrite exponential functions so they can be evaluated.

Logarithm base ***b*** of a positive number ***x*** satisfies the following definition.

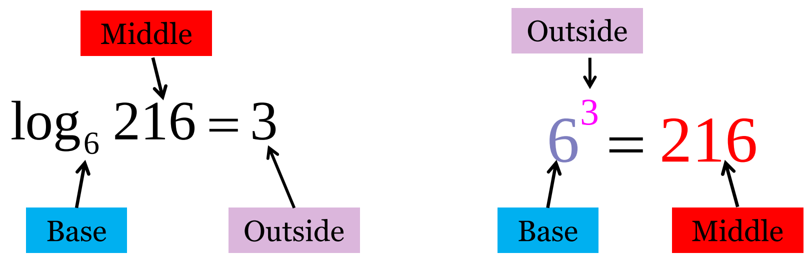
For $b > 0, b \neq 1, log_b x = y$ if and only if $b^y = x$.

(Read log base ***b*** of ***x***)

Writing Logarithms in Exponential Form

- Use the BOM method

B -Base
O - to the Outside
M -equals the Middle



Basic Properties of Logarithms

If $b > 0, \mathrm{b} \neq 0$ and x is a real number, then the following statements are true:

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x, x > 0$

Evaluate each expression:

1) $\log_8 512 = \log_8 8^3$
$= 3$

2) $22^{\log_{22} 15.2} = 15.2$

Evaluating a Logarithm

1. Write a logarithmic equation
2. Write in exponential form and evaluate

1). $\log_2 16$

$\log_2 16 = y$

$2^y = 16$

$2^y = 2^4$

$y = 4$

2). $\log_{10} \sqrt[3]{10}$

$\log_{10} \sqrt[3]{10} = y$

$10^y = \sqrt[3]{10}$

$10^y = 10^{\frac{1}{3}}$

$y = \frac{1}{3}$

3). $\log_4 32$

$\log_4 32 = y$

$4^y = 32$

$\left(2^2\right)^y = 2^5$

$2^{2y} = 2^5$

$2y = 5$

$y = \frac{5}{2}$

Common Logarithmic Function

- The inverse of $y = 10^x$ is the Common Logarithm.

If $x = 10^y$, then $y = \log_{10} x$

The inverse is $y = \log x$

$y = 10^x$

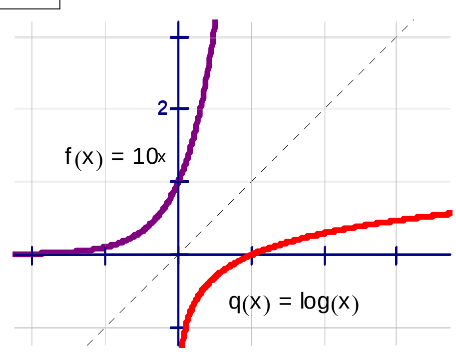
Domain: All Reals

Range: $y > 0$

$y = \log x$

Domain: $x > 0$

Range: All Reals



Natural Logarithmic Function

- The inverse of $y = e^x$ is the Natural Logarithm.

If $y = e^x$, then $x = \log_e y = \ln y$

The inverse is $y = \ln x$

$y = e^x$

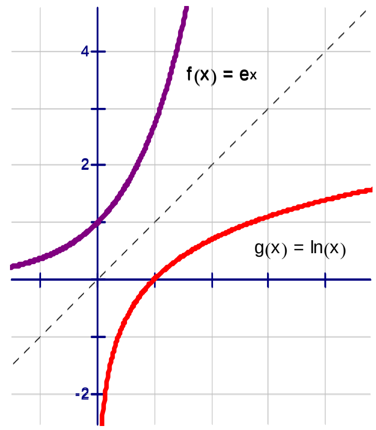
Domain: All Reals

Range: $y > 0$

$y = \ln x$

Domain: $x > 0$

Range: All Reals



Logarithmic Scales

- Used to cover a wide range of values
- Reported measurements logs of values not values themselves
  - Richter Scale, Decibel Scale, pH scale are all logarithmic scales

The Richter scale measures the intensity R of an earthquake. The Richter scale uses the formula $R = \log\left(\frac{a}{T}\right) + B$, where a is the amplitude (in microns) of the vertical ground motion, T is the period of the seismic wave in seconds and B is a factor that accounts for the weakening of seismic waves.

a. A city is not concerned about earthquakes with an intensity of less than 3.5. An earthquake occurs with an amplitude of 125 microns, a period of 0.33 seconds and $B = 1.2$. What is the intensity of the earthquake? Should this earthquake be a concern for the city?

$$\log\left(\frac{125}{0.33}\right) + 1.2 = 3.778$$

The city should be concerned.

b. Earthquakes with an intensity of 6.1 or greater can cause considerable damage to those living within 100 km of the earthquake's center. Determine the amplitude of an earthquake whose intensity is 6.1 with a period of 3.5 seconds and $B = 3.7$.

Solve in calculator using a window of $840 \le x \le 900$ and $6 \le y \le 6.3$.

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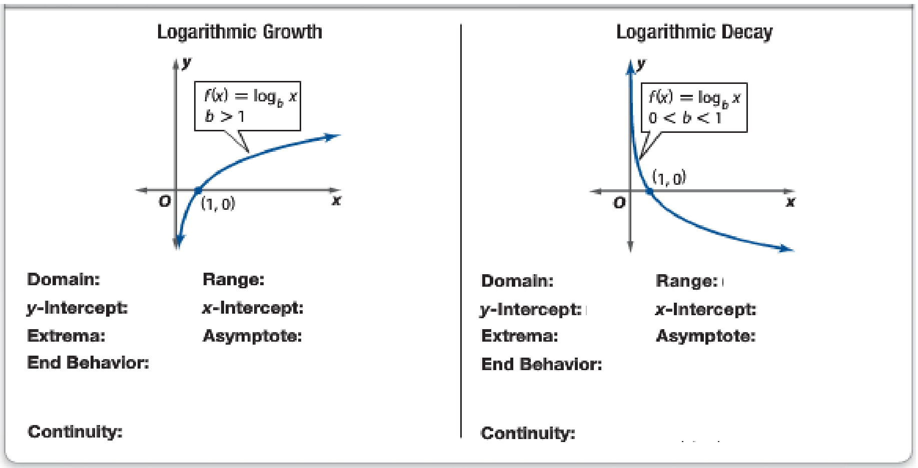
Properties of Logarithmic Functions



Graphing a Logarithmic Function

$y = log_b x$ is the *inverse* of the exponential function $y = b^x$

$y = 10^x$

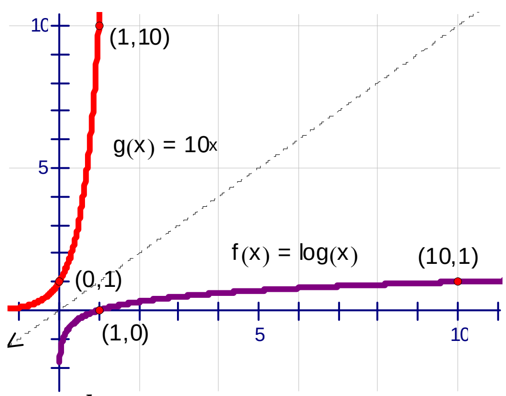
Domain: All Reals

Range: $y > 0$

$y = log_b x$

Domain: $x > 0$

Range: All Reals



$y = log_b x$

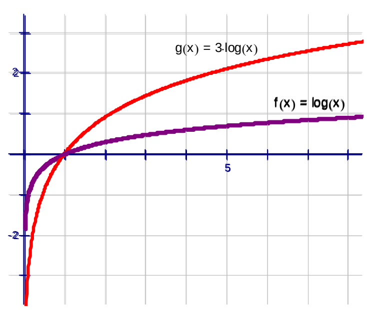
No y-intercept

y=axis is an asymptote

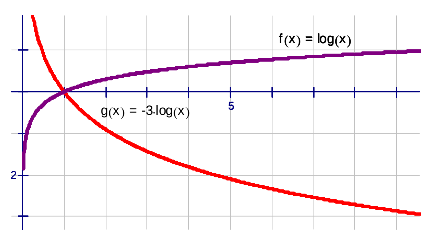
Families of Logarithmic Functions

Families of Logarithmic Functions	
Parent Function	$y = \log_b x; b > 0, b \neq 1$
Stretch ($\lvert a \rvert > 1$) Compression (Shrink) ($0 < \lvert a \rvert < 1$) Reflection($a < 0$) in x-axis	$y = a \log_b x$
Translations (Horizontal by h; Vertical by k)	$y = \log_b (x - h) + k$
All transformations combined	$y = a \log_b (x - h) + k$

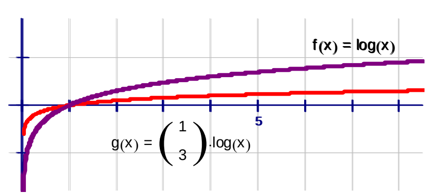
Families of Logarithmic Functions



Stretch ($\lvert a \rvert > 1$)

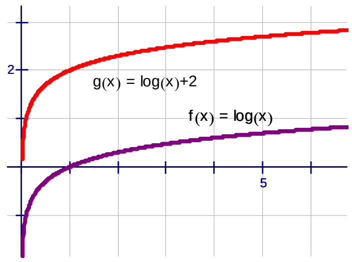


Reflection ($a < 0$)

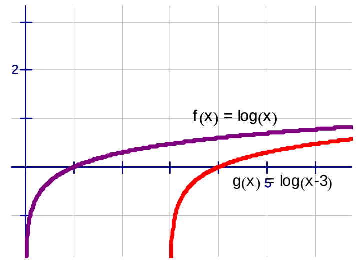


Compression ($0 < \lvert a \rvert < 1$)

Families of Logarithmic Functions



Vertical
Translation
$y = log_b x + k$



Horizontal
Translation
$y = log_b(x - h)$

All Transformations combined gives the form:
$y = alog_b(x - h) + k$

Homework: p. 178 #29-41 odd