# ABSOLUTE VALUE BARS

- Absolute value bars are needed when we simplify radical expressions with variables with an even index.
- This ensures that an even root is always nonnegative – Looking for the principal root
- Although there are 2 real nth roots of b when n is even, only the positive root is principle root. So when asked to give the root, only give the one positive root.

$\sqrt{x^4} = x^2$ $\sqrt{x^4 y^6} = x^2 \left| y^3 \right|$ $\sqrt{x^2 y} \cdot \sqrt{xy^2} = xy\sqrt{xy}$

Remember that when combining radicals (products or quotients) we are working with **real numbers**, so absolute value bars aren't needed when we assume variables are nonnegative

# WARM-UP

Simplify each radical

A) $\sqrt{25a^4}$

$5a^2$

B) $\sqrt[4]{p^{12}q^4}$

$\left| p^3 q \right|$

C) $\dfrac{\sqrt{21x^{10}}}{\sqrt{7x^5 y}}$

$\dfrac{\sqrt{3x^5}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} \quad = \dfrac{x^2\sqrt{3xy}}{y}$

D) $2\sqrt[3]{5} \cdot 3\sqrt[3]{25}$

$6\sqrt[3]{125}$

$30$

# 6-3 BINOMIAL RADICAL EXPRESSIONS

Ms. Miller

## COMBINING RADICAL EXPRESSIONS: SUMS AND DIFFERENCES

**Like radicals** are radicals that have the same index and radicand

We can combine like radicals using properties of real numbers. Combine these like radicals using the Distributive Property:

A) $2\sqrt{3}+\sqrt{3}$

$(2+1)\sqrt{3}$

$3\sqrt{3}$

B) $4\sqrt[5]{5}-7\sqrt[5]{5}$

$(4-7)\sqrt[5]{5}$

$-3\sqrt[5]{5}$

## COMBINING RADICAL EXPRESSIONS: SUMS AND DIFFERENCES

Use the distributive Property to add or subtract like radicals

$a\sqrt[n]{x}+b\sqrt[n]{x}=(a+b)\sqrt[n]{x}$ $a\sqrt[n]{x}-b\sqrt[n]{x}=(a-b)\sqrt[n]{x}$

Simplify:

A) $7\sqrt[3]{5}-4\sqrt{5}$

Cannot combine – Different indexes

B) $3x\sqrt{xy}+4x\sqrt{xy}$

$(3x+4x)\sqrt{xy}$

$7x\sqrt{xy}$

C) $17\sqrt[5]{3x^2}-15\sqrt[5]{3x^2}$

$(17-15)\sqrt[5]{3x^2}$

$2\sqrt[5]{3x^2}$

## SIMPLIFYING BEFORE ADDING OR SUBTRACTING

When you have a sum or difference of radical expressions, you should simplify each expression so that you can find all the like radicals

**What is the simplest form of** $\sqrt[3]{250}+\sqrt[3]{54}-\sqrt[3]{16}$ **?**

- Simplify each radical:

$\sqrt[3]{2^1\cdot 5^3}+\sqrt[3]{2^1\cdot 3^3}-\sqrt[3]{2^3\cdot 2^1}$ $=5\sqrt[3]{2}+3\sqrt[3]{2}-2\sqrt[3]{2}$

- Combine all like radicals:

$5\sqrt[3]{2}+3\sqrt[3]{2}-2\sqrt[3]{2}$ $=(5+3-2)\sqrt[3]{2}$ $=6\sqrt[3]{2}$

## MULTIPLYING BINOMIAL RADICAL EXPRESSIONS

Use FOIL or distribute to multiply binomials that have radical expressions.

Find the products of the following binomial:

$$\left(3+2\sqrt{5}\right)\left(2+4\sqrt{5}\right) = 6+12\sqrt{5}+4\sqrt{5}+2\sqrt{5}\cdot 4\sqrt{5}$$

$$= 6+12\sqrt{5}+4\sqrt{5}+8\sqrt{25}$$

$$= 6+(12+4)\sqrt{5}+8\sqrt{25}$$

$$= 6+(12+4)\sqrt{5}+8\cdot 5$$

$$= 6+16\sqrt{5}+40$$

$$= 46+16\sqrt{5}$$

## TRY THESE

What is the simplest form of:

A) $14a\sqrt{7bc}+5a\sqrt{7bc}$

$19a\sqrt{7bc}$

B) $3\sqrt[5]{x}-\sqrt[5]{3x}$

Cannot combine – Different radicands

C) $\sqrt{28}-\sqrt{175}+\sqrt{63}$

$\sqrt{4\cdot 7}-\sqrt{25\cdot 7}+\sqrt{9\cdot 7}$

$2\sqrt{7}-5\sqrt{7}+3\sqrt{7}$

$0$

D) $\left(1+2\sqrt{7}\right)\left(4-3\sqrt{7}\right)$

$4+8\sqrt{7}-3\sqrt{7}-6\sqrt{7\cdot 7}$

$4+5\sqrt{7}-6\cdot 7$

$-38+5\sqrt{7}$

## MULTIPLYING CONJUGATES

$\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are conjugates. When a and b are rational numbers, the product of two radical conjugates is a rational number.

$$\left(6-\sqrt{12}\right)\left(6+\sqrt{12}\right) = 36-6\sqrt{12}+6\sqrt{12}-\sqrt{12\cdot 12}$$
$$= 36-\sqrt{144}$$
$$= 36-12 \quad = 24$$

$$\left(3+\sqrt{8}\right)\left(3-\sqrt{8}\right) = 9+3\sqrt{8}-3\sqrt{8}-\sqrt{8\cdot 8}$$
$$= 9-\sqrt{64}$$
$$= 9-8 \quad = 1$$

## RATIONALIZING THE DENOMINATOR

Sometimes a denominator is a sum or difference involving radicals. If the expressions are square roots, you can rationalize the denominator by multiplying the numerator and denominator by the conjugate of the denominator

Rationalize the denominator: $\dfrac{2\sqrt{7}}{\sqrt{3}-\sqrt{5}}$

$$\frac{2\sqrt{7}}{\sqrt{3}-\sqrt{5}}\cdot\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{2\sqrt{7}\left(\sqrt{3}+\sqrt{5}\right)}{\left(\sqrt{3}-\sqrt{5}\right)\left(\sqrt{3}+\sqrt{5}\right)} = \frac{2\sqrt{21}+2\sqrt{35}}{\sqrt{9}-\sqrt{25}} = \frac{2\sqrt{21}+2\sqrt{35}}{3-5}$$

$$= \frac{2\sqrt{21}+2\sqrt{35}}{-2} \qquad = -\sqrt{21}-\sqrt{35}$$

# TRY THESE

What is the simplest form of:

A) $\left(5+3\sqrt{2}\right)\left(5-3\sqrt{2}\right)$

$$25+15\sqrt{2}-15\sqrt{2}-9\sqrt{2\cdot 2}$$

$$25-9\cdot 2$$

$$7$$

B) $\dfrac{11}{6+\sqrt{3}} \cdot \dfrac{6-\sqrt{3}}{6-\sqrt{3}}$

$$\frac{66-11\sqrt{3}}{36-3} = \frac{66-11\sqrt{3}}{33}$$

$$= \frac{6-\sqrt{3}}{3}$$