6-1 ROOTS AND RADICAL EXPRESSIONS

Ms. Miller

THE NTH ROOT

If $a^n = b$, with ***a*** and ***b*** being real numbers and ***n*** a positive integer, then ***a*** is an *nth root* of ***b.***

If ***n*** is odd, there is 1 real nth root of ***b***, noted in radical form as $\sqrt[n]{b}$

If ***n*** is even and ***b*** is positive, there are two real roots of ***b***

- The positive root is called the principle root and its symbol is $\sqrt[n]{b}$
- The negative root is its opposite $-\sqrt[n]{b}$
- If ***b*** is negative, there are no real *nth* roots of ***b***

- The only *nth* root of 0 is 0
- The focus of this chapter will be real roots

THE NTH ROOT

You can use the **radical sign** to indicate a root. The **index** is the degree of the root.

Radical Sign

Index

$$\sqrt[n]{a}$$

Radicand

Corresponding to every power, there is a root. Just as there are squares (square powers), there are square roots. Just as there are cubes (third powers), there are cube roots. And so on.

FINDING ALL REAL ROOTS

What are all the real square roots?

A) 0.01

$$\sqrt{\frac{1}{100}} = \pm\frac{1}{10}$$

B) -1

None

C) $\frac{36}{121}$

$$\frac{\sqrt{36}}{\sqrt{121}} = \pm\frac{6}{11}$$

FINDING ROOTS

What are the real-number roots?

A) $\sqrt[3]{-27}$

$(?)^3 = -27$

$\sqrt[3]{-27} = -3$

B) $\sqrt[4]{-81}$

None

C) $\sqrt{(-7)^2} \quad = \sqrt{49}$

$(?)^2 = 49$

$\sqrt{49} = 7$

When the directions say "Find the root," we are looking for principle root.

SIMPLIFYING RADICAL EXPRESSIONS

For any real number ***a***, $\sqrt[n]{a^n} = \begin{cases} a \text{ if n is odd} \\ |a| \text{ if n is even} \end{cases}$

Simplify the following:

A) $\sqrt{81x^4}$

$\sqrt{(9)^2(x^2)^2}$

$|9x^2|$

$9x^2$

B) $\sqrt[3]{a^{12}b^{15}}$

$\sqrt[3]{(a^4)^3(b^5)^3}$

a^4b^5

C) $\sqrt[4]{x^{12}y^{16}}$

$\sqrt[4]{(x^3)^4(y^4)^4}$

$|x^3y^4|$

$|x^3|y^4$

ABSOLUTE VALUE BARS

Absolute value bars are needed when the index is even.

This ensures that an even root is always nonnegative.

Although there are 2 real nth roots of b when n is even, only the positive root is principle root. So when asked to give the root, only give the one positive root.

Technically, $\sqrt{x^2} = x$ is false when x is negative, while $\sqrt{x^2} = |x|$ is true.