Study Guide and Review - Chapter 1

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A <u>function</u> assigns every element of its domain to exactly one element of its range.

SOLUTION:

A function assigns every element of its domain to exactly one element of its range. When more than one element of the range is assigned to one element of the domain, then the relation is not a function.

2. Graphs that have <u>point symmetry</u> can be rotated 180° with respect to a point and appear unchanged.

SOLUTION:

Graphs that have point symmetry can be rotated 180° with respect to a point and appear unchanged. An example of this is the graph of $f(x) = x^3$ which is symmetric with respect to the origin.

3. An <u>odd function</u> has a point of symmetry.

SOLUTION:

An example of this is the graph of $f(x) = x^3$ which is symmetric with respect to the origin. A function is odd when $f(-x) = -f(x)$.

4. The graph of a <u>continuous function</u> has no breaks or holes.

SOLUTION:

The graph of a continuous function has no breaks or holes. If a function has a break or a hole, then it will not be continuous.

8. The <u>translation</u> of a graph produces a mirror image of the graph with respect to a line.

SOLUTION:

The reflection of a graph produces a mirror image of the graph with respect to a line. A translation produces a duplicate image of the graph, just moved vertically or horizontally.

State the domain of each function.

17. $f(x) = 5x^2 - 17x + 1$

SOLUTION:

There is no value of x that will make the function undefined, so $D = \{x \mid x \in \mathbb{R}\}$.

18. $g(x) = \sqrt{6x-3}$

SOLUTION:

The function is undefined when $6x - 3 < 0$.

$$\begin{aligned} 6x - 3 &< 0 \\ 6x &< 3 \\ x &< \frac{1}{2} \\ x &< 0.5 \end{aligned}$$

$D = \{x \mid x \geq 0.5, x \in \mathbb{R}\}$

19. $h(a) = \dfrac{5}{a+5}$

SOLUTION:

The function is undefined when $a + 5 = 0$.

$D = \{a \mid a \neq -5, a \in \mathbb{R}\}$

20. $v(x) = \dfrac{x}{x^2 - 4}$

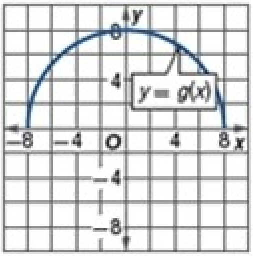
SOLUTION:

The function is undefined when $x^2 - 4 = 0$.

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

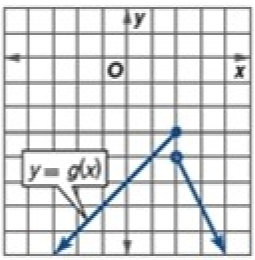
$D = \{x \mid x \neq \pm 2, x \in \mathbb{R}\}$

Use the graph of g to find the domain and range of each function.

21. 

SOLUTION:

The x-values range from −8 to 8 and the y-values range from 0 to 8. D = [−8, 8], R = [0, 8]

22. 

SOLUTION:

The arrows indicate that the x-values extend to negative infinity and positive infinity. The y-values extend to negative infinity and reach a maximum of −3.

D = $\{x \mid x \in \mathbb{R}\}$, R = $(-\infty, -3)$

Find the y-intercept(s) and zeros for each function.

23. $f(x) = 4x - 9$

SOLUTION:

$$f(0) = 4(0) - 9 = 0 - 9 = -9$$

$$4x - 9 = 0$$
$$4x = 9$$
$$x = \frac{9}{4}$$

24. $f(x) = x^2 - 6x - 27$

SOLUTION:

$$f(0) = (0)^2 - 6(0) - 27 = 0 - 0 - 27 = -27$$

$$x^2 - 6x - 27 = 0$$
$$(x - 9)(x + 3) = 0$$
$$x = 9 \text{ or } -3$$

25. $f(x) = x^3 - 16x$

SOLUTION:

$$f(0) = (0)^3 - 16(0) = 0 - 0 = 0$$

$$x^3 - 16x = 0$$
$$x(x^2 - 16) = 0$$
$$x(x + 4)(x - 4) = 0$$
$$x = 0, 4, \text{ or } -4$$

26. $f(x) = \sqrt{x + 2} - 1$

SOLUTION:

$$f(0) = \sqrt{0 + 2} - 1 = \sqrt{2} - 1$$

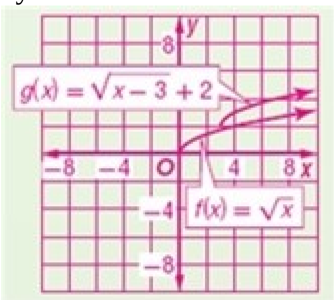
$$\sqrt{x + 2} - 1 = 0$$
$$\sqrt{x + 2} = 1$$
$$x + 2 = 1$$
$$x = -1$$

Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

38. $g(x) = \sqrt{x-3} + 2$

SOLUTION:

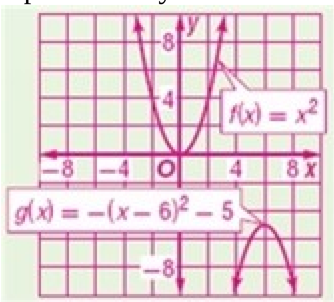
$g(x) = f(x-3) + 2$, so $g(x)$ is the graph of $f(x) = \sqrt{x}$ translated 3 units to the right and 2 units up. The translation right is represented by the subtraction of 3 inside the function. The translation up is represented by the addition of 2 on the outside of $f(x)$.



39. $g(x) = -(x-6)^2 - 5$

SOLUTION:

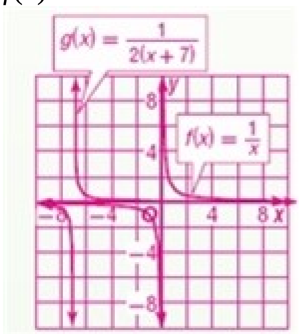
$g(x) = -f(x-6) - 5$, so $g(x)$ is the graph of $f(x) = x^2$ reflected in the x-axis and translated 6 units to the right and 5 units down. The translation right is represented by the subtraction of 6 inside the function. The reflection is represented by the negative coefficient of $f(x)$. The translation down is represented by the subtraction of 5 outside of $f(x)$.



40. $g(x) = \frac{1}{2(x+7)}$

SOLUTION:

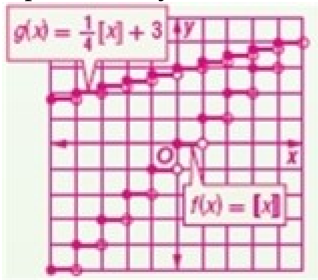
$g(x) = \frac{1}{2}f(x+7)$, so $g(x)$ is the graph of $f(x) = \frac{1}{x}$ translated 7 units to the left and is compressed vertically by a factor of $\frac{1}{2}$. The translation is represented by the addition of 7 inside the function. The compression is represented by the coefficient of $f(x)$.



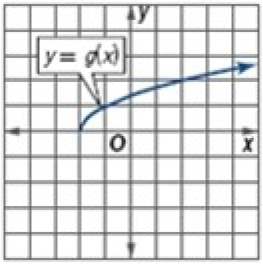
41. $g(x) = \frac{1}{4}[[x]] + 3$

SOLUTION:

$g(x) = \frac{1}{4}f(x) + 3$, so $g(x)$ is the graph of $f(x) = [[x]]$ compressed vertically by a factor of $\frac{1}{4}$ and translated 3 units up. The compression is represented by the coefficient of $f(x)$. The translation is represented by the addition of 3 outside the function.



Describe how the graphs of $f(x) = \sqrt{x}$ and $g(x)$ are related. Then write an equation for $g(x)$.

42. 

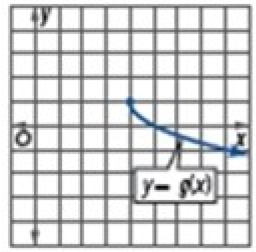
SOLUTION:

The central characteristic of $f(x) = \sqrt{x}$ is the endpoint or starting point. For our purposes here, it can be considered as an endpoint or a critical point. This point is at (0, 0) for the parent function. Identifying where it shifts will help you identify $g(x)$. Note that these are translations only. For reflections and dilations, we will have to consider more aspects of the graph.

The endpoint is at (−2, 0), so the endpoint is translated 2 unit to the left. Therefore, the graph of $f(x)$ is also translated 2 unit to the left.

The x-coordinate tells us what changed inside the square root symbols. Treat this like a zero for a linear equation. If the coordinate is −2, the expression inside the radical should be $x + 2$.

Thus, $g(x) = f(x + 2) = \sqrt{x+2}$.

43. 

SOLUTION:

The central characteristic of $f(x) = \sqrt{x}$ is the endpoint or starting point. For our purposes here, it can be considered as an endpoint or a critical point. This point is at (0, 0) for the parent function. Identifying where it shifts will help you identify $g(x)$. Note that these are translations only. For reflections and dilations, we will have to consider more aspects of the graph.

The endpoint is at (4, 1), so the endpoint is translated 4 units right and 1 unit to the left. Therefore, the graph of $f(x)$ is also translated 2 unit to the left.

The graph is also reflected in the x-axis. This is done by multiplying $f(x)$ by −1

The x-coordinate tells us what changed inside the square root symbols. Treat this like a zero for a linear equation. If the coordinate is 4, the expression inside the radical should be $x + 4$.

The y-coordinate tells us what was added outside of the radical symbols. It describes the vertical shift from the origin. The y-coordinate is 1, so we need to add 1.

Thus, $f(x)$. $g(x) = -f(x - 4) + 1 = -\sqrt{x-4} + 1$.

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

44. $f(x)=x+3$

$g(x)=2x^2+4x-6$

SOLUTION:

$$(f+g)(x)=x+3+2x^2+4x-6$$
$$=2x^2+5x-3$$
$$(f-g)(x)=x+3+-(2x^2+4x-6)$$
$$=-2x^2-3x+9$$
$$(f\cdot g)(x)=(x+3)(2x^2+4x-6)$$
$$=2x^3+4x^2-6x+6x^2+12x-18$$
$$=2x^3+10x^2+6x-18$$
$$\left(\frac{f}{g}\right)(x)=\frac{x+3}{2x^2+4x-6}$$
$$=\frac{x+3}{(2x-2)(x+3)}$$
$$=\frac{1}{2x-2}$$

D = $(-\infty,\infty)$ for all of the functions except $\left(\frac{f}{g}\right)(x)$, for which $D=(-\infty,-3)\cup(-3,1)\cup(1,\infty)$.

Even though there appears to be no restriction of −3 in the simplified function, there is in the original.

45. $f(x)=4x^2-1$

$g(x)=5x-1$

SOLUTION:

$$(f+g)(x)=4x^2-1+5x-1$$
$$=4x^2+5x-2$$
$$(f-g)(x)=4x^2-1-(5x-1)$$
$$=4x^2-1-5x+1$$
$$=4x^2-5x$$
$$(f\cdot g)(x)=(4x^2-1)(5x-1)$$
$$=20x^3-4x^2-5x+1$$
$$\left(\frac{f}{g}\right)(x)=\frac{4x^2-1}{5x-1}$$

D = $(-\infty,\infty)$ for all of the functions except $\left(\frac{f}{g}\right)(x)$, for which $D=\left(-\infty,\frac{1}{5}\right)\cup\left(\frac{1}{5},\infty\right)$.

46. $f(x)=x^3-2x^2+5$

$g(x)=4x^2-3$

SOLUTION:

$$(f+g)(x)=x^3-2x^2+5+4x^2-3$$
$$=x^3+2x^2+2$$
$$(f-g)(x)=x^3-2x^2+5-(4x^2-3)$$
$$=x^3-6x^2+8$$
$$(f\cdot g)(x)=(x^3-2x^2+5)(4x^2-3)$$
$$=4x^5-3x^3-8x^4+6x^2+20x^2-15$$
$$=4x^5-8x^4-3x^3+26x^2-15$$
$$\left(\frac{f}{g}\right)(x)=\frac{x^3-2x^2+5}{4x^2-3}$$

D = $(-\infty,\infty)$ for all of the functions except $\left(\frac{f}{g}\right)(x)$, for which

$$D=\left(-\infty,-\frac{\sqrt{3}}{2}\right)\cup\left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right)\cup\left(\frac{\sqrt{3}}{2},\infty\right).$$

47. $f(x) = \frac{1}{x}$
$g(x) = \frac{1}{x^2}$

SOLUTION:

$$(f+g)(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x}{x^2} + \frac{1}{x^2} = \frac{x+1}{x^2}$$

$$(f-g)(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x}{x^2} - \frac{1}{x^2} = \frac{x-1}{x^2}$$

$$(f \cdot g)(x) = \left(\frac{1}{x}\right)\left(\frac{1}{x^2}\right) = \frac{1}{x^3}$$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{1}{x}}{\frac{1}{x^2}} = \frac{1}{x} \cdot \frac{x^2}{1} = x$$

$D = (-\infty, 0) \cup (0, \infty)$ for every function.

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](2)$.

48. $f(x) = 4x - 11$; $g(x) = 2x^2 - 8$

SOLUTION:

$[f \circ g](x)$
$= 4\left(2x^2 - 8\right) - 11$
$= 8x^2 - 32 - 11$
$= 8x^2 - 43$

$[g \circ f](x)$
$= 2(4x - 11)^2 - 8$
$= 2\left(16x^2 - 88x + 121\right) - 8$
$= 32x^2 - 176x + 242 - 8$
$= 32x^2 - 176x + 234$

$[f \circ g](2)$
$= 8(2)^2 - 43$
$= 32 - 43$
$= -11$

49. $f(x) = x^2 + 2x + 8$; $g(x) = x - 5$

SOLUTION:

$[f \circ g](x)$
$= (x-5)^2 + 2(x-5) + 8$
$= x^2 - 10x + 25 + 2x - 10 + 8$
$= x^2 - 8x + 23$

$[g \circ f](x)$
$= \left(x^2 + 2x + 8\right) - 5$
$= x^2 + 2x + 3$

$[f \circ g](2)$
$= (2)^2 - 8(2) + 23$
$= 4 - 16 + 23$
$= 11$

50. $f(x) = x^2 - 3x + 4$; $g(x) = x^2$

SOLUTION:

$[f \circ g](x) =$
$= (x^2)^2 - 3(x^2) + 4$
$= x^4 - 3x^2 + 4$

$[g \circ f](x)$
$= (x^2 - 3x + 4)^2$
$= x^4 - 3x^3 + 4x^2 - 3x^3 + 9x^2$
$\quad - 12x + 4x^2 - 12x + 16$
$= x^4 - 6x^3 + 17x^2 - 24x + 16$

$[f \circ g](2)$
$= (2)^4 - 3(2)^2 + 4$
$= 16 - 12 + 4$
$= 8$

Find $f \circ g$.

51. $f(x) = \frac{1}{x-3}$
$g(x) = 2x - 6$

SOLUTION:

The domain of $f(x)$ is $x \neq 3$. In order for the range of $g(x)$ to correspond with this domain, $g(x) \neq 3$.

$2x - 6 = 3$
$2x = 9$
$x = \frac{9}{2}$

$[f \circ g](x) = \frac{1}{(2x-6)-3}$
$= \frac{1}{2x-9}$

There are no more restrictions.

$[f \circ g](x) = \frac{1}{2x-9}$ for $x \neq \frac{9}{2}$

52. $f(x) = \sqrt{x-2}$
$g(x) = 6x - 7$

SOLUTION:

The domain of $f(x) = x > 2$. In order for the range of $g(x)$ to correspond with this domain, $g(x)$ must be greater than or equal to 2.

$6x - 7 \geq 2$
$6x \geq 9$
$x \geq \frac{3}{2}$

$[f \circ g](x) = \sqrt{(6x-7)-2}$
$= \sqrt{6x-9}$

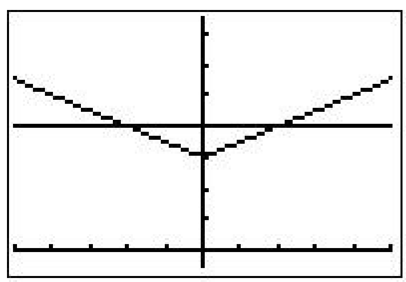
There are no more restrictions.

$[f \circ g](x) = \sqrt{6x-9}$ for $x \geq \frac{3}{2}$

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

53. $f(x) = |x| + 6$

SOLUTION:

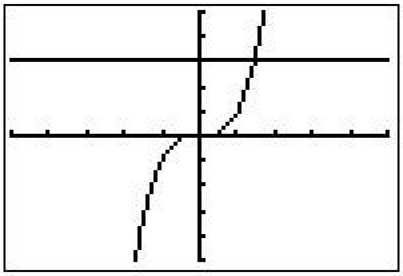


[−5, 5] scl: 1 by [−1, 15] scl: 2

This graph fails the Horizontal Line Test.

54. $f(x) = x^3$

SOLUTION:

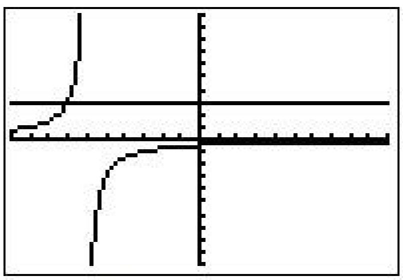


[−5, 5] scl: 1 by [−5, 5] scl: 1

This graph passes the Horizontal Line Test.

55. $f(x) = -\frac{3}{x+6}$

SOLUTION:

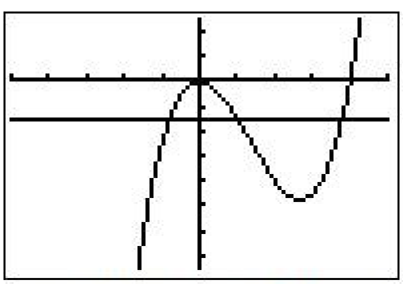


[−10, 10] scl: 1 by [−10, 10] scl: 1

This graph passes the Horizontal Line Test.

56. $f(x) = x^3 - 4x^2$

SOLUTION:



[−5, 5] scl: 1 by [−15,5] scl: 2

This graph fails the Horizontal Line Test.

Find the inverse function and state any restrictions on the domain.

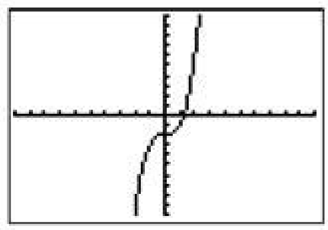
57. $f(x) = x^3 - 2$

SOLUTION:

Replace $f(x)$ with y, interchange the variables, then solve for y.

$$\begin{aligned} f(x) &= x^3 - 2 \\ y &= x^3 - 2 \\ x &= y^3 - 2 \\ x + 2 &= y^3 \\ \sqrt[3]{x+2} &= y \\ f^{-1}(x) &= \sqrt[3]{x+2} \end{aligned}$$

The domain of the inverse function is equal to the range of the original function. Graph $f(x) = x^3 - 2$.



[-10, 10] scl: 1 by [-10, 10] scl: 1

From the graph, the range of $f(x)$ is all real numbers. Thus the domain of f^{-1} has no restrictions.

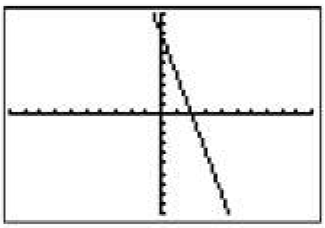
58. $g(x) = -4x + 8$

SOLUTION:

Replace $g(x)$ with y, interchange the variables, and solve for y.

$$\begin{aligned} g(x) &= -4x + 8 \\ y &= -4x + 8 \\ x &= -4y + 8 \\ x - 8 &= -4y \\ -\frac{1}{4}x + 2 &= y \\ g^{-1}(x) &= -\frac{1}{4}x + 2 \end{aligned}$$

The domain of the inverse function is equal to the range of the original function. Graph $g(x) = -4x + 8$.



[-10, 10] scl: 1 by [-10, 10] scl: 1

From the graph, the range of $g(x)$ is all real numbers. Thus, the domain of $g^{-1}(x)$ has no restrictions.

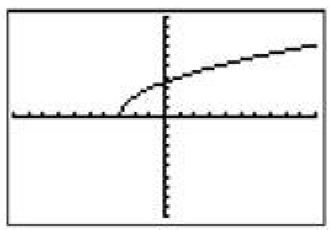
59. $h(x) = 2\sqrt{x+3}$

SOLUTION:

Replace $h(x)$ with y, interchange the variables, and solve for y.

$$\begin{aligned} h(x) &= 2\sqrt{x+3} \\ y &= 2\sqrt{x+3} \\ x &= 2\sqrt{y+3} \\ \frac{x}{2} &= \sqrt{y+3} \\ \frac{x^2}{4} &= y + 3 \\ \frac{x^2}{4} - 3 &= y \\ h^{-1}(x) &= \frac{1}{4}x^2 - 3 \end{aligned}$$

The domain of the inverse function is equal to the range of the original function.
Graph $h(x) = 2\sqrt{x+3}$.



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From the graph, the range of $h(x)$ is $\{y \mid y \geq 0\}$. Thus, the domain of $h^{-1}(x)$ will be restricted to $\{x \mid x \geq 0\}$.

Therefore, $h^{-1}(x) = \frac{1}{4}x^2 - 3, x \geq 0$.

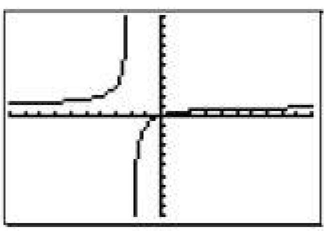
60. $f(x) = \frac{x}{x+2}$

SOLUTION:

Replace $f(x)$ with y, interchange the variables, and solve for y.

$$f(x) = \frac{x}{x+2}$$
$$y = \frac{x}{x+2}$$
$$x = \frac{y}{y+2}$$
$$xy + 2x = y$$
$$xy - y = -2x$$
$$y(x-1) = -2x$$
$$y = \frac{-2x}{x-1}$$
$$f^{-1}(x) = \frac{-2x}{x-1}$$

The domain of the inverse function is equal to the range of the original function. Graph $f(x) = \frac{x}{x+2}$



[-10, 10] scl: 1 by [-10, 10] scl: 1

From the graph, the range of $f(x)$ is $\{y \mid y \in R, y \neq 1\}$. Thus, the domain of $f^{-1}(x)$ is restricted to $\{x \mid x \neq 0\}$.

Therefore, $f^{-1}(x) = \frac{-2x}{x-1}$, $x \neq 1$.