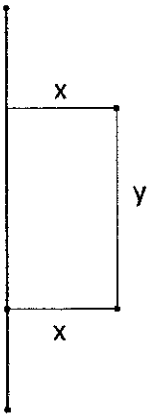
Unit 2, Lesson 3: Graph Optimization

In this section, you'll be presented with WORD PROBLEMS that will ask you to *maximize* or *minimize* some value. When completing these word problems, follow the steps below.

1. Read the problem carefully. Distinguish constants from variables.
2. Decide what is being maximized or minimized. This will be represented by the *dependent* variable (a.k.a. the "y-coordinate")
3. Write the quantity as a function of <u>one</u> variable.
4. Determine an appropriate domain, where necessary.
5. Graph your equation and determine maxima or minima. **You may need to adjust your window!**
6. Make sure that you answer the question that was asked.

<u>Ex. 1:</u> 100 feet of fencing is used to build a fence along the side of a wall. (see picture) Let x be the width of the fence, and let y be the length. What dimensions should the fence have that will give the maximum area enclosed?



<u>Ex. 2:</u> A company has 1000 widgets and they determine that the following equation will determine how many widgets will be sold at a given *price x*.

Number of widgets sold = $1010 - 10x$

How many widgets should they sell to maximize the profit? How much revenue will they bring in?

Write an equation for profit: Graph this equation:

How many widgets should the sell? _____

What is the maximum amount of money brought in? _____

Ex. 3: Find two non-negative numbers whose sum is 16 and whose product is as large as possible.

Ex. 4: A company wants to construct a box. They take a 50" by 75" piece of cardboard, and cut out a square of length x from each corner. The cardboard is then folded up into a box with no lid. For what value of x will the volume be maximized? What is the maximum volume?

Draw a picture:

Equation for volume:

Graph the polynomial:

What is the value for x that will maximize the volume? _____

What is the maximum volume?

_____

Optimization Problems Pre Calc

How to solve these problems:

1) Read the problem carefully. Distinguish constants from variables.

2) Decide what is being maximized or minimized. This gets the dependent variable.

3) Write the quantity as a function of 1 variable. Use other information in the problems and your knowledge of formulas to help you do this.

4) Determine an appropriate domain if necessary.

5) Graph your equation. Determine maxima and / or minima. You may need to adjust your viewing window on your calculator to see this. Keep the domain in mind as well.

6) Make sure you answer the question that was asked.

EX 1: A yard is to be enclosed with one side along a building, so it only requires 3 sides of fencing. What is the largest area you can enclose with 1200 feet of fencing?

EX 2: Suppose you want to enclose a 5000 square foot yard. What is the least amount of fencing you can use?

EX 3: A long rectangular sheet of metal 12 inches wide is to be made into a rain gutter by turning up two sides so they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

EX 4: An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16" wide and 21" long by cutting a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume. Disregard the thickness of the cardboard.

EX 5: Find the dimensions that will make a can that holds 1000 cubic centimeters (1 liter) and uses the least amount of material.

EX 6: Find two non-negative numbers who sum is 16 and whose product is as large as possible.

Optimization WS

Solve the following. Show all work.

1. The sum of two nonnegative numbers is 20. Find the numbers if:
   (a) the sum of their squares is as large as possible
   (b) the sum of their squares is as low as possible.

2. What is the largest possible area for a right triangle who hypotenuse is 5 cm. long and what are its dimensions? (Hint: call the two legs x and y. Use the Pythagorean Theorem to solve for y. Use the formula for area of a triangle to finish the problem).

3. What is the smallest perimeter possible for a rectangle whose area is 16 in^2 and what are its dimensions?

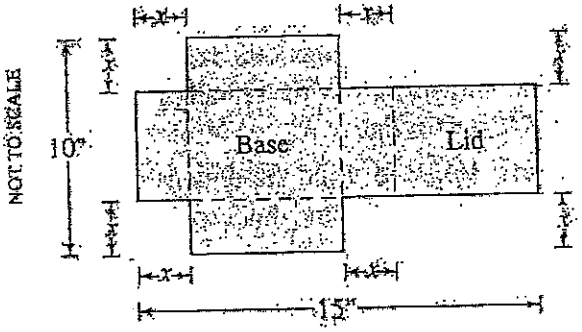
4. You are planning to make an open rectangular box from an 8 inch by 15 inch piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way and what is its volume?

5. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand electric fence. With 800m of wire at your disposal, what is the largest area you can enclose and what are its dimensions?

6. A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

7. What are the dimensions of the lightest <u>open top</u> right circular cylindrical can that will hold a volume of 1000 cm^3?

8. A piece of cardboard measures 10 by 15 inches. Two equal squares are removed from the corners of a 10 inch side as shown in the figure. Two equal rectangles are removed from the other corners so that the tabs can be folded to form a rectangular box with a lid.



(a) Write a formula V(x) for the volume of the box.
(b) Find the domain of V for this problem and graph V over this domain.
(c) Find the maximum volume and the value of x that gives it.