Complex Numbers

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Precalculus

Definition of an imaginary unit: $i = \sqrt{-1}$

If $a > 0$:

$$\sqrt{-a} = \sqrt{-1 \cdot a} = \sqrt{-1}\sqrt{a} = i\sqrt{a}$$

$$i = \sqrt{-1}$$
$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$
$$i^3 = i^2 \times i = -1\sqrt{-1} = -\sqrt{-1}$$
$$i^4 = i^2 \times i^2 = -1(-1) = 1$$

This is a common pattern which can be used to find any power of ***i****.*

Calculate: a. i^{11} b. i^{24} c. i^{74}

a. $-\sqrt{-1}$ b. 1 c. -1

Simplify each expression:

1. $\sqrt{-7} = \sqrt{-1} \times \sqrt{7} = i\sqrt{7}$

2. $\sqrt{-121} = \sqrt{-1} \times \sqrt{121} = i\sqrt{121} = 11i$

3. $-\sqrt{-81} = -\sqrt{-1} \times \sqrt{81} = -i\sqrt{81} = -9i$

4. $-\sqrt{-96} = -\sqrt{-1} \times \sqrt{96} = -i\sqrt{16} \times \sqrt{6} = -4i\sqrt{6}$

You MUST take care of i before taking the square root.

Adding & Subtracting Imaginary Numbers

1. $3i + 4i = 7i$

2. $10i - 4i = 6i$

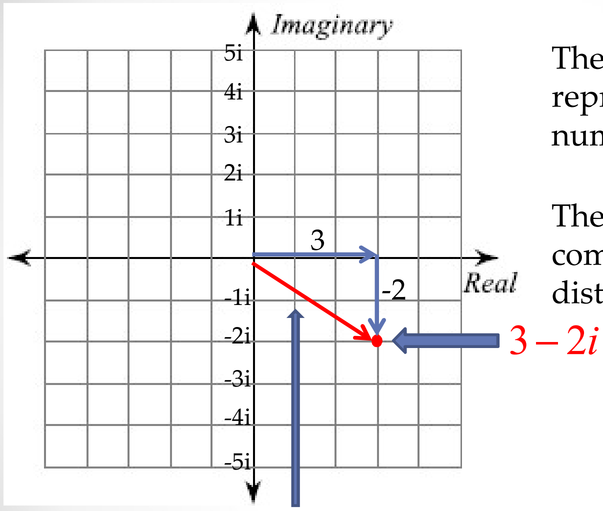
3. $-\sqrt{36} + \sqrt{-36} = -6 + i\sqrt{36} = -6 + 6i$

4. $\sqrt{-6} \bullet \sqrt{-3} = i\sqrt{6} \times i\sqrt{3} = i^2\sqrt{18} = -1\sqrt{9 \times 2} = -3\sqrt{2}$

5. $\sqrt{-25} \bullet \sqrt{-16} = 5i \times 4i = 20i^2 = -20$

6. $i\left(\sqrt{25} + \sqrt{-49}\right) = i(5 + 7i) = 5i + 7i^2 = 5i - 7 = -7 + 5i$

Complex Number Plane



$$|3-2i| = \sqrt{3^2+2^2} = \sqrt{13}$$

The point $(\boldsymbol{a}, \boldsymbol{b})$ represents the complex number $\boldsymbol{a} + \boldsymbol{bi}$.

The absolute value of a complex number is its distance from the origin.

Simplifying Complex Numbers:
Add and simplify:

a) $(6-5i)+(3+4i) = 6-5i+3+4i = 9-i$

b) $(2+i)-(7-2i) = 2+i-7+2i = -5+3i$

Simplifying Complex Numbers:
Multiply and simplify:

a) $3i(12-3i) = 36i-9i^2 = 36i-9(-1) = 9+36i$

b) $(5+9i)(2-7i) = 10-17i-63(-1) = 73-17i$

c) $(1+i)(1-i) = 1-i+i-i^2 = 1-(-1) = 1+1 = 2$

Complex Conjugate

- In general they are of the form $(a+bi)$ paired with $(a-bi)$.
- Complex conjugates are useful when dividing complex numbers.

$$(a+bi)(a-bi) = a^2 - abi + abi - (bi)^2$$
$$= a^2 - (-1)b^2 = a^2 + b^2$$

- To divide two complex numbers multiply the numerator and denominator by the **conjugate** of the *denominator*.

Example: Write $\frac{3-4i}{2+5i}$ in $a + bi$ form.

$$\frac{3-4i}{2+5i} \times \frac{2-5i}{2-5i} = \frac{6-15i-8i+20i^2}{4-10i+10i-25i^2}$$
$$= \frac{6-23i+20(-1)}{4-25(-1)}$$
$$= \frac{6-23i-20}{4+25}$$
$$= \frac{-14-23i}{29} \quad = \frac{-14}{29} - \frac{23}{29}i$$

Solve $3x^2 + 12$ by using factoring:

$$3x^2 + 12 = 0$$

$$3(x^2 + 4) = 0$$ ← Factor out the GCF

$$3(x + 2i)(x - 2i) = 0$$ ← Use $a^2 + b^2 = (a + bi)(a - bi)$ to factor.

$$x + 2i = 0 \qquad x - 2i = 0$$
$$x = -2i \qquad x = 2i$$ ← Use the Zero Product Property to solve.

Using the Quadratic Formula

Solve $-x^2 + 4x - 5 = 0$ by using the quadratic formula:

$$-x^2 + 4x - 5 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 20}}{-2} = \frac{-4 \pm \sqrt{-4}}{-2}$$

$$x = \frac{-4 \pm 2i}{-2} = 2 \pm i$$

Homework: Complex Numbers WS evens