5-5: DOUBLE ANGLE IDENTITIES

CP Precalculus

Mr. Gallo

Double Angle Identities

Derivation of *sin 2θ* :

$$\begin{aligned}\sin 2\theta &= \sin(\theta+\theta)\\ &= \sin\theta\cos\theta+\sin\theta\cos\theta\\ &= 2\sin\theta\cos\theta\end{aligned}$$

Derivation of *cos 2θ* :

$$\begin{aligned}\cos 2\theta &= \cos(\theta+\theta)\\ &= \cos\theta\cos\theta-\sin\theta\sin\theta\\ &= \cos^2\theta-\sin^2\theta\end{aligned}$$

$$\begin{aligned}\cos^2\theta-\sin^2\theta &= \cos^2\theta-(1-\cos^2\theta)\\ &= 2\cos^2\theta-1\end{aligned}$$

$$\begin{aligned}\cos^2\theta-\sin^2\theta &= (1-\sin^2\theta)-\sin^2\theta\\ &= 1-2\sin^2\theta\end{aligned}$$

Double Angle Identities

Derivation of *tan 2θ* :

$$\tan 2\theta = \tan(\theta+\theta)$$
$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$
$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$$

Given $sin\theta = -\frac{8}{17}$ and $cos\theta < 0$, evaluate the following:

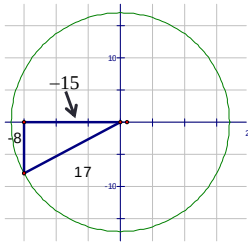
$\sin 2\theta$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(-\frac{8}{17}\right)\left(-\frac{15}{17}\right)$$
$$= \frac{240}{289}$$

$\cos 2\theta$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(-\frac{15}{17}\right)^2 - 1$$
$$= \frac{450}{289} - \frac{289}{289} = \frac{161}{289}$$

First, find cosine and tangent



$$\cos\theta = -\frac{15}{17}$$

$$\tan\theta = \frac{8}{15}$$

Given $sin\theta = -\frac{8}{17}$ and $cos\theta < 0$, evaluate the following:

$\tan 2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{8}{15}\right)}{1-\left(\frac{8}{15}\right)^2} = \frac{\frac{16}{15}}{1-\left(\frac{64}{225}\right)} = \frac{\frac{16}{15}}{\frac{161}{225}} = \frac{240}{161}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} = \frac{2\left(-\frac{8}{17}\right)\left(-\frac{15}{17}\right)}{2\left(-\frac{15}{17}\right)^2 - 1} = \frac{\frac{240}{289}}{\frac{161}{289}} = \frac{240}{289}\left(\frac{161}{289}\right) = \frac{240}{161}$$

Find the exact value of the following expression:

a. $2\sin 15^\circ\cos 15^\circ$

$$2\sin 15^\circ\cos 15^\circ = \sin 2(15) = \sin 30 = \frac{1}{2}$$

b. $\cos^2 105^\circ - \sin^2 105^\circ$

$$\cos^2 105^\circ - \sin^2 105^\circ = \cos 2(105) = \cos 210 = -\frac{\sqrt{3}}{2}$$

c. $\dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$$\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} = \tan 2\left(\frac{\pi}{8}\right) = \tan\frac{\pi}{4} = 1$$

Use Double Angles to help verify this identity.

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$$

$$\frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-\left(1-2\sin^2\theta\right)}{2\sin\theta\cos\theta}$$
$$= \frac{1-1+2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta$$

Homework: 5-5 Double Angle Identity Homework WS