5-2: VERIFYING TRIGONOMETRIC IDENTITIES

CP Precalculus
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VERIFYING TRIGONOMETRIC IDENTITIES

How do you verify an identity?

- You **prove** the left side is equal to the right side
  - Transform one side of the equation (usually the more complicated) into the other side.
  - DO NOT move anything from one side of the equation to the other side of the equation.

Verify the following identity: $\dfrac{\tan^2 x + 1}{1 - \sin^2 x} = \sec^4 x$

$$\frac{\sec^2 x}{\cos^2 x} =$$

$$\frac{\frac{1}{\cos^2 x}}{\cos^2 x} =$$

$$\frac{1}{\cos^2 x}\left(\frac{1}{\cos^2 x}\right) = \frac{1}{\cos^4 x} =$$

$$\frac{1}{\cos^4 x} = \sec^4 x = \sec^4 x$$

VERIFYING TRIGONOMETRIC IDENTITIES

Strategies to try:

- Start with the more complicated side of the identity and work to transform it into the simpler side, keeping the other side of the identity in mind as your goal.
- Use reciprocal, quotient, Pythagorean, and other basic trigonometric identities.
- Use algebraic operations such as combining fractions, rewriting fractions as sums or differences, multiplying expressions, or factoring expressions.
- Convert a denominator of the form $1 \pm u$ or $u \pm 1$ to a single term using its conjugate and a Pythagorean Identity.
- Work each side separately to reach a common intermediate expression.
- If no other strategy presents itself, try converting the entire expression to one involving only sines and cosines.

Verify the following identities:

$$\sec^2\theta\cot^2\theta - 1 = \cot^2\theta$$

$$\frac{1}{\cos^2\theta}\left(\frac{\cos^2\theta}{\sin^2\theta}\right) - 1 =$$

$$\frac{1}{\sin^2\theta} - 1 =$$

$$\csc^2\theta - 1 =$$

$$\cot^2\theta = \cot^2\theta$$

$$\tan^2\alpha = \sec\alpha\csc\alpha\tan\alpha - 1$$

$$= \frac{1}{\cos\alpha}\left(\frac{1}{\sin\alpha}\right)\left(\frac{\sin\alpha}{\cos\alpha}\right) - 1$$

$$= \frac{1}{\cos^2\alpha} - 1$$

$$= \sec^2\alpha - 1$$

$$\tan^2\alpha = \tan^2\alpha$$

VERIFY BY COMBINING FRACTIONS

Verify the following identity: $-2\cot x = \dfrac{\sin x}{1+\cos x} - \dfrac{\sin x}{1-\cos x}$

$$= \frac{\sin x(1-\cos x) - \sin x(1+\cos x)}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin x - \sin x\cos x - \sin x - \sin x\cos x}{1-\cos^2 x}$$

$$= \frac{-2\sin x\cos x}{\sin^2 x}$$

$$= \frac{-2\cos x}{\sin x}$$

$$-2\cot x = -2\cot x$$

VERIFY BY MULTIPLYING

Verify the following identity: $\dfrac{\sin x}{\sec x - 1} = \cos x\cot x + \cot x$

$$\frac{\sin x}{\sec x - 1}\left(\frac{\sec x + 1}{\sec x + 1}\right) =$$

$$\frac{\sin x\sec x + \sin x}{\sec^2 x - 1} =$$

$$\frac{\frac{\sin x}{\cos x} + \sin x}{\tan^2 x} = \frac{\sin x\left(\frac{1}{\cos x}\right) + \sin x}{\tan^2 x} =$$

$$\frac{\tan x + \sin x}{\tan^2 x} =$$

$$\frac{\tan x}{\tan^2 x} + \frac{\sin x}{\tan^2 x} =$$

$$\frac{1}{\tan x} + \frac{\sin x}{\frac{\sin x}{\cos x}} =$$

$$\cot x + \frac{\cos^2 x}{\sin x} =$$

$$\cot x + \cos x\left(\frac{\cos x}{\sin x}\right) =$$

$$\cot x + \cos x\cot x = \cos x\cot x + \cot x$$

VERIFY BY FACTORING

Verify the following identity: $\cos x \sec^2 x \tan x - \cos x \tan^3 x = \sin x$

$$\cos x\left(\frac{1}{\cos^2 x}\right)\tan x - \cos x \tan^3 x =$$

$$\sec x \tan x - \cos x \tan^3 x =$$

$$\tan x\left(\sec x - \cos x \tan^2 x\right) =$$

$$\tan x\left(\sec x - \cos x\left(\frac{\sin^2 x}{\cos^2 x}\right)\right) =$$

$$\tan x\left(\sec x - \left(\frac{1}{\cos x}\right)\sin^2 x\right) =$$

$$\tan x\left(\sec x - \sec x \sin^2 x\right) =$$

$$\tan x \sec x\left(1 - \sin^2 x\right) =$$

$$\frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)\left(\cos^2 x\right) =$$

$$\frac{\sin x}{\cos^2 x}\left(\cos^2 x\right) =$$

$$\sin x = \sin x$$

VERIFY BY WORKING ON EACH SIDE

Verify the following identity:

$$\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$$

$$\sec^2 x\left(\sec^2 x - 1\right) =$$

$$\sec^2 x \tan^2 x =$$

$$\tan^2 x\left(\tan^2 x + 1\right) =$$

$$= \tan^2 x\left(\tan^2 x + 1\right)$$

$$= \tan^2 x \sec^2 x$$

$$\tan^4 x + \tan^2 x = \tan^4 x + \tan^2 x$$

Verify the following identity: $\frac{\cos\alpha}{1+\sin\alpha}+\frac{\cos\alpha}{1-\sin\alpha}=2\sec\alpha$

$$\frac{\cos\alpha(1-\sin\alpha)+\cos\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1-\sin\alpha)}=$$

$$\frac{\cos\alpha-\sin\alpha\cos\alpha+\cos\alpha+\sin\alpha\cos\alpha}{1-\sin^2\alpha}=$$

$$\frac{2\cos\alpha}{\cos^2\alpha}=$$

$$\frac{2}{\cos\alpha}=$$

$$2\sec\alpha=2\sec\alpha$$

Verify the following identity: $\frac{\tan x}{\sec x+1}=\csc x-\cot x$

$$\frac{\tan x\sec x-\tan x}{\sec^2 x-1}=\frac{\tan x}{\sec x+1}\bullet\frac{\sec x-1}{\sec x-1}=$$

$$\frac{\frac{\sin x-\sin x\cos x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}=\frac{\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)-\frac{\sin x}{\cos x}}{\tan^2 x}=$$

$$\frac{\sin x(1-\cos x)}{\cos^2 x}\left(\frac{\cos^2 x}{\sin^2 x}\right)=$$

$$\frac{1}{\sin x}-\frac{\cos x}{\sin x}=\frac{1-\cos x}{\sin x}=$$

$$\csc x-\cot x=\csc x-\cot x$$

Verify the following identity: $\sin^2 x \tan^2 x \csc^2 x + \cos^2 x \tan^2 x \csc^2 x$

$$\tan^2 x \csc^2 x \left(\sin^2 x \cos^2 x\right) =$$

$$\frac{\sin^2 x}{\cos^2 x}\left(\frac{1}{\sin^2 x}\right) =$$

$$\frac{1}{\cos^2 x} =$$

$$\sec^2 x = \sec^2 x$$

HOMEWORK: P. 324 #1-17 ODD