5-1: TRIGONOMETRIC IDENTITIES

CP Precalculus
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TRIGONOMETRIC IDENTITIES

What is an identity?

- An equation in which the left side is equal to the right side:

$$\frac{x^2-9}{x-3} = x+3$$

Identities already known:

Reciprocal Identities			Quotient Identities
$\sin\theta =$	$\cos\theta =$	$\tan\theta =$	$\tan\theta =$
$\csc\theta =$	$\sec\theta =$	$\cot\theta =$	$\cot\theta =$

Examples:

If $\cos x = \frac{3}{4}$, find $\sec x$

$\frac{4}{3}$

If $\sec x = \frac{5}{4}$ and $\tan x = \frac{3}{4}$, find $\sin x$

$\frac{3}{5}$

PYTHAGOREAN IDENTITIES

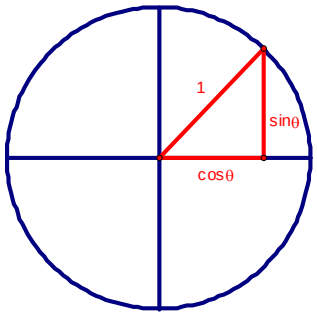
Based on definition of trigonometric functions on the unit circle and the Pythagorean Theorem

- Three Pythagorean Identities:

$a = \sin\theta \quad b = \cos\theta \quad c = 1$

$a^2 + b^2 = c^2$

$\sin^2\theta + \cos^2\theta = 1$



$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

EXAMPLE 1:

If $\theta = \frac{\pi}{6}$, then $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$.

Show the Pythagorean identity holds:

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$\frac{1}{4} + \frac{3}{4} = 1$

EXAMPLE 2:

If $\sin\theta = \frac{1}{3}$, use the Pythagorean Identity to find $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

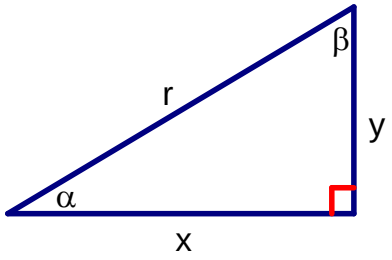
$\cos^2\theta = \frac{8}{9}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{8}{9}}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

COFUNCTION IDENTITIES

A trigonometric function f is a **cofunction** of another trigonometric function g if $f(\alpha) = g(\beta)$ when α and β are complementary angles.



$$\sin\alpha = \cos\beta = \cos(90-\alpha) = \frac{y}{r}$$

$$\tan\alpha = \cot\beta = \cot(90-\alpha) = \frac{y}{x}$$

$$\sec\alpha = \csc\beta = \csc(90-\alpha) = \frac{r}{y}$$

Cofunction Identities:

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

ODD-EVEN IDENTITIES

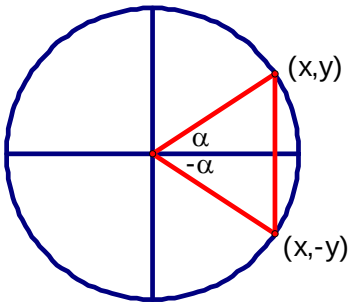
Each of the trigonometric are either **odd** or **even**

- Can see on the unit circle

$$\sin\alpha = y \qquad \sin(-\alpha) = -y$$

$$\cos\alpha = x \qquad \cos(-\alpha) = x$$

$$\tan\alpha = \frac{y}{x} \qquad \tan(-\alpha) = -\frac{y}{x}$$



Odd-Even Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

EXAMPLE 3:

If $\sin x = -0.37$, find $\cos\left(x - \frac{\pi}{2}\right)$.

$$\cos\left(x-\frac{\pi}{2}\right) = \cos\left(-\left(\frac{\pi}{2}-x\right)\right)$$
$$= \cos\left(\frac{\pi}{2}-x\right)$$
$$= \sin x$$
$$= -0.37$$

EXAMPLE 4:

If $\cos x = -.75$, find $\sin\left(x - \frac{\pi}{2}\right)$.

$$\sin\left(x-\frac{\pi}{2}\right) = \sin\left(-\left(\frac{\pi}{2}-x\right)\right)$$
$$= -\sin\left(\frac{\pi}{2}-x\right)$$
$$= -\cos x$$
$$= 0.75$$

HOMEWORK: P. 317 #1-21 ODD

USING IDENTITIES TO SIMPLIFY AND REWRITE TRIGONOMETRIC EQUATIONS

Various methods to try:

- Simplify by rewriting using only sine and cosine
- Simplify by factoring
- Simplify by combining fractions
- Rewrite so fractions are eliminated

To be successful:

- Know the identities
- Persevere and don't give up after only a brief try

REWRITE USING ONLY SINE AND COSINE

Simplify $\frac{1}{\cos x}\left(1-\sin^2 x\right)$

$$= \frac{1}{\cos x}\left(\cos^2 x\right)$$

$$= \frac{\cos^2 x}{\cos x} = \cos x$$

Simplify $\sec x - \tan x \sin x$

$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} \bullet \sin x$$

$$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$= \frac{1-\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x} = \cos x$$

SIMPLIFY BY FACTORING

Simplify $\cos x \tan x - \sin x \cos^2 x$

$$= \cos x\left(\frac{\sin x}{\cos x}\right) - \sin x \cos^2 x$$
$$= \sin x - \sin x \cos^2 x$$
$$= \sin x\left(1 - \cos^2 x\right)$$
$$= \sin x\left(\sin^2 x\right) = \sin^3 x$$

Simplify $-\csc\left(\frac{\pi}{2} - x\right) - \tan^2 x \sec x$

$$= -\sec x - \frac{\sin^2 x}{\cos^2 x}\left(\frac{1}{\cos x}\right)$$
$$= -\frac{1}{\cos x} - \frac{\sin^2 x}{\cos^3 x}$$
$$= \frac{-\cos^2 x - \sin^2 x}{\cos^3 x}$$
$$= -\left(\frac{1}{\cos^3 x}\right) = -\sec^3 x$$

SIMPLIFY BY COMBINING FRACTIONS

Simplify $\dfrac{\sec x}{1 - \sec x} - \dfrac{\sec x}{1 + \sec x}$

$$= \frac{\sec x + \sec^2 x - \sec x + \sec^2 x}{1 - \sec^2 x}$$
$$= -\left(\frac{2\sec^2 x}{\sec^2 x - 1}\right) = -\left(\frac{2\sec^2 x}{\tan^2 x}\right)$$
$$= \frac{-\dfrac{2}{\cos^2 x}}{\dfrac{\sin^2 x}{\cos^2 x}} = -\frac{2}{\sin^2 x}$$
$$= -2\csc^2 x$$

Simplify $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$

$$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x\left(1 + \sin x\right)}$$
$$= \frac{\cos^2 x + \sin^2 x + 1 + 2\sin x}{\cos x\left(1 + \sin x\right)}$$
$$= \frac{1 + 1 + 2\sin x}{\cos x\left(1 + \sin x\right)} = \frac{2 + 2\sin x}{\cos x\left(1 + \sin x\right)}$$
$$= \frac{2\left(1 + \sin x\right)}{\cos x\left(1 + \sin x\right)}$$
$$= \frac{2}{\cos x} = 2\sec x$$

REWRITE TO ELIMINATE FRACTIONS

Simplify $\dfrac{1+\tan^2 x}{\csc^2 x}$

$$= \frac{\sec^2 x}{\dfrac{1}{\sin^2 x}}$$

$$= \frac{\dfrac{1}{\cos^2 x}}{\dfrac{1}{\sin^2 x}}$$

$$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

Simplify $\dfrac{4}{\sec x + \tan x} \cdot \dfrac{\sec x - \tan x}{\sec x - \tan x}$

$$= \frac{4\sec x - 4\tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{4\sec x - 4\tan x}{1}$$

$$= 4\sec x - 4\tan x$$

HOMEWORK: P. 317 #23-35 ODD, 39-47 ODD