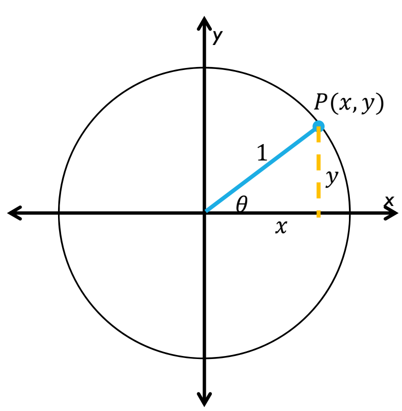
4-3: TRIGONOMETRIC FUNCTIONS ON THE UNIT CIRCLE

Precalculus
Mr. Gallo

ANGLES AND THE UNIT CIRCLE

Trig functions of angles are used to describe coordinates



To find the coordinates of *P*:

1. Draw height of triangle back to *x*-axis (creates reference angle, θ)
2. Hypotenuse has length of 1 unit, height = *y* and base = *x*
3. Use trig functions to find *x* and *y*

$$\cos\theta = \frac{x}{1} \qquad \sin\theta = \frac{y}{1}$$

$$\cos\theta = x \qquad \sin\theta = y$$

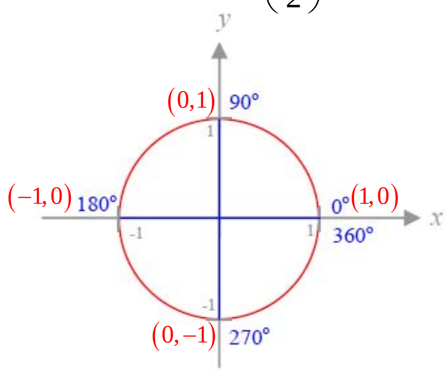
4. Rewrite the coordinates of *P* as:

$$P(\cos\theta, \sin\theta)$$

FINDING COSINE AND SINE OF QUADRANTAL ANGLES

Quadrantal Angles

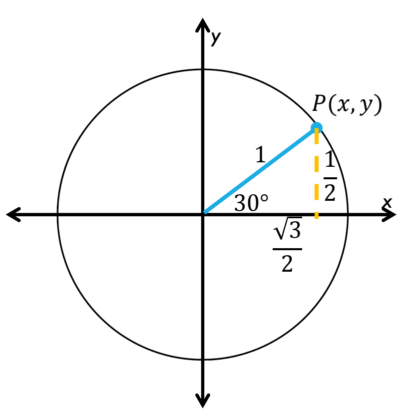
- Terminal side of angle falls on the *x*- or *y*-axis
- $0° \; (0),\; 90° \left(\frac{\pi}{2}\right),\; 180° \; (\pi),\; 270° \left(\frac{3\pi}{2}\right),\; 360° \; (2\pi)$



Angles		Coordinates
$0°$	0	$(1,0)$
$90°$	$\frac{\pi}{2}$	$(0,1)$
$180°$	π	$(-1,0)$
$270°$	$\frac{3\pi}{2}$	$(0,-1)$
$360°$	2π	$(1,0)$

FINDING EXACT VALUES OF COSINE AND SINE

Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of *P*:

1. Draw height of triangle back to *x*-axis (creates reference angle, 30°)
2. Hypotenuse has length of 1 unit, height = *y* and base = *x*
3. Use trig functions to find *x* and *y*

$$\cos 30 = \frac{\frac{\sqrt{3}}{2}}{1} \qquad \sin 30 = \frac{\frac{1}{2}}{1}$$

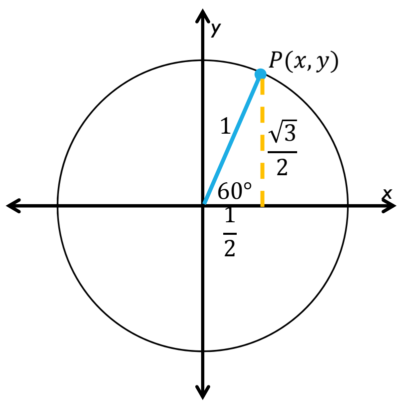
$$\cos 30 = \frac{\sqrt{3}}{2} \qquad \sin 30 = \frac{1}{2}$$

4. Rewrite the coordinates of *P* as:

$$P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

FINDING EXACT VALUES OF COSINE AND SINE

Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of *P*:

1. Draw height of triangle back to *x*-axis (creates reference angle, 60°)
2. Hypotenuse has length of 1 unit, height = *y* and base = *x*
3. Use trig functions to find *x* and *y*

$$\cos 60 = \frac{\frac{1}{2}}{1} \qquad \sin 60 = \frac{\frac{\sqrt{3}}{2}}{1}$$

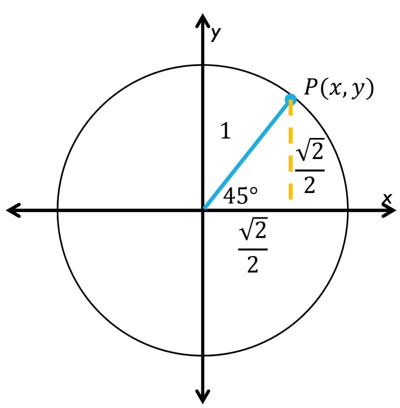
$$\cos 60 = \frac{1}{2} \qquad \sin 60 = \frac{\sqrt{3}}{2}$$

4. Rewrite the coordinates of *P* as:

$$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

FINDING EXACT VALUES OF COSINE AND SINE

Use special right triangles and trig functions to calculate the coordinates.



To find the coordinates of *P*:

1. Draw height of triangle back to *x*-axis (creates reference angle, 45°)
2. Hypotenuse has length of 1 unit, height = *y* and base = *x*
3. Use trig functions to find *x* and *y*

$$\cos 45 = \frac{\frac{\sqrt{2}}{2}}{1} \qquad \sin 45 = \frac{\frac{\sqrt{2}}{2}}{1}$$

$$\cos 45 = \frac{\sqrt{2}}{2} \qquad \sin 45 = \frac{\sqrt{2}}{2}$$

4. Rewrite the coordinates of *P* as:

$$P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

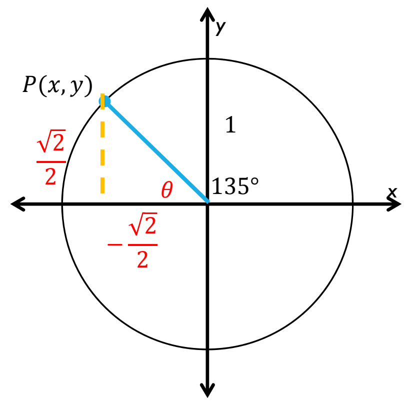
REFERENCE ANGLES

Can be used to find the values of any angle θ.

- Always drawn back to x-axis

What are the coordinates of a $135°$ angle?

1. Create a reference angle between the terminal side of the angle and the y-axis
2. $\theta = 180° - 135° = 45°$ so the coordinates are the same as the triangle formed by a $45°$angle.
3. $P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

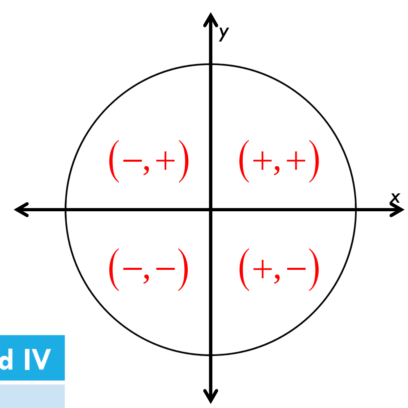


Complete Guided Practice 3A-3C p.244

3A. $\frac{\pi}{4}$ 3B. $60°$ 3C. $30°$

SIGNS OF TRIGONOMETRIC FUNCTIONS

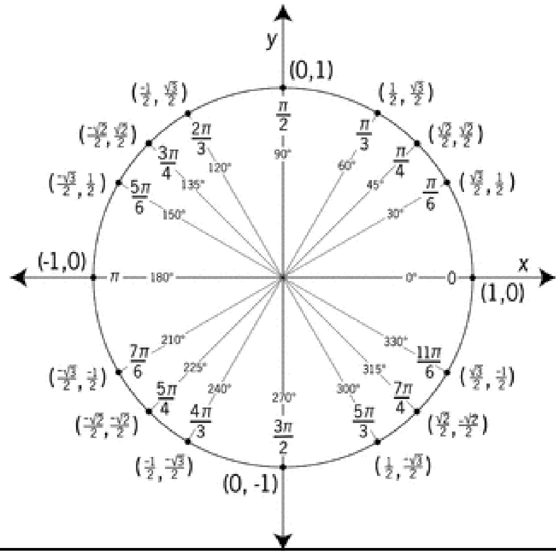
- Look at the sign of the x and y coordinates in each quadrant
- Remember $cos\theta = x$ and $sin\theta = y$



	Quad I	Quad II	Quad III	Quad IV
sin	+	+	−	−
cos	+	−	−	+

COMPLETE UNIT CIRCLE

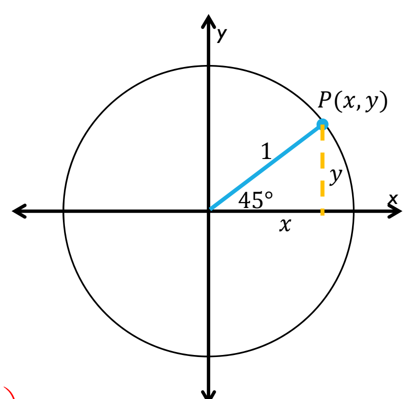
Use reference angles to complete the other quadrants.



FINDING TANGENT

Remember:

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

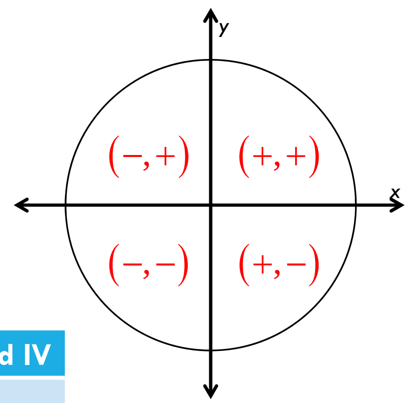


What is the $\tan 45°$?

$$\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2}\left(\frac{2}{\sqrt{2}}\right) = 1$$

SIGNS OF TRIGONOMETRIC FUNCTIONS

- Look at the sign of the x and y coordinates in each quadrant
- Remember $tan\theta = \frac{sin\theta}{cos\theta}$



	Quad I	Quad II	Quad III	Quad IV
sin	+	+	−	−
cos	+	−	−	+
tan	+	−	+	−

Homework: p.251 #9, 10, 13-31 odd, 41

SIX TRIGONOMETRIC FUNCTIONS ON THE UNIT CIRCLE

Sine, Cosine and Tangent can be used to find the reciprocal trigonometric functions:

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$\sin t = y$ | $\cos t = x$ | $\tan t = \frac{y}{x}, x \neq 0$

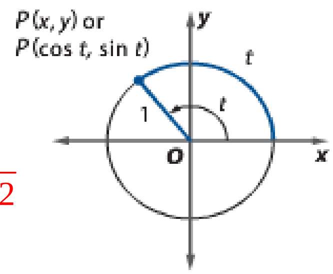
$\csc t = \frac{1}{y}, y \neq 0$ | $\sec t = \frac{1}{x}, x \neq 0$ | $\cot t = \frac{x}{y}, y \neq 0$

Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.

Find the following:

$\cot 210°$ $\qquad$ $\sec \frac{7\pi}{4}$

$$\cot 210 = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3} \qquad \sec \frac{7\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$$



TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

Apply concept of unit circle to finding coordinates of circle of any size radius:

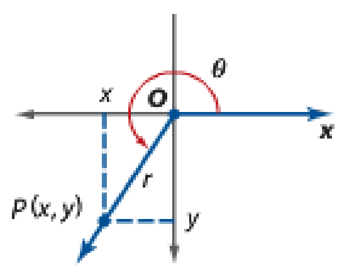
Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ. Let r represent the nonzero distance from P to the origin.

That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$\sin \theta = \frac{y}{r}$ | $\csc \theta = \frac{r}{y}, y \neq 0$

$\cos \theta = \frac{x}{r}$ | $\sec \theta = \frac{r}{x}, x \neq 0$

$\tan \theta = \frac{y}{x}, x \neq 0$ | $\cot \theta = \frac{x}{y}, y \neq 0$



EVALUATING TRIGONOMETRIC FUNCTIONS GIVEN A POINT

Let $(-4,3)$ be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ

$$r=\sqrt{x^2+y^2}=\sqrt{(-4)^2+3^2}=\sqrt{25}=5$$

$$\sin\theta=\frac{y}{r}=\frac{3}{5} \qquad \cos\theta=\frac{x}{r}=\frac{-4}{5} \qquad \tan\theta=\frac{y}{x}=\frac{3}{-4}$$

$$\csc\theta=\frac{r}{y}=\frac{5}{3} \qquad \sec\theta=\frac{r}{x}=\frac{5}{-4} \qquad \cot\theta=\frac{x}{y}=\frac{-4}{3}$$

Complete Guided Practice 1A-1B p.242

1A. $\sin\theta=\frac{3}{5}$ $\cos\theta=\frac{4}{5}$ $\tan\theta=\frac{3}{4}$

$\csc\theta=\frac{5}{3}$ $\sec\theta=\frac{5}{4}$ $\cot\theta=\frac{4}{3}$

1B. $\sin\theta=\frac{-\sqrt{5}}{5}$ $\cos\theta=\frac{-2\sqrt{5}}{5}$ $\tan\theta=\frac{1}{2}$

$\csc\theta=\sqrt{5}$ $\sec\theta=\frac{\sqrt{5}}{-2}$ $\cot\theta=2$

Homework: p.251 #1-7 odd, 11, 12-16 even, 33-39 odd, 43-57odd