2-5: RATIONAL EXPRESSIONS

PRECALCULUS

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RATIONAL EXPRESSIONS

- Quotient of two polynomials
  - Can think of as ratios, fractions or quotients

- Simplest Form
  - Numerator and denominator have no common divisors
  - Rational expression and simplest form **must** have same domain.

$$\frac{x^2+8x+16}{x^2-2x-24}=\frac{(x+4)(x+4)}{(x-6)(x+4)}=\frac{(x+4)}{(x-6)}$$

Domain: $\{x: x \neq -4 \text{ and } x \neq 6\}$.

This is because both -4 and 6 would cause the denominator of the original function to equal 0

SIMPLIFYING A RATIONAL EXPRESSION

What is $\frac{9x^2+6x}{36x+2}$ in simplest form? State any restrictions on the variables.

$$\frac{9x^2+6x}{36x+24} = \frac{3x(3x+2)}{12(3x+2)} = \frac{x}{4}$$

Domain: $\left\{x: x \neq -\frac{2}{3}\right\}$.

This is because $-\frac{2}{3}$ would cause the denominator of the original function to equal 0

What is the simplest form of the following? Be sure to state any restrictions.

$=\frac{3x(3x+2)}{12(3x+2)}=\frac{x}{4}$

$\frac{12-4x}{x^2-9}$

$\frac{x+4}{x-3}; x \neq 2 \text{ or } x \neq 3$

$\frac{-4}{x+3}; x \neq \pm 3$

MULTIPLYING RATIONAL EXPRESSIONS

What is the product $\frac{x^2-3x+2}{x+2} \cdot \frac{x^2-3}{x^2+5x-6}$ in simplest form? State any restrictions on the variable.

$$\frac{x^2-3x+2}{x+2} \bullet \frac{x^2-36}{x^2+5x-6} = \frac{(x-2)(x-1)}{x+2} \bullet \frac{(x-6)(x+6)}{(x+6)(x-1)}$$

$$= \frac{(x-2)(x-6)}{x+2}$$

Domain: $\{x: x \neq -6, x \neq -2, x \neq 1\}$.

This is because $-6, -2, 1$would cause the denominator of the original function to equal 0

What is the product below in simplest form? Be sure to state any restrictions.

$\frac{2x-8}{x^2-16} \bullet \frac{x^2+5x+4}{x^2+8x+16}$

$\frac{2(x+1)}{(x+4)^2}; x \neq \pm 4$

DIVIDING RATIONAL EXPRESSIONS

What is the quotient $\frac{6x-3}{36-x^2}^2 \div \frac{x^3-x^2-2x}{x^2-5x-6}$ in simplest form? State any restrictions on the variable.

$$\frac{6x-3x^2}{36-x^2} \div \frac{x^3-x^2-2x}{x^2-5x-6} = \frac{6x-3x^2}{36-x^2} \cdot \frac{x^2-5x-6}{x^3-x^2-2x}$$

$$= \frac{3x(x-2)}{(6-x)(6+x)} \cdot \frac{(x-6)(x+1)}{x(x-2)(x+1)}$$

$$= \frac{3(x-2)}{(x-6)(6+x)} \cdot \frac{(x-6)}{x-2}$$

$$= \frac{3}{6+x}$$

Domain: $\{x: x \neq 0, x \neq \pm 6, x \neq 2, x \neq -1\}$

This is because $0, \pm 6, 2, -1$ would cause the denominator of the original function to equal 0

What is the product below in simplest form? Be sure to state any restrictions.

$$\frac{x^2+5x+4}{x^2+x-12} \div \frac{x^2-1}{2x^2-6x}$$

$$\frac{2x}{x-1}; x \neq 0, \pm 1, -4, 3$$

Homework: WS HW #1 & 2 odds

ADDING RATIONAL EXPRESSIONS

- Find LCM of denominators and use as the Least Common Denominator (LCD).
  - Use LCD just like you did to add fractions.

$$\frac{1}{8}+\frac{1}{10}=\frac{1}{2^3}+\frac{1}{2\cdot5}=\frac{5}{5}\cdot\frac{1}{2^3}+\frac{1}{2\cdot5}\cdot\frac{2^2}{2^2}=\frac{5}{40}+\frac{4}{40}=\frac{9}{40}$$

  - Make sure answer is in simplest form

ADDING RATIONAL EXPRESSIONS

What is the sum $\frac{4}{x^2+3x}+\frac{x-2}{x^2+6x+}$ in simplest form? State any restriction on the variables.

$$\frac{4}{x^2+3x}+\frac{x-2}{x^2+6x+9}=\frac{4}{x(x+3)}+\frac{x-2}{(x+3)^2}$$

LCM: $x(x+3)^2$

$$=\frac{(x+3)}{(x+3)}\cdot\frac{4}{x(x+3)}+\frac{x-2}{(x+3)^2}\cdot\frac{x}{x}$$

$$=\frac{4x+12}{x(x+3)^2}+\frac{x^2-2x}{x(x+3)^2}$$

$$=\frac{x^2+2x+12}{x(x+3)^2}\qquad x\neq0 \text{ or } -3$$

Find the following sum. State any restrictions

$$\frac{x}{x^2-4}+\frac{1}{x+2}\qquad \frac{2x-2}{(x-2)(x+2)};\ x\neq\pm2$$

SUBTRACTING RATIONAL EXPRESSIONS

What is the difference $\frac{x+1}{x^2+2x-} - \frac{x}{4x-8}$ in simplest form? State any restrictions on the variable.

See next slide for solution

$$\frac{x+1}{x^2+2x-8} - \frac{x}{4x-8} = \frac{x+1}{(x+4)(x-2)} - \frac{x}{4(x-2)}$$

$$= \frac{4}{4}\cdot\frac{x+1}{(x+4)(x-2)} - \frac{x}{4(x-2)}\cdot\frac{x+4}{x+4}$$

LCM: $4(x-2)(x+4)$

$$= \frac{4x+4}{4(x+4)(x-2)} - \frac{x^2+4x}{4(x+4)(x-2)}$$

$$= \frac{4x+4-(x^2+4x)}{4(x+4)(x-2)}$$

$$= \frac{-x^2+4}{4(x+4)(x-2)} = \frac{-(x^2-4)}{4(x+4)(x-2)}$$

$$= \frac{-(x-2)(x+2)}{4(x+4)(x-2)} = \frac{-(x+2)}{4(x+4)}$$

$x \neq -4$ or 2

SIMPLIFYING A COMPLEX FRACTION

Complex Fractions:

- A rational expression with at least one fraction in the numerator, denominator or both.

- Two methods to simplify complex fractions:
  1. Multiply both numerator and denominator by the LCD of **all** the rational expressions.
  2. Combine the fractions in the numerator and the fractions in the denominator. Multiply the new numerator by the reciprocal of the new denominator.

SIMPLIFYING A COMPLEX FRACTION

What is a simpler form of $\dfrac{3x-\dfrac{1}{y}}{\dfrac{y^2}{x}+x}$

- Method 1:

$$\frac{3x-\frac{1}{y}}{\frac{y^2}{x}+x} = \frac{\left(3x-\frac{1}{y}\right)xy}{\left(\frac{y^2}{x}+x\right)xy}$$

Multiply by the LCD

LCD of all rational expressions:

xy

$$= \frac{(3x)xy-\left(\frac{1}{y}\right)xy}{\left(\frac{y^2}{x}\right)xy+(x)xy}$$

Distribute and Simplify

$$= \frac{3x^2y-x}{y^3+x^2y}$$

SIMPLIFYING A COMPLEX FRACTION

- Method 2:

$$\frac{3x-\frac{1}{y}}{\frac{y^2}{x}+x} = \frac{\frac{y}{y}(3x)-\frac{1}{y}}{\frac{y^2}{x}+(x)\frac{x}{x}}$$

Combine rational expressions in numerator and denominator

LCD of numerator: y

LCD of denominator: x

$$= \frac{\frac{3xy-1}{y}}{\frac{y^2+x^2}{x}}$$

Simplify

$$= \frac{3xy-1}{y}\left(\frac{x}{y^2+x^2}\right)$$

Multiply by the reciprocal

$$= \frac{3x^2y-x}{y^3+x^2y}$$

Homework: HW #3 & 4 WS odds

SOLVING RATIONAL EQUATIONS

Rational Equations

- Contain at least <u>one</u> rational expressions

$$\frac{2x}{x-2}+\frac{1}{x+2}=\frac{3x+1}{x^2-4}$$

Rational Equation

$$3x+\frac{1}{2}=\frac{3}{4}$$

Not A Rational Equation

- Solve by clearing equation of denominators first.
  - Use LCD of the rational expressions
- Must check for <u>extraneous solutions</u>.

SOLVING A RATIONAL EQUATION

What are the solutions of this rational equation?

$$\frac{4}{x+2}=\frac{5}{2x+3}$$

$$\frac{4}{x+2}=\frac{5}{2x+3}$$ ← Factor the denominators to find the LCD

$$(x+2)(2x+3)\left(\frac{4}{x+2}\right)=(x+2)(2x+3)\left(\frac{5}{2x+3}\right)$$ ← Multiply by the LCD

$$4(2x+3)=5(x+2)$$
$$8x+12=5x+10$$
$$3x=-2$$
$$x=-\frac{2}{3}$$

← Simplify and solve

Make sure to check for extraneous solutions

$$\frac{4}{-\frac{2}{3}+2}=\frac{5}{2\left(-\frac{2}{3}\right)+3}$$

$$3=3$$

SOLVING A RATIONAL EQUATION

What are the solutions of this rational equation?

$$\frac{1}{x^2-5x}+\frac{x-7}{x}=\frac{4}{x^2-5x}$$

$$x\left(x^2-5x\right)\left(\frac{1}{x^2-5x}+\frac{x-7}{x}\right)=x\left(x^2-5x\right)\left(\frac{4}{x^2-5x}\right)$$

$$x+\left(x^2-5x\right)(x-7)=4x$$

$$x+x^3-5x^2-7x^2+35x=4x$$

$$x^3-12x^2+32x=0$$

$$x\left(x^2-12x+32\right)=0$$

$$x(x-8)(x-4)=0$$

$$x=0 \quad x=8 \quad x=4$$

After checking for extraneous solutions, $x=4$ and $x=8$ are the solutions. If $x=0$, there would be division by 0 in the original problem.

USING A GRAPHING CALCULATOR TO SOLVE

1. Enter one side of the equation as Y_1 and the other side of the equation as Y_2.
2. Graph the equations.
3. Use the INTERSECTION function to find the solutions.

Solve

$$\frac{x+2}{1-2x}=5 \qquad x=0.\overline{27}$$

Homework: Solving Rational Equations HW1

LIMITS AND ASYMPTOTES

Find the vertical and horizontal asymptotes for each of the following and describe the behavior at each vertical asymptote.

A.) $f(x)=\frac{2x^2-1}{x^2+3}$

-V.A.: None

-H.A.: $y = 2$ Why?

$$\lim_{x\to\pm\infty} f(x)=\frac{\frac{2x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}=$$

$$\lim_{x\to\pm\infty}\frac{2-\frac{1}{x^2}}{1+\frac{3}{x^2}}=\frac{2-\frac{1}{\pm\infty}}{1+\frac{3}{\pm\infty}}=\frac{2-0}{1+0}=2$$

B.) $f(x)=\frac{x-3}{x+3}$

- V.A.: $x = -3$

- H.A.: $y = 1$

$$\lim_{x\to -3^+}=\frac{x-3}{x+3}=\frac{-}{+}=-\infty$$

$$\lim_{x\to -3^-}=\frac{x-3}{x+3}=\frac{-}{-}=\infty$$

$$\lim_{x\to\pm\infty}=\frac{x-3}{x+3}=\frac{\frac{x}{x}-\frac{3}{x}}{\frac{x}{x}+\frac{3}{x}}=$$

$$\lim_{x\to\pm\infty}=\frac{1-\frac{3}{x}}{1+\frac{3}{x}}=\frac{1-0}{1+0}=1$$

SLANT ASYMPTOTES

The **end behavior** asymptote of a rational function when the highest power is in the numerator.

Find the slant asymptote of $f(x)=\frac{x^2+2x-3}{x+2}$

$$x+2\overline{)x^2+2x-3} = x-\frac{3}{x+2}$$

Therefore, the line $y = x$ is the **slant asymptote** of the graph of $f(x)$.

This means the graph of $f(x)$ will act like the line $y = x$ as x approaches $\pm\infty$

Using intercepts and asymptotes graph $f(x)=\frac{x^2+2x-3}{x+2}$

x-intercepts : (-3, 0) and (1, 0)

y-intercept: (0, -3/2)

V.A.: $x = -2$

H.A.: None

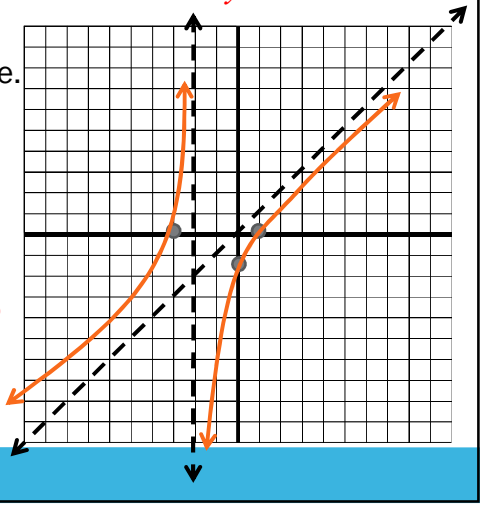
S.A.: $y = x$

How do we graph it?

1. Plot the intercepts on the plane.
2. Draw dashed lines for all asymptotes.
3. Determine the behaviors at each V.A.

$$\lim_{x\to -2^+} = \frac{(x+3)(x-1)}{x+2} = \frac{(+)(-)}{(+)} = -\infty$$

$$\lim_{x\to -2^-} = \frac{(x+3)(x-1)}{x+2} = \frac{(-)(-)}{(-)} = \infty$$



Ex – Find all the asymptotes of

$$f(x)=\frac{x^4-2x+1}{x-2}$$

$$x-2\overline{)x^4-2x+1} = x^2+2x+2+\frac{5}{x-2}$$

V.A.: $x = 2$

H.A.: None

S.A.: $y = x^2 + 2x + 2$ (quadratic asymptote)

Analyze and graph $f(x)=\frac{x^2+9x+20}{x-4}$

x-intercepts : (-5, 0) and (-4, 0)

y-intercept: (0, -5)

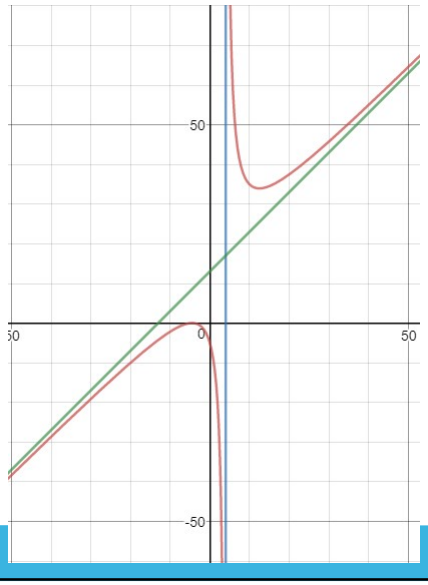
V.A.: $x = 4$

H.A.: None

S.A.: $y = x + 13$

$$\lim_{x\to 4^+}=\frac{(x+5)(x+4)}{x-4}=\frac{(+)(+)}{(+)}=\infty$$

$$\lim_{x\to 4^-}=\frac{(x+5)(x+4)}{x-4}=\frac{(+)(+)}{(-)}=-\infty$$



Homework: Graphing Rational Expressions WS #1-8