2-4: Zeros of Polynomial Functions

Precalculus
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Rational Zero (Root) Theorem

Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with integer coefficients. There are a limited number of possible roots of $P(x) = 0$.

- Integer roots must be factors of a_0.
- Rational roots must have reduced form $\frac{p}{q}$ where p is an integer factor of a_0 and q is an integer factor of a_n.

Factors of the leading coefficient: $\pm 1, \pm 3, \pm 7, \pm 21$

Factors of the constant: $\pm 1, \pm 2, \pm 5, \pm 10$

$$21x^2 + 29x + 10 = 0$$

$$\frac{\text{Factors of constant term } (10)}{\text{Factors of leading coefficient } (21)}$$

Use synthetic division and each combination of factors to find roots

Ex. 1: What are the rational roots of $x^3 - 2x^2 - 5x + 6 = 0$?

Factors of a_0 (6): $\pm 1, \pm 2, \pm 3, \pm 6$

Factors of a_n (1): ± 1

$\frac{\text{Factors of } a_0\ (6)}{\text{Factors of } a_n\ (1)}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}$

Possible Roots: $\pm 1, \pm 2, \pm 3, \pm 6$

Test possible roots using synthetic division & $x^3 - 2x^2 - 5x + 6 = 0$?

-1	1	-2	-5	6
		-1	3	2
	1	-3	-2	8

Remainder $\neq 0$; Not a root.

1	1	-2	-5	6
		1	-1	-6
	1	-1	-6	0

Remainder = 0; Is a root.

Rewrite $x^3 - 2x^2 - 5x + 6 = 0$ as $(x-1)(x^2 - x - 6) = 0$ and use appropriate method to solve the quadratic (factoring, Quadratic Formula, etc.).

Factor $x^2 - x - 6 \Rightarrow (x-3)(x+2)$

Zeroes: x = $-2, 1, 3$

Linear Factors: $(x-1)(x-3)(x+2)$

Rational Zero (Root) Theorem Summary

1. Find all factors of constant term and lead coefficient.
2. Put them all in the ratio $\frac{Constant\ Factors}{Lead\ Coeff.Factors}$
3. Use synthetic division to find the roots
   a) Repeat until you can solve the remaining polynomial or until you have a second-degree polynomial and can solve by factoring or the Quadratic Formula.

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is an irrational root with a and b real, then $a - bi$ is also a root.

Ex. 2: A quintic polynomial $P(x)$ has rational coefficients. If $1, \sqrt{3}$ and $2 - 3i$ are roots of $P(x) = 0$, what are the remaining roots?

$-\sqrt{3}$ and $2 + 31$. 1 is neither irrational or complex so doesn't have a conjugate.

Ex. 3: What polynomial function $P(x)$ with rational coefficients so that $P(x) = 0$ has roots 4 and $3i$? Since $3i$ is a root, so is $-3i$.

Find the linear factors and write the polynomial in factored form.

$$(x-4)(x-3i)(x+3i) = 0$$

Multiply the linear factors and write the polynomial in standard form.

$$(x-4)(x-3i)(x+3i) = 0$$

$$(x-4)(x^2+9) = 0$$

$$x^3 - 4x^2 + 9x - 36 = 0$$

Write a polynomial function of least degree with real coefficients in standard form that has -2, 4, and $3 - i$ as zeros.

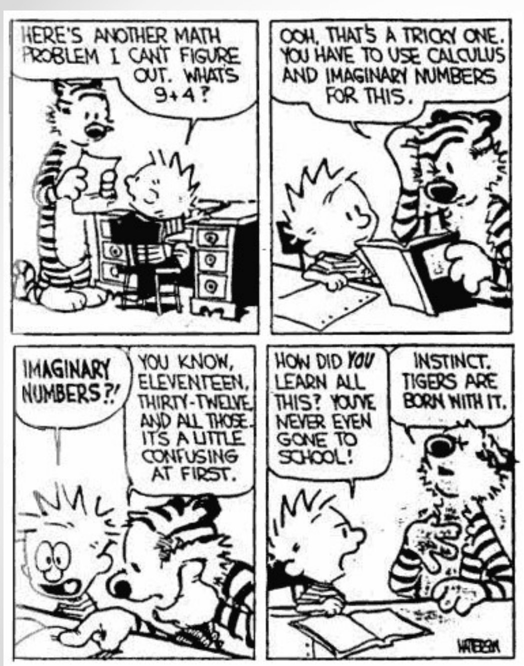
$x = -2 \quad x = 4 \quad x = 3 - i$ and $x = 3 + i$

$$(x+2)(x-4)(x-3-i)(x-3+i)$$

$$(x^2 - 2x - 8)(x^2 - 3x + ix - 3x + 9 - 3i - ix + 3i - i^2)$$

$$x^4 - 6x^3 + 10x^2 - 2x^3 + 12x^2 - 20x - 8x^2 + 48x - 80$$

$$x^4 - 8x^3 + 14x^2 + 28x - 80$$



Homework: p.127 # 1, 3, 7, 10-16 43-46, 49-54