2-3: The Remainder and Factor Theorems

ALGEBRA 2

MR. GALLO

Polynomial Long division

Numerical

$$22\overline{)770}$$

Polynomial

$$2x+1\overline{)6x^2+7x+2}$$

The remainder for each problem is 0, so 22 is a factor of 770 and 2x+1 is a factor of $6x^2+7x+2$.

Using polynomial long division

Use long division to divide $5x^2+2x+3$ by $x+1$

$$
\begin{array}{r}
5x-3 \\
x+1\overline{)\,5x^2+2x+3} \\
-\;5x^2+5x\quad\; \\
\hline
-3x+3 \\
-\;\;-3x-3 \\
\hline
6
\end{array}
$$

The ***degree*** is less than x +1 so this is the ***Remainder***

Divide: $\frac{5x^2}{x}=5x$

Multiply: $5x(x+1)=5x^2+5x$

Subtract to get $-3x$. Bring down 3.

Divide: $\frac{-3x}{x}=-3$

Multiply: $-3(x+1)=-3x-3$

Subtract to get 6.

The quotient is $5x-3+\frac{6}{x+1}$

Division algorithm for polynomials

Let $f(x)$ and $d(x)$ be polynomials such that the degree of $d(x)$ is less than or equal to the degree of $f(x)$ and $d(x) \neq 0$. Then there exist unique polynomials $q(x)$ and $r(x)$ such that:

$$\frac{f(x)}{d(x)}=q(x)+\frac{r(x)}{d(x)} \quad \text{or} \quad f(x)=d(x)\cdot q(x)+r(x)$$

where $r(x)=0$ or the degree of $r(x)$ is less than the degree of $d(x)$. If $r(x)=0$, then $d(x)$ ***divides evenly*** into $f(x)$.

Checking factors

Is x^2+1 a factor of $3x^4-4x^3+12x^2+5$?

$$
\begin{array}{r}
3x^2 - 4x + 9 \\
x^2+0x+1 \overline{)\, 3x^4-4x^3+12x^2+0x+5} \\
-\ \underline{3x^4+0x^3+3x^2} \\
-4x^3+9x^2+0x \\
-\ \underline{-4x^3+0x^2-4x} \\
9x^2-4x+5 \\
-\ \underline{9x^2+0x+9} \\
-4x-4
\end{array}
$$

The ***degree*** is less than the divisor so this is the ***Remainder***

The quotient is: $3x^2-4x+9+\frac{-4x-4}{x^2+1}$

The remainder is not 0 so x^2+1 is not a factor of $3x^4-4x^3+12x^2+5$

Is x^2-2 a factor of $f(x)=x^4-x^2-2$? If it is, write $f(x)$ as a product of two factors.

$$
\begin{array}{r}
x^2+0x+1 \\
x^2+0x-2 \overline{)\, x^4+0x^3-x^2+0x-2} \\
-\ \underline{x^4+0x^3-2x^2} \\
x^2+0x-2 \\
-\ \underline{x^2+0x-2} \\
0
\end{array}
$$

$f(x)=(x^2+1)(x^2-2)$

Homework: p.115 #1-18

Synthetic Division

Can only use synthetic division when you divide a polynomial by a ***linear factor*** “x – c”

Write out ALL coefficients (including zeroes!) of polynomial when in standard form.

For a divisor, use “c.” (Reverse the sign of the number in “x – c.”)

Example 1

$$(x^3 - 14x^2 + 51x - 54) \div (x + 2)$$

-2	1	-14	51	-54
		-2	32	-166
	1	-16	83	-220

Final Number in Bottom Row is the Remainder

The quotient is $x^2 - 16x + 83 + \frac{-220}{x+2}$

Example 2

$$(x^3 - 57x + 56) \div (x - 7)$$

7	1	0	-57	56
		7	49	-56
	1	7	-8	0

The quotient is $x^2 + 7x - 8$

Homework: p.115 #18-26 & Complete the evens on the 2-3: Polynomial Synthetic Division WS

Remainder Theorem

If you divide a polynomial $f(x)$ by x – c, then the remainder is $r = f(c)$.

Reason:

$$f(x) = d(x)q(x) + r(x)$$

$$f(x) = (x-c)q(x) + r(x)$$

$$f(c) = (c-c)q(c) + r(c)$$

$$f(c) = 0 + r(c)$$

$$f(c) = r(c)$$

Example 3

Find P(3) for $P(x) = x^5 - 2x^3 - x^2 + 2$ using synthetic division.

3	1	0	-2	-1	0	2
		3	9	21	60	180
	1	3	7	20	60	182

$P(3) = 182$

Factor Theorem

For all polynomials $p(x)$, $(x - c)$ is a factor of $p(x)$ iff $p(c) = 0$. For any polynomial $p(x)$, the following are logically equivalent:

a. $(x - c)$ is a factor of $p(x)$
b. $p(c) = 0$
c. c is an x-intercept of the graph of $p(x)$
d. c is a zero of $p(x)$
e. The remainder when $p(x)$ is divided by $(x - c)$ is 0

Homework: p.115 #31-45 odd, 53-56, 64-66