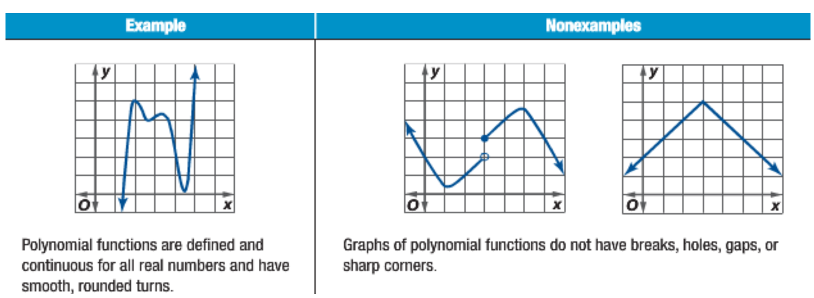
2-2: Polynomial Functions

CP PRECALCULUS

MR. GALLO

Polynomial Graphs

Characteristics



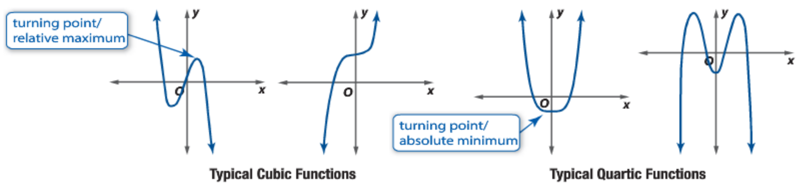
Importance of Lead Coefficient

- Determines end behavior of graph (Leading Term Test)
- Tells number of zeros and turning points
  - Relative Minimums and Maximums

Zeros and Turning Points

Turning Points

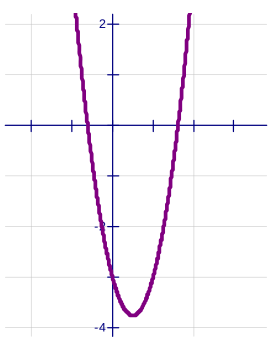
- Where graph changes from increasing to decreasing and vice versa (y-values)
- Relative Minimums and/or Relative Maximums occur here



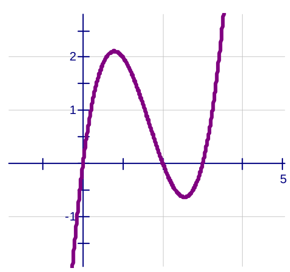
Zeros

- Solutions, x-intercepts, roots

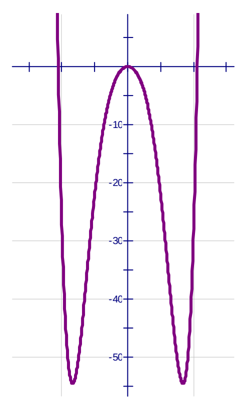
Zeros and Turning Points



$f(x) = 3x^2 - 3x - 3$



$f(x) = x^3 - 5x^2 + 6x$



$f(x) = 3x^6 - 10x^4 - 15x^2$

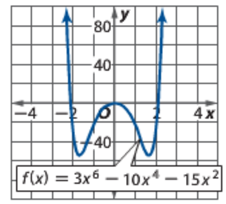
Can you relate the number of zeros and turning points to the degree (n) of the polynomial functions?

Possible Number of Real Zeros = n Number of Turning Points = $n - 1$

Zeros and Turning Points

A polynomial function f of degree $n \geq 1$ has at most n distinct real zeros and at most $n - 1$ turning points.

Example Let $f(x) = 3x^6 - 10x^4 - 15x^2$. Then f has at most 6 distinct real zeros and at most 5 turning points. The graph of f suggests that the function has 3 real zeros and 3 turning points.



Finding Zeros

- Factoring
- Quadratic Form

Words A polynomial expression in x is in **quadratic form** if it is written as $au^2 + bu + c$ for any numbers a, b, and c, $a \neq 0$, where u is some expression in x.

Symbols $x^4 - 5x^2 - 14$ is in quadratic form because the expression can be written as $(x^2)^2 - 5(x^2) - 14$. If $u = x^2$, then the expression becomes $u^2 - 5u - 14$.

Quadratic Form

$$x^4 - 6x^2 = 16$$
$$x^4 - 6x^2 - 16 = 0$$
$$a = x^2 \quad a^2 - 6a - 16 = 0$$
$$(a-8)(a+2) = 0$$
$$(x^2-8)(x^2+2) = 0$$

The solutions are: $x = \pm 2\sqrt{2}$ $\quad x = \pm i\sqrt{2}$

$$2x^4 - 20x^2 + 50 = 0$$
$$2(x^4 - 10x^2 + 25) = 0$$
$$x^4 - 10x^2 + 25 = 0$$
$$a = x^2 \quad a^2 - 10a + 25 = 0$$
$$(a-5)^2 = 0$$
$$(x^2-5)^2 = 0$$

The solutions are: $x = \pm\sqrt{5}$

Try These:

$$g(x) = x^4 - 9x^2 + 18$$

4 Real Zeros and 3 Turning Points;
$\pm\sqrt{3}, \pm\sqrt{6}$

$$h(x) = x^5 - 6x^3 - 16x$$

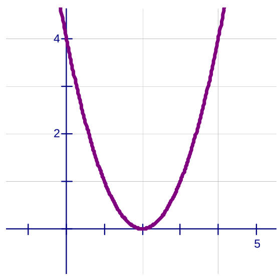
5 Real Zeros and 4 Turning Points;
$0, \pm\sqrt{8}$

Repeated Zeros *(Multiplicity)*

The factor $(x - c)$ occurs more than once when in factored form

Zero occurs an even number of times
$(x - c)^n$; n is even number:

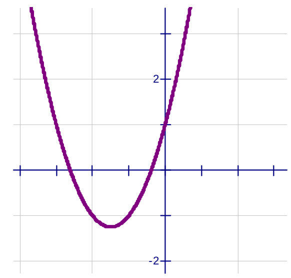
- Graph tangent to x-axis at that point



$f(x) = x^2 - 4x + 4 = (x-2)^2$

Zero occurs an odd number of times
$(x - c)^n$; n is odd number:

- Graph crosses x-axis at that point



$f(x) = x^2 + 3x + 1 = (x+1)(x+2)$

Summary

Graphing Polynomial Functions $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where $a_n \neq 0$:

- Degree: n
- Maximum number of Turning Points: $n - 1$
- At a zero of odd multiplicity: The graph crosses the x-axis
- At a zero of even multiplicity: The graph touches the x-axis
- Between zeros: The graph is either above or below the x-axis
- The end behavior is determined by: The lead coefficient

For $f(x) = x(3x+1)(x-2)^2$,

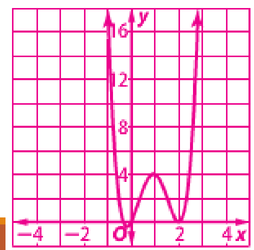
a. Apply the leading-term test
b. Determine the zeros and state the multiplicity of any repeated zeros
c. Find a few additional points
d. Graph the function

a. The degree is 4 and the leading coefficient is 3, so: $\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$

b. The zeros are $x = 0$, $x = 2$ and $x = -\frac{1}{3}$. The zero at $x = 2$ has a multiplicity of 2

c. $(-1,18), (1,4), (3,30)$

d.



Homework: p.104 #1-5, 8, 12-15, 25, 26, 29, 33-35, 64-67