

1-7: Inverse Relations and Functions

Precalculus
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Inverse of a Relation

- Relation pairs an element from its domain with its range (a, b)
- Inverse relation "reverses or undoes" relation and pairs (b, a)

x	y
-2	2
-1	0
0	3
1	0

Relation

x	y
2	-2
0	-1
3	0
0	1

Inverse Relation

Equations of Inverse Relations

- Switch x and y and solve for y .

$$y = 5x^2 + 2$$

$$x = 5y^2 + 2$$

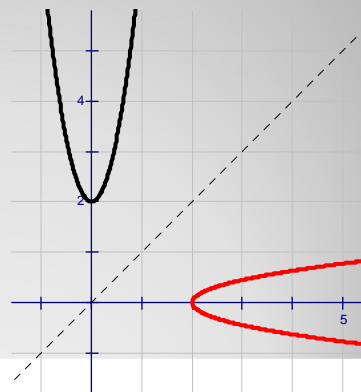
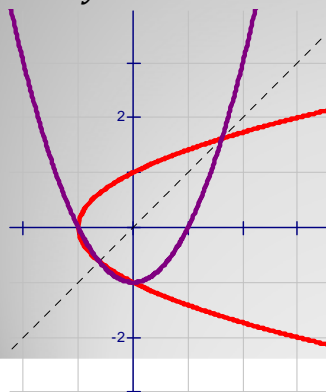
$$x - 2 = 5y^2$$

$$\frac{x - 2}{5} = y^2$$

$$\pm \sqrt{\frac{x - 2}{5}} = y$$

Graphs of Inverse Relations

- Found by switching the x and y coordinates
- Reflections of each other over the equation $x = y$.



Finding an Inverse Function

Consider the function $g(x) = -\frac{2}{3}x + 7$.

- What is the domain and range of g ?
- What is g^{-1} ?
- What are the domain and range of g^{-1} ?
- Is g^{-1} a function?

a. The domain and range are both all real numbers.

$$b. \quad y = -\frac{2}{3}x + 7$$

c. The domain and range are both all real numbers.

$$x = -\frac{2}{3}y + 7$$

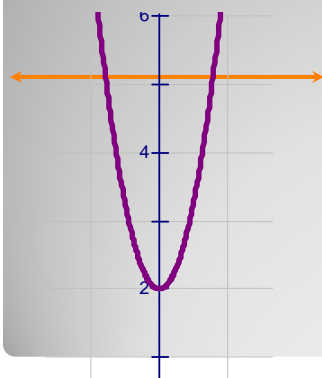
d. Yes, for each x in the domain, there is only one value for y in the range.

$$x - 7 = -\frac{2}{3}y$$

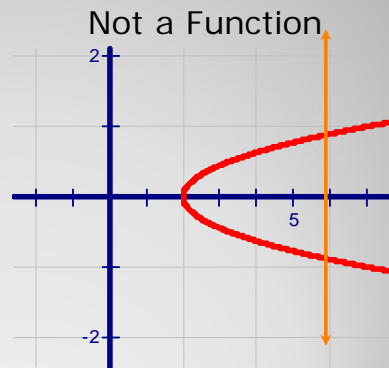
$$-\frac{3x + 21}{2} = y$$

Graphs of Functions

- If the graph of a function passes the Horizontal Line Test, the **inverse** is also a function.



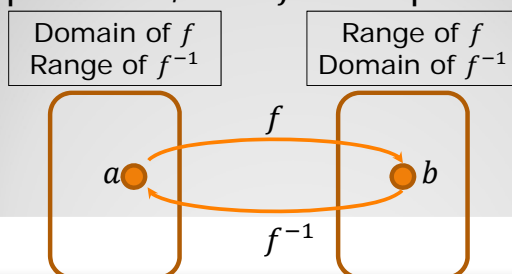
Function



Inverse

One to One Functions

- Each member of the domain corresponds to one member in the **range**.
- Each member of the range corresponds to one member in the **domain**.
- Has an inverse which is a function.
- f maps a to b , and f^{-1} maps b to a .



Composition of Inverse Functions

If f and f^{-1} are inverse functions, then

$(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1} , respectively.

Let $f(x) = -2x - 3$. What is each of the following?

a). $f^{-1}(x)$ b). $(f \circ f^{-1})(x)$ c). $(f^{-1} \circ f)(x)$

$$\begin{array}{lll}
 x = -2y - 3 & = -2\left(\frac{x+3}{-2}\right) - 3 & = \frac{(-2x-3)+3}{-2} \\
 x+3 = -2y & = x+3-3 & = \frac{-2x}{-2} \\
 \frac{x+3}{-2} = y & = x & = x
 \end{array}$$

Proves whether a function is an inverse of another function.

Homework: p. 70 #1, 11, 13, 17, 21, 25,
27, 29, 33, 37, 45, 59, 61