1-6: Function Operations

Mr. Gallo

Precalculus

Function Operations

Addition	$(f+g)(x)=f(x)+g(x)$
Subtraction	$(f-g)(x)=f(x)-g(x)$
Multiplication	$(f \cdot g)(x)=f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$

The domains of the sum, difference, product and quotient functions consist of the ***x***-values that are in the domains of **<u>both</u>** ***f*** and ***g***. Also, the domain of the quotient function does not contain any ***x***-value for which $g(x)=0$.

Adding and Subtracting Functions

Let $f(x) = 5x^3 + 1$ and $g(x) = x^2 - 4$. What are $f + g$ and $f - g$? What are their domains?

$$(f+g)(x) = f(x) + g(x) = (5x^3+1) + (x^2-4) = 5x^3 + x^2 - 3$$

$$(f-g)(x) = f(x) - g(x) = (5x^3+1) - (x^2-4) = 5x^3 - x^2 + 5$$

The domains of both functions are all real numbers.

Complete Guided Practice 1A on p.58 for $(f+g)(x)$ and $(f-g)(x)$

$(f+g)(x) = x - 4 + \sqrt{9-x^2}$ Domain: $[-3, 3]$

$(f-g)(x) = x - 4 - \sqrt{9-x^2}$ Domain: $[-3, 3]$

Multiplying and Dividing Functions

Let $f(x) = x^2 + x - 6$ and $g(x) = x - 2$. What are $f \cdot g$ and f/g? What are their domains?

$$(f \cdot g)(x) = f(x) \cdot g(x) = (x^2+x-6)(x-2) = x^3 - x^2 - 8x + 12$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(x^2+x-6)}{(x-2)} = x + 3$$

The domain of $(f \cdot g)(x)$ is all real numbers;
The domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers except $x = 2$.

Complete Guided Practice 1A on p.58 for $(f \cdot g)(x)$ and $(f \div g)(x)$

$(f \cdot g)(x) = x\sqrt{9-x^2} - 4\sqrt{9-x^2}$ Domain: $[-3, 3]$

$\left(\frac{f}{g}\right)(x) = \frac{x-4}{\sqrt{9-x^2}}$ Domain: $(-3, 3)$

Homework: p. 61 #1-13 odd, 14, 61-66

Composite Function

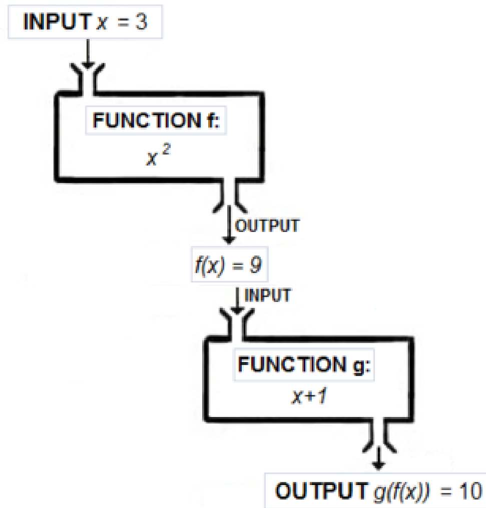
- The **composite $g \circ f$** of two functions *f* and *g* is the function that maps *x* onto *g(f(x))*, and whose domain is the set of all values in the domain of *f* for which *f(x)* is in the domain of *g*.
  - *g(f(x))* can also be written as:

$$g \circ f(x) \qquad or \qquad (g \circ f)(x)$$

    - This notation tells the user to apply ___x___ to the function ___$f(x)$___ and then use the result to apply to the function ___$g(x)$___.

Let $g(x)=x+1$ and $f(x)=x^2$. Evaluate $g\circ f(3)$.

$$g(f(3))=g(3^2)$$
$$=9+1$$
$$=10$$



Given $f(x)=2x^2-1$ and $g(x)=x+3$, find each of the following:

a. $[f\circ g](x)$ $=2(x+3)^2-1=2x^2+12x+17$

b. $[g\circ f](x)$ $=(2x^2-1)+3=2x^2+2$

c. $[f\circ g](2)$ $=2(2)^2+12(2)+17=49$

Domains of Composed Functions

- When domains of f or g are restricted, the domain of $f \circ g$ is restricted to all x values in the domain of g whose range values, $g(x)$ are in the domain of f.

Find $f \circ g$ and its domain.

$$f(x)=\sqrt{x-1} \qquad g(x)=(x-1)^2$$

$$[f \circ g](x)=\sqrt{(x-1)^2-1}=\sqrt{x^2-2x}$$

$$D=(-\infty,0]\cup[2,\infty)$$

$$f(x)=\frac{1}{x} \qquad g(x)=\sqrt{x^2-1}$$

$$[f \circ g](x)=\frac{1}{\sqrt{x^2-1}}=\frac{\sqrt{x^2-1}}{x^2-1}$$

$$D=(-\infty,-1)\cup(1,\infty)$$

An animator starts with an image of a circle with a radius of 25 pixels. The animator then increases the radius by 10 pixels per second.

a. Find functions to model the radius (R) and area (A).

$$R(t)=25+10t \qquad A(R)=\pi R^2$$

b. Find $A \circ R$. What does the function represent?

$$A(R(t))=\pi(25+10t)^2=100\pi t^2+500\pi t+625\pi$$

The area of the circle as a function of time.

c. How long does it take for the circle to quadruple its original size? $\text{initial}=625\pi \quad \text{quad}=2500\pi$

$$2500\pi=100\pi t^2+500\pi+625\pi$$

$$2500=100t^2+500+625$$

$$0=100t^2+500-1875 \qquad t=2.5 \text{ seconds}$$

Is Function Composition Commutative?

Let $g(x) = \frac{1}{x}$ and $f(x) = 2x - 1$.

Find $f(g(5))$

$g(5) = \frac{1}{5}$

$f\left(\frac{1}{5}\right) = 2\left(\frac{1}{5}\right) - 1 = -\frac{3}{5}$

Find $g(f(5))$

$f(5) = 2(5) - 1 = 9$

$g(9) = \frac{1}{9}$

Composition of functions is **not** commutative.
It matters which function you evaluate first.

Decomposing a Composite Function

- Separating a function into two simpler functions
  - When composed, the two simpler functions create the original function

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$.

a. $h(x) = \frac{1}{(x+2)^2}$

$f(x) = \frac{1}{x^2}$

$g(x) = x + 2$

b. $h(x) = 3x^2 - 12x + 12$

$f(x) = 3x^2$

$g(x) = x - 2$

Homework: p. 61 #15-29 odd, 35, 37, 40, 42, 53, 55