1-4: Extrema and Average Rates of Change

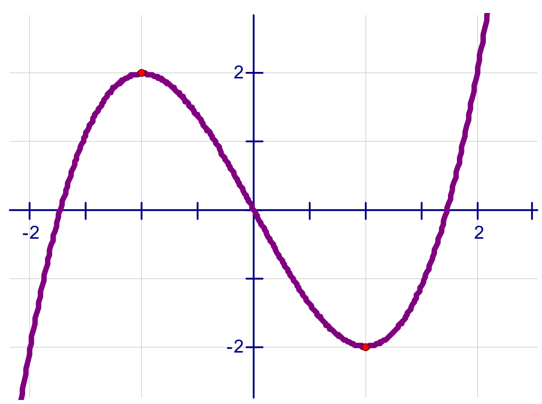
ADVANCED PRECALCULUS

MR. GALLO

Increasing, Decreasing, Positive and Negative Behavior

Behavior of the y values over specific intervals of x values.

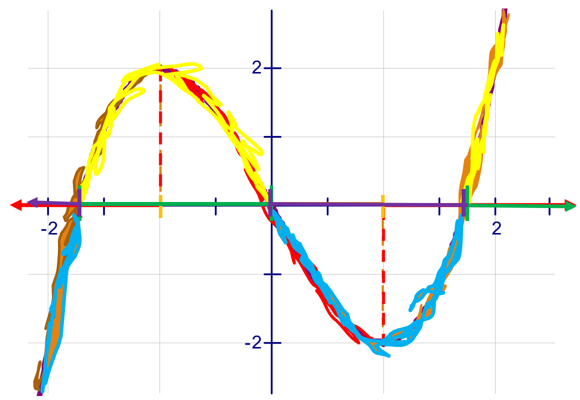
- Increasing
  - As x values ***increase***, y values ***increase***.
- Decreasing
  - As x values ***increase***, y values ***decrease***.
- Positive
  - The y values are positive
  - The graph is ***above*** the x-axis
- Negative
  - The y values are negative
  - The graph is ***below*** the x-axis.



Increasing, Decreasing, Positive and Negative Behavior

Identify the intervals the function:

1. Increases
   $(-\infty,-1)$ and $(1,\infty)$
2. Decreases
   $(-1,1)$
3. Is Positive
   $(-1.73,0)$ and $(1.73,\infty)$
4. Is Negative
   $(-\infty,-1.73)$ and $(0,1.73)$



Important Points of a Function

Critical Points

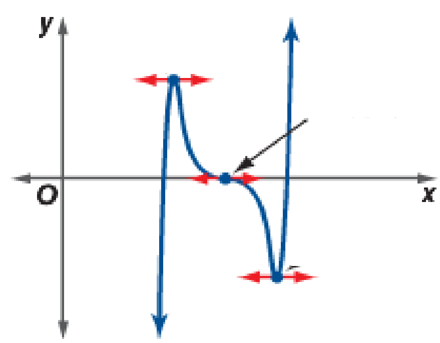
- Points at which a tangent line to the curve would be horizontal or vertical

Extrema

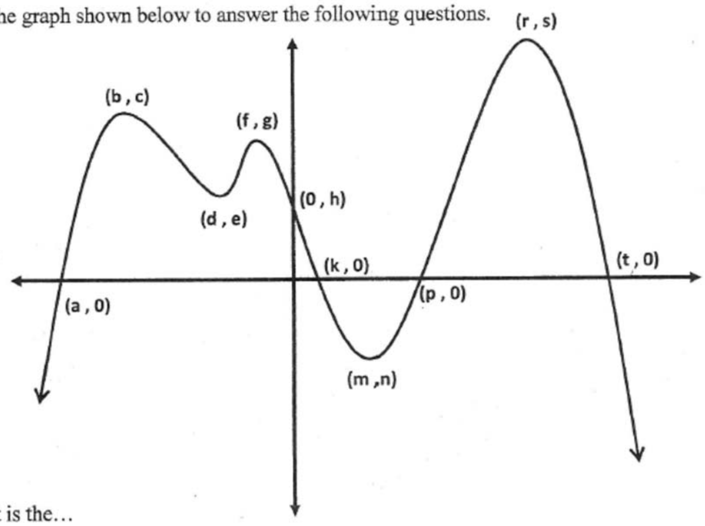
- Point at which function changes its increasing or decreasing behavior
  - Absolute Maximum
  - Relative Maximum
  - Absolute Minimum
  - Relative Minimum

Point of Inflection

- Point where graph changes shape
- Doesn't change increasing or decreasing behavior



Ex. 1: Use the graph shown below to answer the following questions.



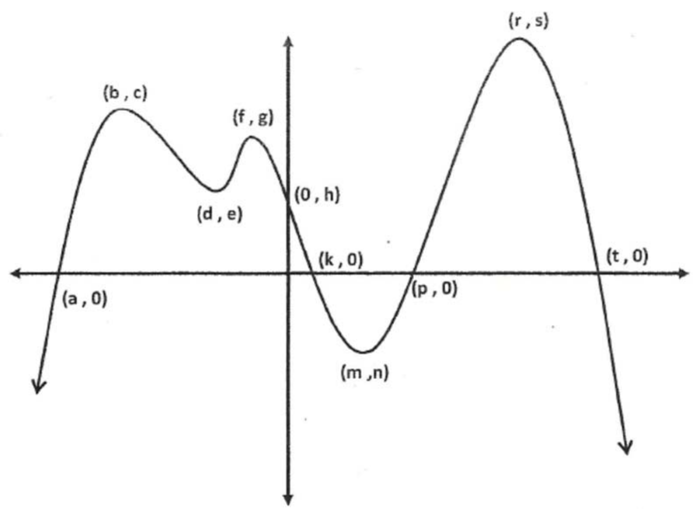
What is the…

Absolute max: $y=s$

Absolute min: $None$

Relative max: $y=c$ $y=g$ $y=s$

Relative min: $y=e$ $y=n$



Where is the graph…

Positive: (a,k) (p,t)

Negative: $(-\infty,a)(k,p)(t,\infty)$

Increasing: $(-\infty,b)(d,f)(m,r)$

Decreasing: $(b,d)(f,m)(r,\infty)$

A designer wants to make a rectangular prism box with maximum volume, while keeping the sum of its length, width and height 12 in. The length must be 3 times the height. What should each dimension be?

x = height of box 3*x* = length of box

12-(x+3x) = width of box

$$V = l \times w \times h$$

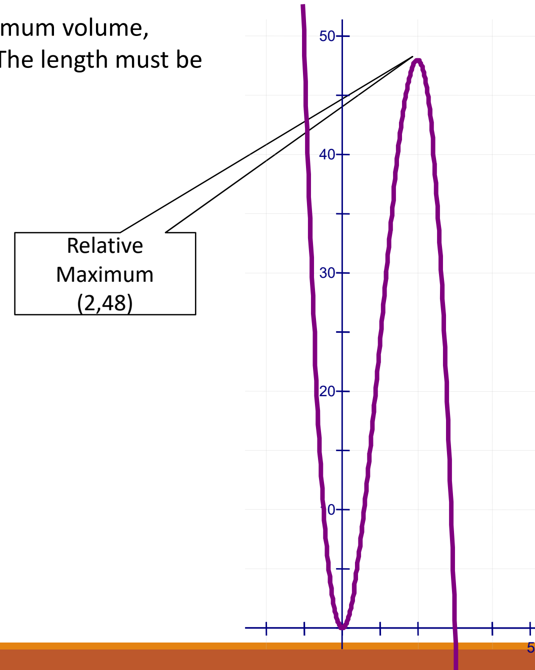
$$V = x(3x)(12-(x+3x))$$

$$V = x(3x)(12-4x)$$

$$V = 3x^2(12-4x)$$

$$V = 36x^2 - 12x^3$$

2 in.= height
6 in.= length
4 in = width



Average Rate of Change

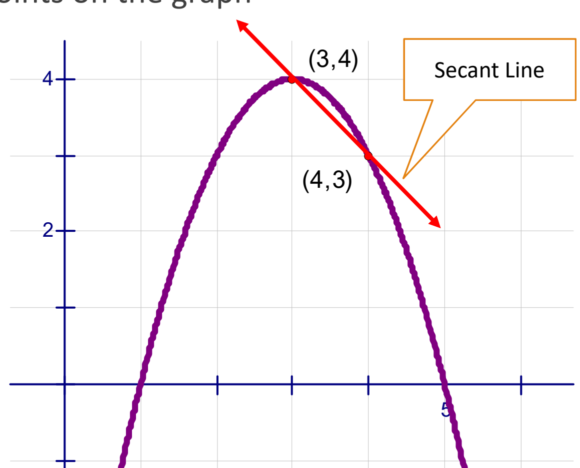
Used for nonlinear functions

- Slope of the line between any two points on the graph
  - Secant Line

$$m_{\sec ant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$m_{\sec ant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{3-4}{4-3} = -1$$

The average rate of change between (3,4) and (4,3) is -1.

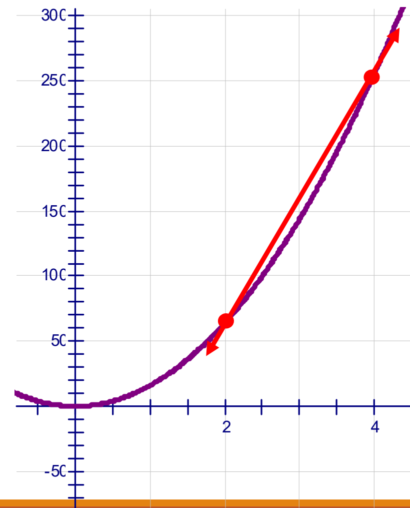


If wind resistance is ignored, the distance $d(t)$ in feet an object travels when dropped from a high place is given by $d(t) = 16t^2$, where t is the time in seconds after the object is dropped. Find and interpret the average speed of the object from 2 to 4 seconds.

$$m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{256 - 64}{4 - 2} = 96$$

The average speed of the object is 96 ft/sec.

The distance the object traveled increased over that time interval.



The formula for the distance traveled by falling objects on the Moon is $d(t) = 2.7t^2$, where $d(t)$ is the distance in feet and t is the time in seconds. Find and interpret the average speed of the object for each time interval.

a. 1 to 2 seconds

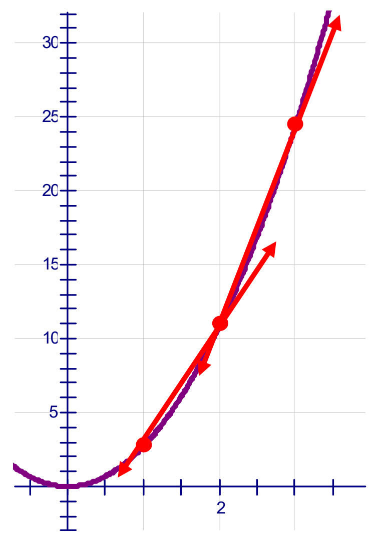
$$m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{10.8 - 2.7}{2 - 1} = 8.1$$

The average speed of the object is 8.1 ft/sec.

b. 2 to 3 seconds

$$m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{24.3 - 10.8}{3 - 2} = 13.5$$

The average speed of the object is 13.5 ft/sec.



Homework: p.41 #1-45 (every other odd), 46, 48