Realtors in a metropolitan area student the average home price per square foot as a function of total square footage. Their evaluation yielded the following piecewise-defined function. Find the average price per square foot for a home with the given square footage.

$$p(a)=\begin{cases}\dfrac{a-1000}{40}+75 & \text{If } 1000 \le a < 2600\\ \dfrac{-(a-2600)}{100}+110 & \text{If } 2600 \le a < 4000\\ \dfrac{a-4000}{25}+98 & \text{If } a \ge 4000\end{cases}$$

a). 1400 square feet

$$p(1400)=\frac{1400-1000}{40}+75$$
$$=\$85$$

b). 3200 square feet

$$p(3200)=\frac{-(3200-2600)}{100}+110$$
$$=\$104$$

1-2: Analyzing Graphs of Functions and Relations

Precalculus

Mr. Gallo

Analyzing Function Graphs

- You should be able to:
  1. Estimate a value from the graph.
  2. Find the domain and range from the graph.
  3. Find the y-intercepts.
  4. Find the zeroes from the graph.

The function $f(x) = -5x^2 + 50x$ approximates the profit at a toy company, where x is the marketing costs and $f(x)$ is profit. Both costs and profits are measured in tens of thousands of dollars.

a. Use the graph to estimate the profit when marketing costs are $30,000. Confirm your estimate algebraically.

$f(30,000) = \$1,050,000$

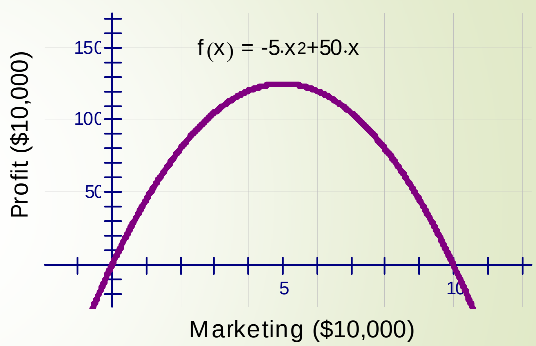
b. What is the domain and range of the function?

D: $[0, 100,000]$ R: $[0, 125,000]$

c. Use the graph to estimate the y-intercept of the function. Confirm your estimate algebraically.

$y = 0$

d. Use the graph to estimate the zeroes of the function. Confirm your estimate algebraically. $x = 0$ and $x = 100,000$

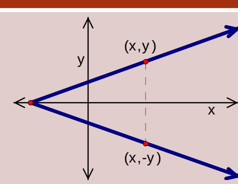
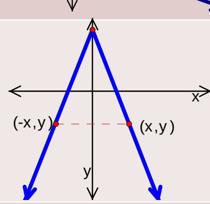
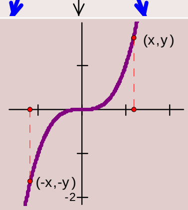


Types of Symmetries

- Line Symmetry
  - Can be folded along a line so the two halves match exactly
- Point Symmetry
  - Can be rotated 180° with respect to a point and appear unchanged.

- Three types of symmetries:
  1. With respect to the x–axis
  2. With respect to the y–axis
  3. With respect to the **origin**

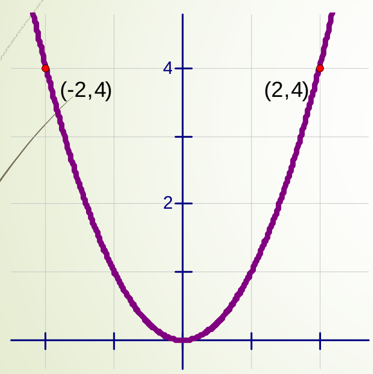
Tests for Symmetry (on p.16 in book)

Graphical Test	Model	Algebraic Test
Symmetric with respect to the *x*-axis iff. for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph		Replacing y with $-y$ produces an equivalent equation.
Symmetric with respect to the *y*-axis iff. for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.		Replacing x with $-x$ produces an equivalent equation.
Symmetric with respect to the origin iff. for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.		Replacing x with $-x$ and y with $-y$ produces an equivalent equation.

Show that $y = x^2$ is symmetric to the y-axis-graphically and algebraically.

If symmetric to the y-axis, then $(-x)$ will produced the same value as (x).

Graphically:



Algebraically:

$y = x^2$

$f(x) = x^2$

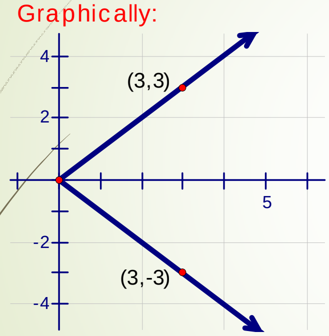
$$f(-x) = (-x)^2 = (-1 \cdot x)^2$$
$$= (-1)^2 (x)^2$$
$$= 1 \cdot x^2$$
$$= x^2$$

The answers are the same so $y = x^2$ is symmetric with respect to the y-axis.

Show that $x = |y|$ is symmetric to the x-axis-graphically and algebraically.

If symmetric to the x-axis, then $(-y)$ will produced the same value as (y).

Graphically:



Algebraically:

$x = |y|$

$f(y) = |y|$

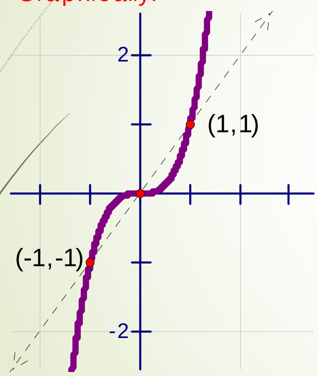
$$x = |y|$$
$$f(y) = |y|$$

The answers are the same so $x = |y|$ is symmetric with respect to the x-axis.

Show that $y = x^3$ is symmetric to the origin-graphically and algebraically.

If symmetric to the origin, then $(-x)$ will produced $(-y)$.

Graphically:



Algebraically:

$y = x^3$

$f(x) = x^3$

$$y = x^3$$

$$f(x) = x^3$$

The answers are the opposites so $y = x^3$ is symmetric with respect to the origin.

Show that $y = 2x^5 + x^3 + x$ is symmetric to the origin-algebraically.

If symmetric to the origin, then $(-x)$ will produced $(-y)$.

Algebraically:

$f(x) = 2x^5 + x^3 + x$

$$f(-x) = 2(-x)^5 + (-x)^3 + (-x)$$
$$= 2(-x^5) + (-x^3) + (-x)$$
$$= -2x^5 - x^3 - x$$

factor out a -1:

$$= -(2x^5 + x^3 + x)$$

The answers are the opposites so $y = 2x^5 + x^3 + x$ is symmetric with respect to the origin.

Odd and Even Functions

- Power Functions
  - Functions which take the form $y = x^n$, where n is an integer and $n \geq 2$.
- Even and Odd Functions

Type of Function	Algebraic Test
Functions which are symmetric with respect to the y-axis, are called ***Even Functions***.	For every x in the domain of f, $\boldsymbol{f(-x) = f(x)}$
Functions which are symmetric with respect to the origin, are called ***Odd Functions***.	For every x in the domain of f, $\boldsymbol{f(-x) = -f(x)}$

Prove that the following is an even or odd function or neither.

$$f(x) = x^3 - 2x$$

$$f(x) = x^3 - 2x$$

$$f(-x) = (-x)^3 - 2(-x)$$

$$= -x^3 + 2x$$

$$= -(x^3 - 2x)$$

The answer is the opposite of f(x), therefore the function is odd.

Not every function with the highest exponent being even is an even function.

a. Show by counterexample that f(x) = x4 + x is not an even function.

$$f(1) = 1^4 + 1 = 2$$

$$f(-1) = (-1)^4 + (-1) = 1 - 1 = 0$$

The answers are not the same so the function isn't even.

b. Prove that f(x) = x4 + x is not an even function.

$$f(x) = x^4 + x$$

$$f(-x) = (-x)^4 + (-x)$$

$$= x^4 - x$$

The answers are not the same so the function isn't even.

Homework: p.19 #7, 9, 11, 14, 15-41 odd, 45