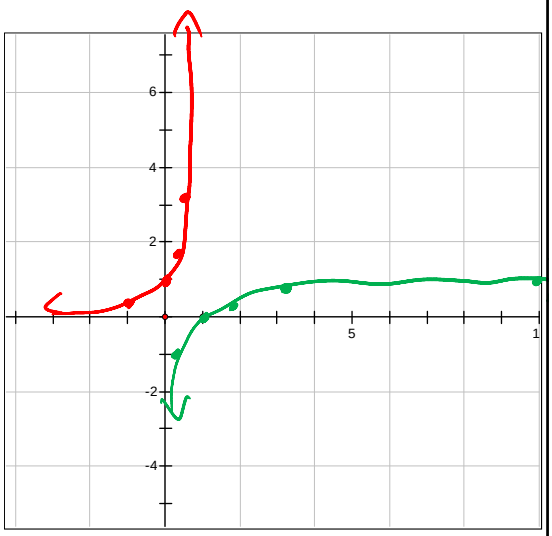
7-3: Logarithmic Functions as Inverses

Algebra 2
Mr. Gallo

Given $y = 10^x$, find and graph its inverse. Inverse is $x = 10^y$

Plot points on a grid. Find $x = 10^y$ simply by switching (x, y) from 10^x.

x	$y = 10^x$	$x = 10^y$
-1	(-1, .1)	(.1, -1)
0	(0, 1)	(1, 0)
0.25	(.25, 1.7)	(1.7, .25)
0.5	(.5, 3.1)	(3.1, .5)
1	(1, 10)	(10, 1)
1.5	(1.5, 31.6)	(31.6, 1.5)
2	(2, 100)	(100, 2)



You have just graphed $y = log_{10}x$. That is the inverse of $y=10^x$.

Logarithm

- Exponent to which ***b*** must be raised to get ***x***.
- Logarithm is inverse of exponential function.
  - Used to rewrite exponential functions so they can be evaluated.

Logarithm base ***b*** of a positive number ***x*** satisfies the following definition.

For $b > 0, b \neq 1, log_b x = y$ if and only if $b^y = x$.

(Read log base ***b*** of ***x***)

Writing Exponentials in Logarithmic Form

What is the logarithmic form of each equation?

$1 = 8^0$ $\qquad$ $64 = 4^3$

Use the definition of logarithm to rewrite the equation.

If $x = b^y$, then $log_b x = y$ $\qquad$ $1 = 8^0$ $\qquad$ $\log_8 1 = 0$

$64 = 4^3$ $\qquad$ $\log_4 64 = 3$

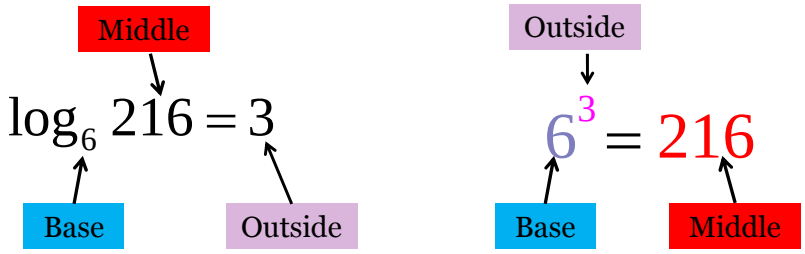
Complete Got It? #1 p.452

a. $log_6 36 = 2$ $\qquad$ b. $log_{\frac{2}{3}} \frac{8}{27} = 3$ $\qquad$ c. $log_3 1 = 0$

Writing Logarithms in Exponential Form

- Use the BOM method

B -Base
O - to the Outside
M -equals the Middle



Complete WS 7-3 K #4-6

Evaluating a Logarithm

1. Write a logarithmic equation
2. Write in exponential form and evaluate

1). $\log_{10} 1000$

$\log_{10} 1000 = x$

$10^x = 1000$

$10^x = 10^3$

$x = 3$

2). $\log_{10} \sqrt[3]{10}$

$\log_{10} \sqrt[3]{10} = x$

$10^x = \sqrt[3]{10}$

$10^x = 10^{\frac{1}{3}}$

$x = \frac{1}{3}$

3). $\log_4 32$

$\log_4 32 = x$

$4^x = 32$

$\left(2^2\right)^x = 2^5$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2}$

Complete WS 7-3 K #7-9

Common Logarithms

- A logarithm with a base of 10

  $\log_{10} 1000$ $\log_{10} \sqrt[3]{10}$

- Usually written without showing the 10

  $\log 1000$ $\log \sqrt[3]{10}$

- Evaluate with the LOG button on the calculator

  $\log 1000$ In Calculator: $log(1000)$

Logarithmic Scales

- Used to cover a wide range of values
- Reported measurements logs of values not values themselves
  - Richter Scale, Decibel Scale, pH scale are all logarithmic scales
- pH Scale
  - pH of a substance can range from 0 to 14
  - Formula: $pH = -\log(H^+)$

H^+ stands for the concentration of hydrogen ions (in moles/liter) of the substance.

If pH< 7 it is acidic
pH = 7 it is neutral
pH > 7 it is alkaline

Seawater has a pH of 8.5

a) Is seawater acidic or alkaline? Because $7 < 8.5$

b) What is the concentration of hydrogen ions in seawater?

$$pH = -\log(H+)$$
$$8.5 = -\log(H+)$$
$$-8.5 = \log(H+)$$
$$10^{-8.5} = H+$$
$$.000000003162 = H+$$

c) Rewrite your answer to part b in scientific notation

3.162×10^{-9} *moles / liter*

Complete Got It? #3 p.453 ≈16 times

Homework: p. 456 #13-35 odd, 47-53 odd, 61-67 odd

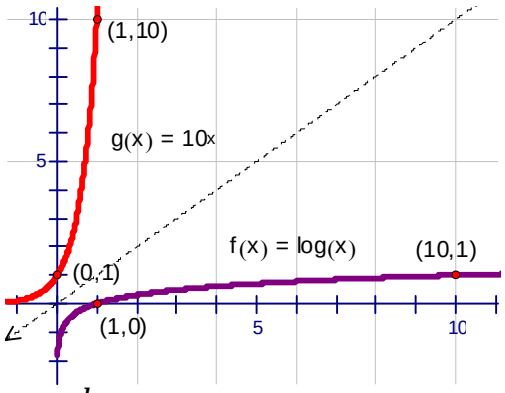
Graphing a Logarithmic Function

$y = log_b x$ is the *inverse* of the exponential function $y = b^x$

$y = 10^x$

Domain: All Reals

Range: $y > 0$



$y = log_b x$

Domain: $x > 0$

Range: All Reals

$y = log_b x$

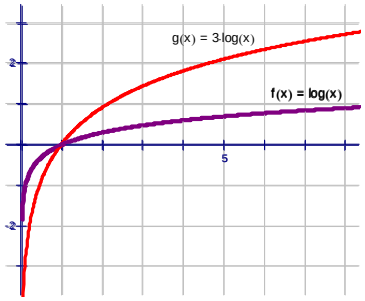
No y-intercept

y=axis is an asymptote

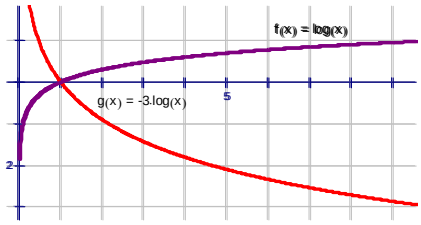
Families of Logarithmic Functions

Families of Logarithmic Functions	
Parent Function	$y = \log_b x; b > 0, b \neq 1$
Stretch ($\|a\| > 1$) Compression (Shrink) ($0 < \|a\| < 1$) Reflection($a < 0$) in x-axis	$y = a \log_b x$
Translations (Horizontal by h; Vertical by k)	$y = \log_b (x - h) + k$
All transformations combined	$y = a \log_b (x - h) + k$

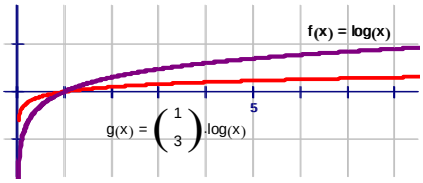
Families of Logarithmic Functions



Stretch
($|a| > 1$)

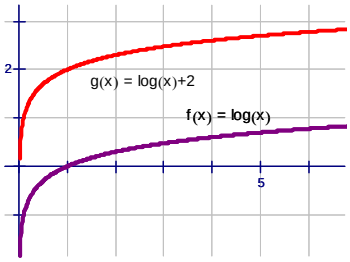


Reflection
($a < 0$)

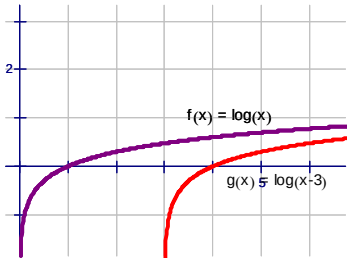


Compression
($0 < |a| < 1$)

Families of Logarithmic Functions



Vertical
Translation
$y = log_b x + k$



Horizontal
Translation
$y = log_b(x - h)$

All Transformations combined gives the form:
$y = alog_b(x - h) + k$

Homework: p. 456 #40-45, 85-94