7-1: Exploring Exponential Models

Algebra 2
Mr. Gallo

You have just won the grand prize on a game show. The game show host tells you that you may choose from one of two cash prizes, but you only have 30 seconds to decide. Your choices are:

Choice #1: $10,000 per day for 30 days.
Choice #2: $.01 the first day. Each of the following day's pay is double the pay of the previous day.

Which choice do you make?

Choice #1: $10,000 per day for 30 days

$10,000 x 30 days = $300,000

Choice #2: Begin with 1¢ and double the amount each day

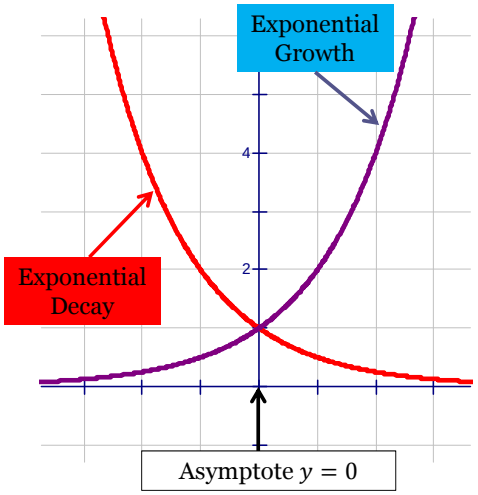
1	$0.01		$0.01
2	$0.02		$0.03
3	$0.04		$0.07
4	$0.08		$0.15
5	$0.16		$0.31
6	$0.32		$0.63
7	$0.64		$1.27
8	$1.28		$2.55
9	$2.56		$5.11
10	$5.12		$10.23
11	$10.24		$20.47
12	$20.48		$40.95
13	$40.96		$81.91
14	$81.92		$163.83
15	$163.84		$327.67
16	$327.68		$655.35
17	$655.36		$1,310.71
18	$1,310.72		$2,621.43
19	$2,621.44		$5,242.87
20	$5,242.88		$10,485.75
21	$10,485.76		$20,971.51
22	$20,971.52		$41,943.03
23	$41,943.04		$83,886.07
24	$83,886.08		$167,772.15
25	$167,772.16		$335,544.31
26	$335,544.32		$671,088.63
27	$671,088.64		$1,342,177.27
28	$1,342,177.28		$2,684,354.55
29	$2,684,354.56		$5,368,709.11
30	$5,368,709.12		$10,737,418.23

If you begin with the penny and then double it each day, you end up with $10.7 million dollars after 30 days.

Choice 2 would be the correct choice.

Exponential Function

- The previous example was an exponential function.
- Types:
  - Exponential Growth
    as ***x*** increases, ***y*** increases
  - Exponential Decay
    as ***x*** increases, ***y*** decreases approaching zero
- Asymptote
  - Line the graph approaches but never reaches.



Exponential Function

For the function $y = ab^x$,

- if $a > 0$ and $b > 1$, the function represents:

exponential growth

- if $a > 0$ and $0 < b < 1$, the function represents:

exponential decay

In either case, the y-intercept is $(0, a)$, the domain is ***all real numbers***, the asymptote is $y = 0$ and the range is $\{y: y > 0\}$

Identify $y = 0.7^x$ as an example of exponential growth or decay.
What is the ***y***- intercept? $a = 1$ $b = 0.7$

Since $a > 0$ and $0 < b < 1$, this is decay. | y-intercept is (0,1)

Exponential Growth and Decay

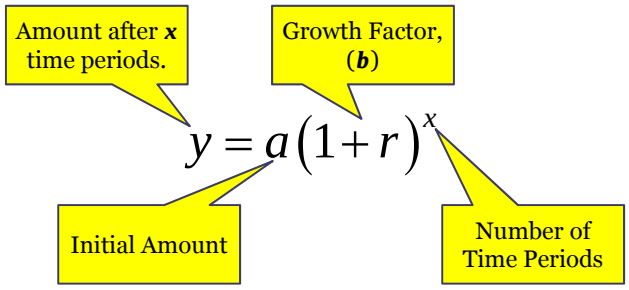
In the equation $y = ab^x$,

- ***a*** is the initial amount.
  - Exponential Growth: $b > 1$
    - ***b*** is the growth factor.
      - Increase written as a decimal is ***r***, rate of increase or growth rate. $b = 1 + r$ for exponential growth
  - Exponential Decay: $0 < b < 1$
    - ***b*** is the decay factor.
    - Decay written as a decimal is ***r***, rate of decay.

$b = 1 + r$ for exponential decay because ***r*** is expressed as a negative quantity.

Exponential Growth or Decay

You can model exponential growth or decay using this model:



For growth or decay to be *exponential*, a quantity changes by a fixed percentage each time period.

You buy a savings bond for $25 that pays a yearly interest rate of 4.2%. What will the savings bond be worth after fifteen years?

Let ***t***= number of years since the money was invested.
Let ***A(t)***= amount in account after ***t*** years.

$A(t) = a(1+r)^t$ $\quad a = 25 \quad r = .042 \quad x = 15$

$$A(t) = 25(1+.042)^{15}$$
$$A(t) = 25(1.042)^{15}$$
$$A(t) \approx 46.339$$

The savings bond will be worth $46.34.

Complete Got It? #3 p.436 It will be worth $593.84

Exponential Functions

- Are often *discrete*.
  - Not a continuous graph
- To model using $y = ab^x$,
  - Use two **consecutive** y values to find r:

$$r = \frac{y_2 - y_1}{y_1}$$

  - This is the percent increase or decrease.

The initial value of a car is \$30,000. After one year, the value of the car is \$20,000. Estimate the value of the car after five years?

Let ***t***= number of years since the car was bought.
Let ***A(t)***= value of the car after ***t*** years.

$A(t) = a(1+r)^t$

$a = 25$ $\quad r = \frac{y_2 - y_1}{y_1} = \frac{20,000 - 30,000}{30,000} = -.33$ $\quad x = 5$

$$A(t) = 30,000\left(1 + \left(\frac{20k - 30k}{30k}\right)\right)^5$$

$A(t) \approx 3950.38$ More Precise

$$A(t) = 30,000(1 + (-.33))^5$$
$$A(t) = 30,000(.67)^5$$
$$A(t) \approx 4050.38$$

The car will be worth \$3950.38.

Complete Got It? #5 p.438 a). ≈3 b). No, the pop. would be negative.

Homework: p. 439 #10, 16, 19-25 odd, 26-29, 31, 32, 46-48, 54-56