6-6: Function Operations

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Algebra 2

Function Operations

Addition	$(f+g)(x)=f(x)+g(x)$
Subtraction	$(f-g)(x)=f(x)-g(x)$
Multiplication	$(f\cdot g)(x)=f(x)\cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$

The domains of the sum, difference, product and quotient functions consist of the ***x***-values that are in the domains of **<u>both</u>** ***f*** and ***g***. Also, the domain of the quotient function does not contain any ***x***-value for which $g(x) = 0$.

Adding and Subtracting Functions

Let $f(x) = 5x^3 + 1$ and $g(x) = x^2 - 4$. What are $f + g$ and $f - g$? What are their domains?

$$(f+g)(x) = f(x) + g(x) = (5x^3+1) + (x^2-4) = 5x^3 + x^2 - 3$$

$$(f-g)(x) = f(x) - g(x) = (5x^3+1) - (x^2-4) = 5x^3 - x^2 + 5$$

The domains of both functions are all real numbers.

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$(f+g)(x) = 2x^2 + x + 5$ Domain: All Reals

$(f-g)(x) = 2x^2 - x + 11$ Domain: All Reals

Multiplying and Dividing Functions

Let $f(x) = x^2 + x - 6$ and $g(x) = x - 2$. What are $f \cdot g$ and f/g? What are their domains?

$$(f \bullet g)(x) = f(x) \bullet g(x) = (x^2+x-6)(x-2) = x^3 - x^2 - 8x + 12$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(x^2+x-6)}{(x-2)} = x + 3$$

The domain of $(f \cdot g)(x)$ is all real numbers;
The domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers except $x = 2$.

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$(f \bullet g)(x) = 9x^3 - 30x^2 - 23x - 4$ Domain: All Reals

$\left(\frac{f}{g}\right)(x) = x - 4$ Domain: All Reals except $x = -\frac{1}{3}$

Homework: p. 401 #9-25 odd, 51, 53, 91-99 odd

Composite Function

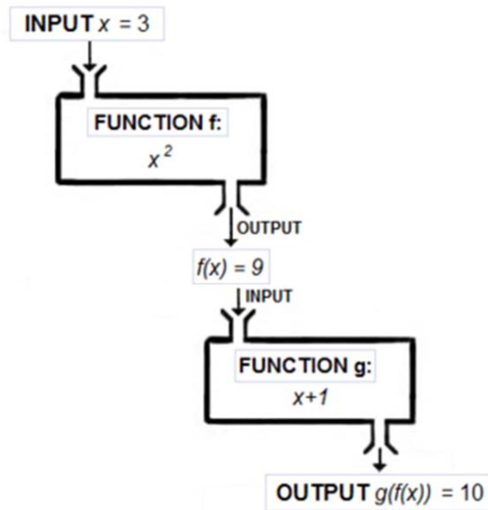
- The **composite** $\boldsymbol{g \circ f}$ of two functions *f* and *g* is the function that maps *x* onto *g(f(x))*, and whose domain is the set of all values in the domain of *f* for which *f(x)* is in the domain of *g*.
  - *g(f(x))* can also be written as:

$$g \circ f(x) \qquad or \qquad (g \circ f)(x)$$

    - This notation tells the user to apply ___ *x* ___ to the function __ $f(x)$ __ and then use the result to apply to the function __ $g(x)$ ____.

Let $g(x) = x + 1$ and $f(x) = x^2$. Evaluate $g \circ f(3)$.

$$g(f(3)) = g(3^2)$$
$$= 9 + 1$$
$$= 10$$



A local car dealer combines two promotions -- they will give you both a 10% discount and a $1,000 cash back. Does it matter in what order they do them?

Suppose you pick out a new car with a sticker price of $20,000. (Remember that a 10% discount means you're paying 90% of the sticker price)

Rebate first, then Discount

$$.90(20,000 - 1000) = 17,100$$

Discount first, then Rebate

$$.90(20,000) - 1000 = 17,000$$

Function Notation

We can do this problem by setting up FUNCTIONS:

1. Let ***x*** = the sticker price of the car $x = 20,000$
2. Let ***r*** be the rebate function $r(x) = x - 1000$
3. Let ***d*** be the discount function $d(x) = .90x$

Rebate first, then Discount

$$d(r(x)) = d(x - 1000)$$
$$= .9(x - 1000)$$
$$= .9x - 900$$
$$.9(20,000) - 900 = 17,100$$

Discount first, then Rebate

$$r(d(x)) = r(.90x)$$
$$= (.90x) - 1000$$
$$= .90x - 1000$$
$$.90(20,000) - 1000 = 17,000$$

Is Function Composition Commutative?

Let $g(x) = \frac{1}{x}$ and $f(x) = 2x - 1$.

Find $f(g(5))$

$$g(5) = \frac{1}{5}$$
$$f\left(\frac{1}{5}\right) = 2\left(\frac{1}{5}\right) - 1 = -\frac{3}{5}$$

Find $g(f(5))$

$$f(5) = 2(5) - 1 = 9$$
$$g(9) = \frac{1}{9}$$

Composition of functions is **not** commutative.
It matters which function you evaluate first.

Homework: p. 402 #27-45 odd, 56, 58, 69-75 odd, 90-98 even