6-4: Rational Exponents

Algebra 2
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Radical Notation for Roots

- The positive nth root of a ____**positive**________number ***x*** can be written as a power of ***x***, namely $x^{\frac{1}{n}}$
- This allows the properties of powers to be used. The allows $\sqrt{\ }$ all of the positive ***nth*** roots to be represented.

Definition

For any **nonnegative** real number x and any integer n $\geq$ 2, $\sqrt[n]{x} = x^{\frac{1}{n}}$

Example 1: Evaluate without using a calculator:

a) $\sqrt[4]{256}$

$256^{\frac{1}{4}}$

$= 4$

b) $\sqrt[3]{27}$

$27^{\frac{1}{3}}$

$= 3$

c) $\sqrt[6]{64}$

$64^{\frac{1}{6}}$

$= 2$

Root of a Power Theorem:
For all positive integers *m* >1 and *n* $\geq$ 2, and all nonnegative real numbers *x*,

$$x^{\frac{1}{n}} = \sqrt[n]{x} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Example 3: Simplify. Assume all variables are nonnegative.

a) $x^{\frac{8}{4}}$

$\sqrt[4]{x^8}$

$= x^4$

b) $y^{\frac{18}{3}}$

$\sqrt[3]{y^{18}}$

$= y^6$

c) $x^{\frac{24}{3}}$

$\sqrt[3]{x^{24}}$

$= x^8$

Properties of Rational Exponents

Property	Example
$a^m \times a^n = a^{m+n}$	$8^{\frac{1}{3}} \times 8^{\frac{2}{3}} = 8^{\frac{1}{3}+\frac{2}{3}} = 8^{\frac{3}{3}} = 8$
$\left(a^m\right)^n = a^{m(n)}$	$\left(5^{\frac{1}{2}}\right)^4 = 5^{\frac{1}{2}(4)} = 5^{\frac{4}{2}} = 5^2 = 25$
$(ab)^m = a^m b^m$	$(4 \bullet 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \bullet 5^{\frac{1}{2}} = 5^{\frac{4}{2}} = 2 \bullet 5^{\frac{1}{2}}$
$a^{-m} = \frac{1}{a^m}$	$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$

Properties of Rational Exponents

Property	Example
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^{\frac{3}{2}}}{7^{\frac{1}{2}}} = 7^{\frac{3}{2}-\frac{1}{2}} = 7^{\frac{2}{2}} = 7^1 = 7$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{27}\right)^{\frac{1}{3}} = \frac{5^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{5^{\frac{1}{3}}}{3}$

Radicals for Roots of Powers

- All properties of powers already discussed apply to radicals.
- What other expressions represent $x^{\frac{n}{m}}$?

$\left(x^{n}\right)^{\frac{1}{m}}$, $\left(x^{\frac{1}{m}}\right)^{n}$, $\left(\sqrt[m]{x}\right)^{n}$, $\sqrt[m]{x^{n}}$

Roots of Roots

By rewriting radicals as rational **exponents** we have a general way of dealing with roots of roots.

Given x $\geq$ 0

$$\sqrt{\sqrt{\sqrt{x}}} = \left(\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{1}{8}}$$

Example 4:

What is $\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$ in simplest form?

$$\frac{\sqrt{x^3}}{\sqrt[3]{x^2}} = \frac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}}$$

$$= x^{\frac{3}{2}-\frac{2}{3}}$$

$$= x^{\frac{9}{6}-\frac{4}{6}}$$

$$= x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

Example 5:

What is $(9y\sqrt{x})^{\frac{3}{2}}$ in simplest form?

$$\left(9y\sqrt{x}\right)^{\frac{3}{2}} = 9^{\frac{3}{2}} \bullet y^{\frac{3}{2}} \bullet \left(x^{\frac{1}{2}}\right)^{\frac{3}{2}}$$

$$= \left(\sqrt{9}\right)^{3} \bullet y^{\frac{3}{2}} \bullet x^{\frac{1}{2}\bullet\frac{3}{2}}$$

$$= 3^{3} \bullet y^{\frac{3}{2}} \bullet x^{\frac{3}{4}} = 27x^{\frac{3}{4}}y^{\frac{3}{2}}$$

Homework: p.386 #11, 15-19 odd, 24, 26, 28, 29, 33, 35, 38-40, 44, 45, 48, 50, 54, 55, 60, 65