

The Mathematics of Finance (Reference Sheet)

Principal = lump sum deposit

Interest Compounded Annually

If a principal P is invested at a fixed annual interest rate r , calculated at the end of each year, then the value of the investment as a function of t years is

$$A(t) = P(1 + r)^t$$

Interest Compounded k Times per Year

If a principal P is invested at a fixed rate r and interest is compounded k times each year, then the value of the investment as a function of t years is

$$A(t) = P \left(1 + \frac{r}{k} \right)^{kt}$$

Interest Compounded Continuously

If a principal P is invested at a fixed rate r and interest is compounded continuously, then the value of the investment as a function of t years is

$$A(t) = Pe^{rt}$$

Annual Percentage Yield (APY): the percentage rate, compounded annually, that would result in the same amount of interest as the annual interest rate with the given compounding rate.

$$AY = 1 \left(1 + \frac{r}{k} \right)^k - 1 \quad \text{or} \quad AY = e^r - 1$$

Annuities – Future Value

Annuity: a sequence of equal periodic payments.

Ordinary Annuity: payments are made at the end of each period at the same time the interest is posted.

Future Value: the total value of the investment returned from an annuity.

$$\text{Future Value} = \text{Total of Monthly Payments} + \text{Total Interest earned}$$

Future Value of an Annuity

The future value FV of an annuity consisting of n equal periodic payments of R dollars at an interest rate i per compounding period (payment interval, usually monthly) is:

$$FV = R \frac{(1+i)^n - 1}{i}; \quad i = \frac{r}{k}; \quad n = \text{TOTAL number of payments, usually } 12t$$

Loans and Mortgages – Present Value

Mortgage: bank loan used to buy a house

Present Value: the net amount of money put into an annuity.

$$\text{Present Value} = \text{Total Loan} + \text{Total Interest Paid on the Loan}$$

Present Value of an Annuity

The present value PV of an annuity consisting of n equal payments of R dollars earning an interest rate I per period (payment interval, usually monthly) is

$$PV = R \frac{1 - (1+i)^{-n}}{i}; \quad i = \frac{r}{k}; \quad n = \text{TOTAL number of payments, usually } 12t$$

Annual Percentage Rate (APR) = the annual interest rate charged on consumer loans