Limits

DAY 2

I. Computational Techniques for Limits

A.) Properties of Limits: If *L*, *M*, *c*, and *k* are real numbers and

$$\lim_{x \to c} f(x) = L \text{ and } \lim_{x \to c} g(x) = M$$, then

1.) ***Sum Rule***: $\lim_{x \to c} \left(f(x) + g(x) \right) = L + M$

2.) ***Difference Rule***: $\lim_{x \to c} \left(f(x) - g(x) \right) = L - M$

3.) ***Product Rule***: $\lim_{x \to c} \left(f(x) \bullet g(x) \right) = L \bullet M$

4.) ***Constant Multiple Rule*:** $\lim_{x\to c}\left(k \bullet f(x)\right) = k \bullet L$

5.) ***Quotient Rule*:** $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}$ $, M \neq 0$

B.) Theorem – Polynomial and Rational Functions-

1.) If $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_0$ is any polynomial fn and c is any real number, then

$$\lim_{x\to c} f(x) = a_n c^n + a_{n-1}c^{n-1} + ... + a_0$$

2.) If $f(x)$ and $g(x)$ are polynomial fns and c is any real number, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}, \text{ if } g(x) \neq 0$$

C.) Limit Properties and Theorems valid for one-sided limits as well.

II. Techniques for Evaluation

A.) Apply the theorems/properties (Substitute)

B.) Modify the expression:

1.) Simplify

2.) Factor

3.) Expand

4.) Multiply by conjugates

C.) Apply thm./prop. again

C.) Evaluate the following limits:

1.) $\lim_{x\to 2} \frac{x^2-4}{x+1} = \frac{2^2-4}{2+1} = \frac{0}{3} = 0$

2.) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{0}{0} = ???$

$$\lim_{x\to 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2} x+2 = 2+2 = 4$$

3.) $\lim_{x\to 2} \frac{x^2-4}{x+2} = \frac{2^2-4}{2+2} = \frac{0}{4} = 0$

4.) $\lim_{x\to 2} \frac{x+2}{x^2-4} = \frac{2+2}{2^2-4} = \frac{4}{0} = \text{DNE}$

$$\lim_{x\to 2^+} \frac{x+2}{x^2-4} = \frac{4}{0^+} = \infty$$

$$\lim_{x\to 2^-} \frac{x+2}{x^2-4} = \frac{4}{0^-} = -\infty$$

5.) $\lim_{x\to 2} \frac{x-3}{(x-2)^2(x+5)} = \frac{-1}{0} = \text{DNE}$

$$\lim_{x\to 2^-} \frac{x-3}{(x-2)^2(x+5)} =$$

$$\lim_{x\to 2^+} \frac{x-3}{(x-2)^2(x+5)} = \frac{-1}{0^+} = -\infty$$

6.) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} = ???$

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{\sqrt{x}-2} =$$

$$\lim_{x\to 4} \sqrt{x}+2 = \sqrt{4}+2 = 4$$

7.) $\lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2}}{x} = \dfrac{0}{0} = ???$

$$\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right) =$$

$$\lim_{x\to 0} \frac{x}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \lim_{x\to 0} \frac{1}{\sqrt{x+2}+\sqrt{2}} =$$

$$\frac{1}{2\sqrt{2}}$$

8.) $\lim_{x\to 4} \dfrac{(3+x)^2-49}{x-4} = \dfrac{0}{0} = ???$

$$\lim_{x\to 4} \frac{(3+x-7)(3+x+7)}{x-4} =$$

$$\lim_{x\to 4} \frac{(x-4)(x+10)}{x-4} = \lim_{x\to 4} x+10 = 14$$

9.) $\lim_{x\to 4} \frac{x^3-64}{x-4} = \frac{0}{0} = ???$

$$\lim_{x\to 4} \frac{(x-4)(x^2+4x+16)}{x-4} =$$

$$\lim_{x\to 4} (x^2+4x+16) = 48$$

III. You Try!

1.) $\lim_{x\to -2} \frac{2x^3+16}{x+2}$

2.) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

3.) $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$

4.) $\lim_{x\to 1} \frac{x^2-x-2}{x^2-1}$

1.) $\lim_{x\to -2} \frac{2x^3+16}{x+2} = \lim_{x\to -2} \frac{2\left(x^3+8\right)}{x+2} =$

$\lim_{x\to -2} \frac{2(x+2)\left(x^2-2x+4\right)}{x+2} =$

$\lim_{x\to -2} 2\left(x^2-2x+4\right) =$

$2\left((-2)^2-2(-2)+4\right) =$

$= 24$

2.) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)} =$

$\lim_{x\to 1} \frac{1}{\sqrt{x}+1} =$

$\frac{1}{\sqrt{1}+1} = \quad = \frac{1}{2}$

$$3.)\ \lim_{x\to0}\frac{\sqrt{x+9}-3}{x}=\lim_{x\to0}\frac{\left(\sqrt{x+9}-3\right)\left(\sqrt{x+9}+3\right)}{x\left(\sqrt{x+9}+3\right)}=$$

$$\lim_{x\to0}\frac{x+9-9}{x\left(\sqrt{x+9}+3\right)}=\lim_{x\to0}\frac{x}{x\left(\sqrt{x+9}+3\right)}=$$

$$\lim_{x\to0}\frac{1}{\left(\sqrt{x+9}+3\right)}=$$

$$\frac{1}{\left(\sqrt{0+9}+3\right)}=\frac{1}{\left(\sqrt{9}+3\right)}\qquad=\frac{1}{6}$$

$$4.)\ \lim_{x\to1}\frac{x^2-x-2}{x^2-1}=\lim_{x\to1}\frac{(x-2)(x+1)}{(x-1)(x+1)}=$$

$$\lim_{x\to1}\frac{(x-2)}{(x-1)}=\frac{1-2}{1-1}=\frac{-1}{0}\qquad\lim_{x\to1^-}\frac{(x-2)}{(x-1)}=\frac{-1}{0^-}=\infty$$

$$\lim_{x\to1^+}\frac{(x-2)}{(x-1)}=\frac{-1}{0^+}=-\infty$$

$$\therefore\lim_{x\to1}\frac{x^2-x-2}{x^2-1}\ \text{D.N.E.}$$