Limits

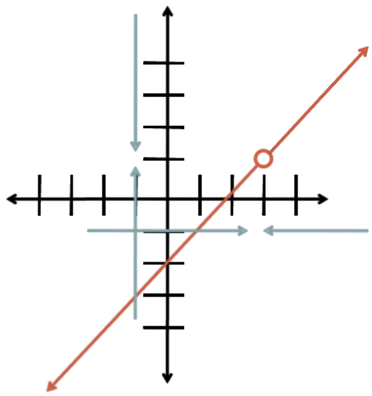
DAY 1

I. Analytical Look at Limits

A.) Given the following function, what happens to $f(x)$ as x gets closer to 3?

$$f(x) = \frac{x^2 - 5x + 6}{x - 3}$$

Solution - GRAPH – ZOOM 4 – or TABLE



$x \to 3^-$	$f(x)$	$x \to 3^+$	$f(x)$
2	0	4	2
2.5	0.5	3.5	1.5
2.9	0.9	3.1	1.1
2.95	0.95	3.05	1.05
2.99	0.99	3.01	1.01
2.999	0.999	3.001	1.001
2.9999	0.9999	3.0001	1.0001

as $x \to 3$, $f(x) \to 1$

B.) Given the following function, what happens to $g(x)$ as x gets closer to 3?

$$g(x) = x - 2$$

as $x \to 3$, $g(x) \to 1$

C.) Given the following function, what happens to $h(x)$ as x gets closer to 3?

$$h(x)=\begin{cases} \dfrac{x^2-5x+6}{x-3}, & x\neq 3 \\ 7, & x=3 \end{cases}$$

as $x \to 3$, $h(x) \to 1$

II. Limit Notation

A.) Two-sided Notation:

$$\lim_{x\to a} f(x)=L$$

Read as "the limit of $f(x)$ as x approaches a is L."

B.) One-sided Notation:

$$\lim_{x \to a^+} f(x) = L$$

Read as "the limit of $f(x)$ as x approaches a from the right is L."

$$\lim_{x \to a^-} f(x) = L$$

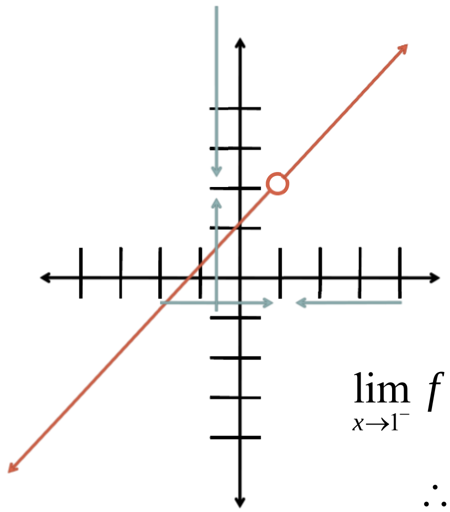
Read as "the limit of $f(x)$ as x approaches a from the left is L."

III. Limit Definition

A.) Def: The function f has a **limit** L as x approaches c iff:

$$\lim_{x \to c} f(x) = L \Leftrightarrow \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

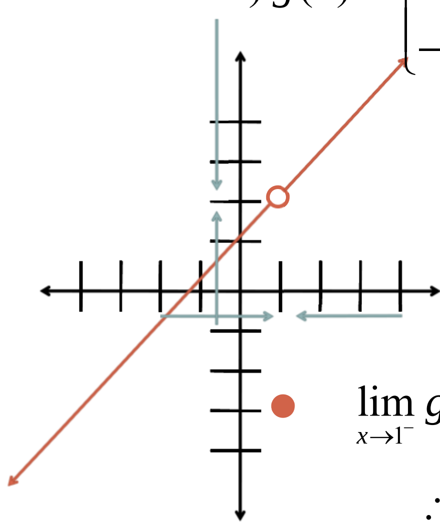
B.) Visual Representations: 1.) $f(x) = \dfrac{x^2 - 1}{x - 1}$ at $x = 1$



$$\lim_{x \to 1^-} f(x) = 2 = \lim_{x \to 1^+} f(x)$$

$$\therefore \lim_{x \to 1} f(x) = 2$$

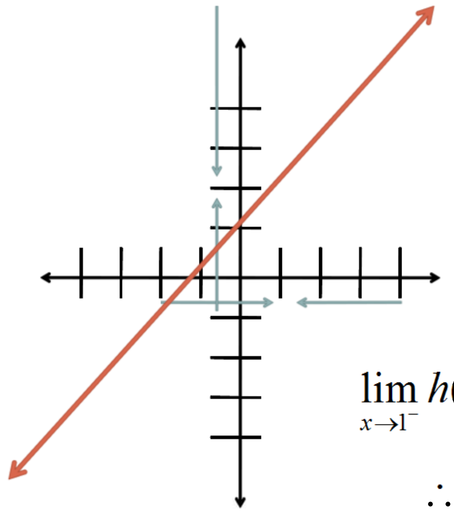
2.) $g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ -3, & x = 1 \end{cases}$ at $x = 1$



$$\lim_{x \to 1^-} g(x) = 2 = \lim_{x \to 1^+} g(x)$$

$$\therefore \lim_{x \to 1} g(x) = 2$$

3.) $h(x) = x + 1$ at $x = 1$



$$\lim_{x \to 1^-} h(x) = 2 = \lim_{x \to 1^+} h(x)$$

$$\therefore \lim_{x \to 1} h(x) = 2$$

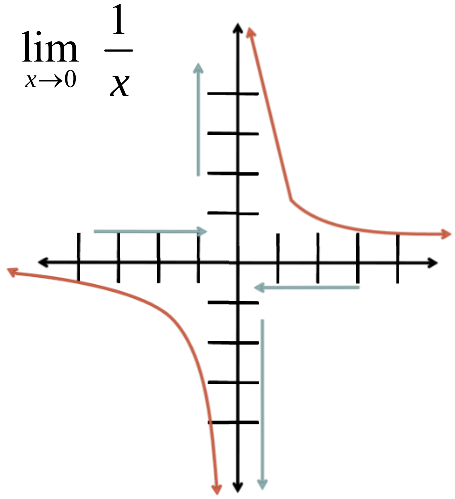
IV. Non-existent Limits

A.) $\lim_{x \to a} f(x)$ fails to exist when:

1) The right-side limit and left-side limit equal different real numbers.
2) The are infinite oscillations .
3) The limit(s) approach $\pm\infty$

B.) Ex. – Evaluate $\lim_{x \to 0} \frac{1}{x}$.

$\lim_{x \to 0} \frac{1}{x}$



$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

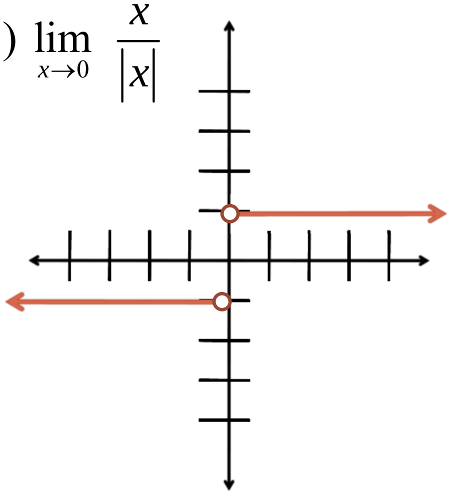
$\therefore \lim_{x \to 0} \frac{1}{x}$ Does Not Exist.

Although limits approaching infinity do not exist, we must still describe the behavior from both/each side(s)!!!

C.) Examples - Graphically evaluate the following limits and determine whether or not they exist.

1.) $\lim_{x\to 0} \frac{x}{|x|}$ 2.) $\lim_{x\to 0} \lfloor x \rfloor$ 3.) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$

1.) $\lim_{x\to 0} \frac{x}{|x|}$

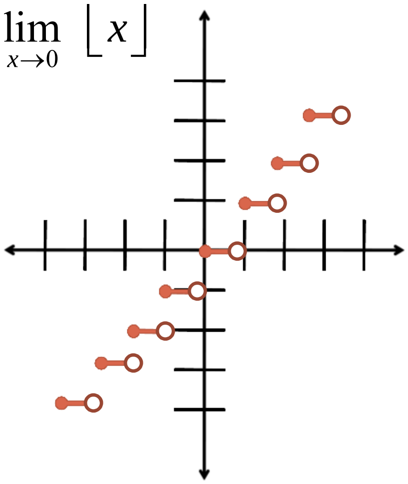


$$\lim_{x\to 0^-} \frac{x}{|x|} = -1$$

$$\lim_{x\to 0^+} \frac{x}{|x|} = 1$$

$\therefore \lim_{x\to 0} \frac{x}{|x|}$ Does Not Exist.

2.) $\lim_{x \to 0} \lfloor x \rfloor$



$$\lim_{x \to 0^-} \lfloor x \rfloor = -1$$

$$\lim_{x \to 0^+} \lfloor x \rfloor = 0$$

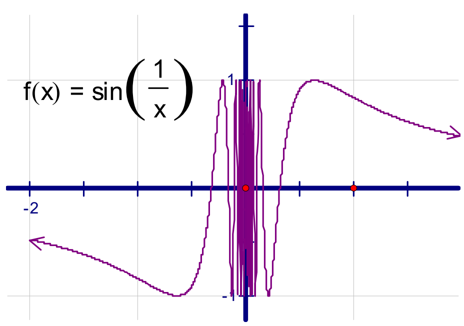
$$\therefore \lim_{x \to 0} \lfloor x \rfloor \text{ Does Not Exist.}$$

3.) $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

Lets use our calculator

$$\lim_{x \to 0^-} \sin\left(\frac{1}{x}\right) = ?$$

$$\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right) = ?$$



$$\therefore \lim_{x \to 0} \sin\left(\frac{1}{x}\right) \text{ Does Not Exist - Infinite Oscillations}$$