Complete p. 225 #1-15 odd

1. $3\sqrt{2}$
3. $-4\sqrt{2}$
5. $\frac{-\sqrt{91}}{13}$
7. $-10\sqrt{2}$
9. 108
11. $|xy|$
13. $\frac{-|x|\sqrt{35}}{5}$
15. $\frac{5\sqrt{14}}{7}$

4-8: Complex Numbers

Mr. Gallo
Algebra 2

Solve $x^2 + 100 = 0$

$$x^2 + 100 = 0$$

$$x^2 = -100$$

$$\sqrt{x^2} = \sqrt{-100} = \sqrt{100 \times -1}$$

$$x = \pm 10\sqrt{-1}$$

When k > 0 the two solutions to $x^2 = -a$ are denoted as: $\sqrt{-a}$ and $-\sqrt{-a}$

New Definition: $i = \sqrt{-1}$. Therefore,

$$x = \pm 10\sqrt{-1} = \pm 10i$$

Complete Got it? #1 p.249 a. $2i\sqrt{3}$ b. $5i$ c. $i\sqrt{7}$ d. $8i \neq -8$

Definition of an imaginary unit: $i = \sqrt{-1}$

If $a > 0$:

$$\sqrt{-a} = \sqrt{-1 \cdot a} = \sqrt{-1}\sqrt{a} = i\sqrt{a}$$

$$i = \sqrt{-1}$$
$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$
$$i^3 = i^2 \times i = -1\sqrt{-1} = -\sqrt{-1}$$
$$i^4 = i^2 \times i^2 = -1(-1) = 1$$

This is a common pattern which can be used to find any power of $\boldsymbol{i}$.

Calculate: a. i^{11} b. i^{24} c. i^{74}

$-\sqrt{-1}$ 1 -1

Simplify each expression:

1. $\sqrt{-7} = \sqrt{-1} \times \sqrt{7} = i\sqrt{7}$

2. $\sqrt{-121} = \sqrt{-1} \times \sqrt{121} = i\sqrt{121} = 11i$

3. $-\sqrt{-81} = -\sqrt{-1} \times \sqrt{81} = -i\sqrt{81} = -9i$

4. $-\sqrt{-96} = -\sqrt{-1} \times \sqrt{96} = -i\sqrt{16} \times \sqrt{6} = -4i\sqrt{6}$

You MUST take care of i before taking the square root.

The properties of Real numbers hold for Imaginary numbers

- Commutative Properties
- Associative Properties
- Distributive Properties

Adding & Subtracting Imaginary Numbers

1. $3i + 4i \ = 7i$

2. $10i - 4i \ = 6i$

3. $-\sqrt{36} + \sqrt{-36} \ = -6 + i\sqrt{36} \ = -6 + 6i$

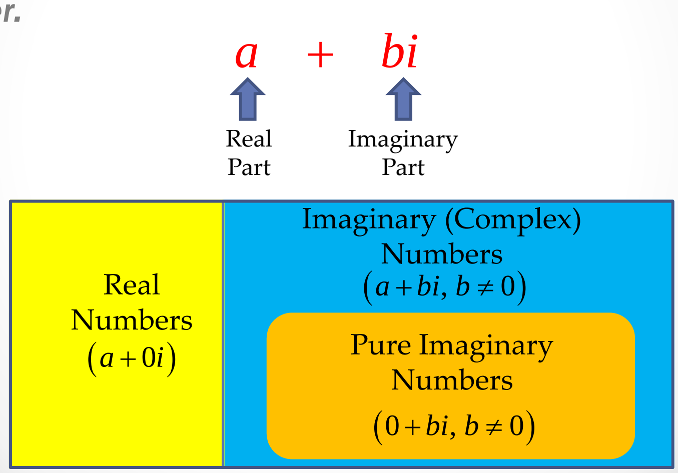
4. $\sqrt{-6} \bullet \sqrt{-3} = i\sqrt{6} \times i\sqrt{3} = i^2\sqrt{18} = -1\sqrt{9 \times 2} = -3\sqrt{2}$

5. $\sqrt{-25} \bullet \sqrt{-16} = 5i \times 4i = 20i^2 = -20$

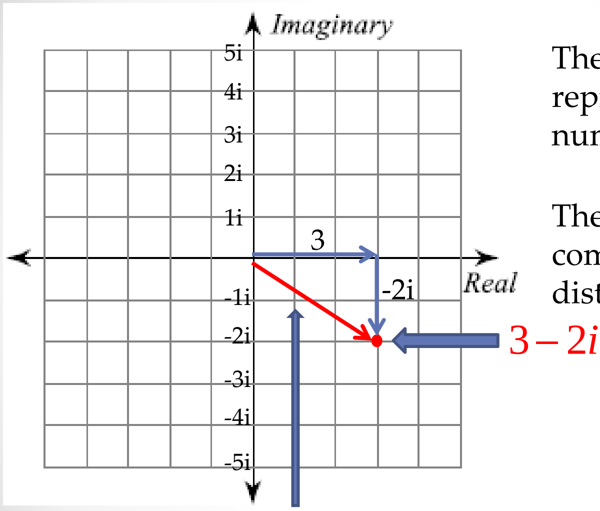
6. $i\left(\sqrt{25} + \sqrt{-49}\right) = i(5 + 7i) = 5i + 7i^2 = 5i - 7 = -7 + 5i$

Complex Numbers

- Written in the form $a + bi$ where a and b are real numbers.
- If $b = 0$, the number $a + bi$ is a real number.
- If $a = 0$ and $b \neq 0$, the number $a + bi$ is a ***pure imaginary number.***



Complex Number Plane



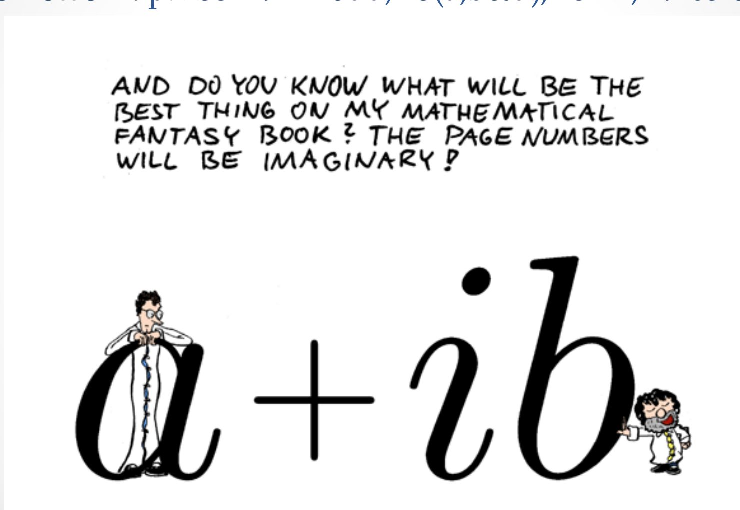
The point $(\boldsymbol{a}, \boldsymbol{b})$ represents the complex number $\boldsymbol{a} + \boldsymbol{bi}$.

The absolute value of a complex number is its distance from the origin.

$$|3-2i| = \sqrt{3^2+2^2} = \sqrt{13}$$

- Complete Got it? #2 p.250 a. $\sqrt{26}$ b. $\sqrt{13}$ c. $\sqrt{17}$

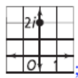
Homework: p. 253 # 9-17 odd, 45(a,b&d), 73-77, 79-85 odd



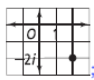
p.253 #9-17 odd, 45(a,b&d), 73-77, 79-85 odd

9. $i\sqrt{7}$

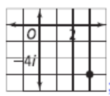
11. $9i$

13. 

; 2

15. 

; $2\sqrt{2}$

17. 

; $3\sqrt{5}$

45a. A: -5; B: $3+2i$; C: $2-i$; D: $3i$; E: $-6-4i$; F: $-1+5i$

45b. $A: 5$; $B: -3-2i$; $C: -2+i$; $D: -3i$; $E: 6+4i$; $F: 1-5i$

45d. A: 5; B: $\sqrt{13}$; C: $\sqrt{5}$; D: 3; E: $2\sqrt{13}$; F: $\sqrt{26}$

73. B

74. G

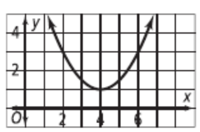
75. A

76. $\pm 2,\ \pm 2i$;

$$x^4 - 16 = (x^2-4)(x^2+4)$$
$$= (x-2)(x+2)(x^2+4)$$
$$x-2=0,\ x+2=0,\ x^2+4=0$$
$$x=2 \qquad x=-2 \qquad x=\pm 2i$$

77. $\dfrac{-3\pm\sqrt{41}}{4}$

79. $\dfrac{-7\pm\sqrt{17}}{2}$

81. 

; axis of symmetry; $x = 4$

83. $y = 3x - 4$

85. $y = -7x + 10$

All properties for real numbers except those for inequalities also hold for the set of complex numbers.

Addition of Complex Numbers:

Example 1) Add and simplify:

a) $(6-5i)+(3+4i) = 6-5i+3+4i = 9-i$

b) $(2+i)-(7-2i) = 2+i-7+2i = -5+3i$

Complete Got it? #3 p.250 a. $4-i$ b. $-2+7i$ c. $12i$ d. $18i$

Distributive Property & Multiplication

Example 2) Simplify:

a) $3i(12-3i) = 36i-9i^2 = 36i-9(-1) = 9+36i$

b) $(5+9i)(2-7i)$

$10-35i+18i-63i^2 = 10-17i-63(-1) = 73-17i$

c) $(1+i)(1-i) = 1-i+i-i^2 = 1-(-1) = 1+1 = 2$

Example 2c is an illustration of a **complex conjugate**.

Complete Got it? #4 p.251 a. -21 b. $23-2i$ c. 41

Complex Conjugate

- In general they are of the form $(a+bi)$ paired with $(a-bi)$.
- Complex conjugates are useful when dividing complex numbers.

$$(a+bi)(a-bi) = a^2 - abi + abi - (bi)^2$$
$$= a^2 - (-1)b^2 = a^2 + b^2$$

- To divide two complex numbers multiply the numerator and denominator by the **conjugate** of the *denominator*.

Example: Write $\frac{3-4i}{2+5i}$ in $a + bi$ form.

$$\frac{3-4i}{2+5i} \times \frac{2-5i}{2-5i} = \frac{6-15i-8i+20i^2}{4-10i+10i-25i^2}$$
$$= \frac{6-23i+20(-1)}{4-25(-1)}$$
$$= \frac{6-23i-20}{4+25}$$
$$= \frac{-14-23i}{29} = \frac{-14}{29} - \frac{23}{29}i$$

- Complete Got it? #5 p.251 a. $\frac{7}{25} - \frac{26}{25}i$ b. $-\frac{1}{6} - \frac{2}{3}i$ c. $\frac{15}{113} - \frac{112}{113}i$

Factoring Using Complex Conjugates

Solve $3x^2 + 12$ by using factoring:

$$3x^2 + 12 = 0$$

$$3(x^2 + 4) = 0$$ ← Factor out the GCF

$$3(x + 2i)(x - 2i) = 0$$ ← Use $a^2 + b^2 = (a + bi)(a - bi)$ to factor.

$$x + 2i = 0 \qquad x - 2i = 0$$

$$x = -2i \qquad x = 2i$$

← Use the Zero Product Property to solve.

Complete Got it? #6 p.252 a. $5(x + 2i)(x - 2i)$

b. $(x + 9i)(x - 9i)$

Using the Quadratic Formula

Solve $-x^2 + 4x - 5 = 0$ by using the quadratic formula:

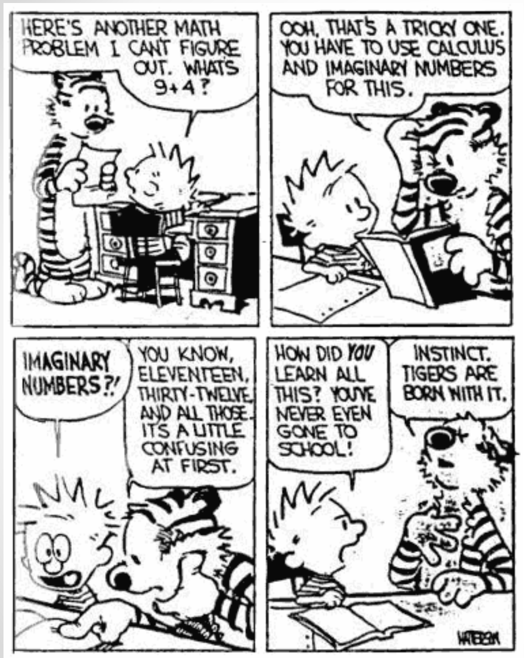
$$-x^2 + 4x - 5 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 20}}{-2} = \frac{-4 \pm \sqrt{-4}}{-2}$$

$$x = \frac{-4 \pm 2i}{-2} = 2 \pm i$$

Complete Got it? #7 p.252 a. $\frac{1}{6} \pm \frac{i\sqrt{23}}{6}$ b. $2 \pm i$



Homework: p. 253 # 19-43 odd, 78-86 even